Jörg Budde*

DISTORTED PERFORMANCE MEASUREMENT AND RELATIONAL CONTRACTS**

ABSTRACT

I analyze the use of alternative performance measures using an agency model that incorporates both formal and informal agreements. I show that under the proper combination of verifiable and unverifiable performance measures, the two types of contract complement each other regardless of the principal's fallback position. To obtain this complementarity, the principal uses an opting-out clause that allows him to replace part of a piece rate by a predefined bonus. My analysis contrasts with earlier studies, and provides a rationale for the use of subjective information in strategic performance measurement systems.

JEL-Classification: D86, M41, M52.

Keywords: Congruity; Distortion; Opting-Out Clause.

1 INTRODUCTION

Since the 1990s, strategic performance measurement has become an increasingly popular device for facilitating decisions and providing incentives. Triggered by the insight that traditional financial measures may not properly represent a firm's current position in a changing environment, several concepts have been proposed to capture the long-term effects of managerial activities.

Most studies suggest the use of non-financial measures to cover these effects. For example, consider the balanced scorecard, a very popular strategic performance measurement system. This device supplements its financial measures with three perspectives: customer, internal, and learning and growth, which firm managers use to incorporate intangibles

* Jörg Budde, Department of Economics, University of Bonn, Adenauerallee 24-42, D-53113 Bonn, phone: +49-228-739247, e-mail: joerg.budde@uni-bonn.de.

** The author would like to thank Christian Riegler, two anonymous referees and participants of the GEABA symposium 2006 in Bielefeld for helpful comments. Financial support by the Deutsche Forschungsgemeinschaft through SFB/TR 15 is gratefully acknowledged.

such as customer satisfaction and employee loyalty (for details, see Kaplan and Norton (1992; 1996)).

Traditionally, the main objective of strategic performance measurement has been to identify, communicate, and implement a firm's goals. Long-time corporate practice has now led us to the understanding that a strategy cannot be enforced without tying employee compensation to respective measures (e.g., Kaplan and Norton (2001)). However, a deficiency of this approach is that non-financial information need not be verifiable by a third party, and thus may not be applicable to formal contracting. Consequently, the use of subjective rewards has been suggested as an alternative (Kaplan and Norton (1996)).

In this paper, I analyze the conditions under which subjective rewards can be credibly promised, and the role that congruity plays in the problem. To do so, I revisit the question of how a firm can optimally integrate subjective rewards into a formal contract. This question is of particular interest in situations in which the firm bases a formal contract on a purely financial performance index. Such an index may be distorted in the sense that it fails to capture all the value-relevant aspects of an agent's actions. To account for this problem, I consider a multitask agency setting in which the agent's impact on the principal's objective differs from his impact on the verifiable performance measure. The first-best allocation in this scenario cannot be induced by a formal contract alone. Subjective rewards may well bring about the desired action, but they are only credible if they are not too high compared to the potential benefits of cooperation.

Baker et al. (1994) analyze a similar question within the context of performance measure distortion due to private predecision information. These authors find that if a verifiable performance measure is sufficiently close to perfect, it can completely rule out subjective rewards. This finding is counterintuitive, but the same result emerges in the present setting when a formal contract and a subjective reward are used side by side. The contradiction is resolved by considering a different arrangement, one in which the relational contract is used in exchange for the formal contract. For that purpose, the firm introduces an opting-out clause into the formal contract, which allows the principal to satisfy (part of) his formal obligations by a predefined payment. By using this payment in the relational contract, the firm prevents explicit and implicit incentives from overlapping and keeps the relevant cost of the subjective reward to a minimum.

This simple modification greatly changes the results. Although it proves that subjective rewards will not always be credible, a more elaborate contract will be credible over a much wider range of rewards. Therefore, formal and relational contracts are complementary, regardless of the principal's fallback position, and the agency always benefits from a more congruent verifiable performance measure.

I derive a similar result in a companion paper (Budde (2007)), in which I study the use of measures from a balanced scorecard in formal and relational contracts. In addition to a formal analysis of the properties of balanced scorecards, I use the verifiable performance measures not only in the formal contract, but also in the relational contract. By doing so, explicit and implicit incentives can be aligned in a similar fashion as it is done by an opting-out clause.

However, the analysis in Budde (2007) relies heavily on the additional assumption that the principal and the agent can observe all measures without noise. Under the opting-out clause, noisy observation is no longer an issue. The situation with noiseless observation serves only as a benchmark for the analysis in this present paper. Its main contribution is to prove that a firm can achieve the same alignment in a more realistic setting with noisy observation, provided the principal makes use of the opting-out clause.

This result illuminates the interplay of formal and relational contracts by providing theoretical evidence that supports Klein's (2002) assertion that "self-enforcement and court enforcement are not alternative enforcement mechanisms, but are complementary instruments used by transactors in combination to guarantee transactor performance". For a wide range of parameter values, the agent's ex post compensation may be entirely based on the relational agreement. Nevertheless, the formal contract is an important ingredient of the arrangement, because it reduces the principal's temptation to break the relational contract. The model thus also supports Klein's (2002) argument that "the fundamental economic motivation for the use of court-enforceable contract terms is to supplement self-enforcement". The key feature of the agreement proposed in the present paper is that the best formal contract is applied not only after a potential defection, but also forms a cornerstone of the informal agreement, reinforcing the relational contract by minimizing the principal's propensity to default. The role of the formal contract is to control the principal's temptation to defect, not to provide direct incentives.

This contractual arrangement is a form of "on equilibrium path" renegotiation, as considered in papers by Hermalin and Katz (1991) and Pearce and Stacchetti (1998). In these models, the possibility of contract renegotiation shields the agent from risk. In the model I construct in the present paper, the firm uses renegotiation to facilitate relational contracting, which in turn encourages the agent to choose actions that cannot be implemented by a formal contract. By doing so, I extend the role of renegotiation to the coordination of the formal and informal agreements that concern credibility issues.

The paper is structured as follows. In Section 2 I describe the stage game of a one-period agency framework and analyze the optimal formal contract. In Section 3 I provide a multi-period extension of the initial agency model and examine the use of subjective rewards. Section 4 concludes. All proofs are in the Appendix.

2 The One-Period Framework

In this section I describe a one-period agency model with a distorted financial performance measurement. I assume that distortion of the measure arises from the multiplicity of tasks the agent has to perform. Formally, this scenario is similar to a setting with post-contractual, pre-decision information (see Holmström and Milgrom (1991)), such as that used by Baker et al. (1994) in a related paper. Using a multitask agency allows me to refer more clearly to the index of congruity of the performance measure, as proposed by Baker (2000; 2002). By slightly modifying this index, the impact of congruity on the interplay between formal and relational contracts will become particularly apparent in the multiperiod model. A multitask setting also better reflects the situations for which strategic performance measurement was designed, situations in which managers perform many tasks of various relevance to the firm's short-term and long-term success.

As a starting point, I consider the situation in which a principal hires an agent to work once on his behalf. The agent's activity $\mathbf{a} \in \Re^n$ has multiple aspects and cannot be legally enforced. The principal seeks to maximize his value V from the agent's action, net of wage payments. I assume that $V(\mathbf{a}) = \mathbf{d}'\mathbf{a}$ is a linear function of the agent's action $\mathbf{a}$, in which $\mathbf{d} = (d_1, \ldots, d_n)'$ are the marginal products of the various activities. I interpret V as the agent's actual contribution to firm value, including both short- and long-term effects. With regard to the firm's long-term financial success, V might be the excess value created by the agent (O'Hanlon and Peasnell (2002)). To analyze the problem of performance measurement distortion in formal contracts, I assume that V is observable to both parties but unverifiable by an outside party. Thus, no formal contract can be based on V.

Since the firm cannot contract on either $\mathbf{a}$ or the principal's objective V, to motivate the agent the principal must rely on some performance measure $P(\mathbf{a}) = \mathbf{y}'\mathbf{a} + \epsilon$. I denote $y_i \in \Re$ as the performance measure's *sensitivity*[1] with respect to action a_i, and $\epsilon \sim N(0, \sigma^2)$ is the normally distributed uncertainty of P. In strategic performance measures, P might represent an aggregate of one or more short-term financial performance measures. But it does not capture all the relevant aspects of a firm's performance, thus, $\mathbf{y} \neq \mathbf{d}$.

The principal offers a linear incentive contract $S = s_b + s_p P$ in which s_b is the base salary and s_p is a share parameter that defines the performance-related payment. By choosing $\mathbf{a}$, the agent incurs a private cost $C(\mathbf{a})$. Similar to most studies on the linear agency framework, I assume that C takes the form $C(\mathbf{a}) = \frac{1}{2}\mathbf{a}'\mathbf{a}$ (Itoh (1991); Feltham and Xie (1994); or Baker (2000)).To isolate this analysis from risk-sharing issues, I assume that both principal and agent are risk neutral. The agent's utility from his compensation S and action choice $\mathbf{a}$ is given by $U^A(S, \mathbf{a}) = S - C(\mathbf{a})$. His outside options are accounted for by a reservation utility $U^R \geq 0$.

The principal's contracting problem in this model is a special case of that analyzed by Feltham and Xie (1994), who consider a risk-averse agent and multiple performance measures. Consequently, I derive the optimal contract from their analysis by setting the degree of risk aversion to zero. By choosing s_p, the principal maximizes the expected total surplus $\Pi = V(\mathbf{a}) - C(\mathbf{a})$, subject to the incentive compatibility constraint $\mathbf{a} = s_p\mathbf{y}$. (I note that a complete characterization of the contracting problem would require me to refer to S instead of $C(a)$. However, since the agent is risk-neutral, the binding participation constraint yields $E[S] - C(\mathbf{a}) = U^R$ or $E[S] = U^R - C(\mathbf{a})$, from which the principal's net profit becomes $E[V - S] = E[V] - C(\mathbf{a}) - U^R$. Thus, $EU^P = \Pi - U^R$.) The principal chooses the base salary s_b to ensure that the agent's reservation utility is obtained.

1 See Banker and Datar (1989) for a definition.

The optimal share parameter is[2]

$$s_p^0 = \frac{\mathbf{d}'\mathbf{y}}{\mathbf{y}'\mathbf{y}}, \tag{1}$$

from which the agency's net surplus becomes

$$\Pi^0 = \Pi(s_p^0) = \frac{1}{2}\frac{(\mathbf{d}'\mathbf{y})^2}{\mathbf{y}'\mathbf{y}} \,. \tag{2}$$

The total profit equals $(\mathbf{y}^0)'\mathbf{y}^0/2$, where $\mathbf{y}^0 = s_p^0\mathbf{y}$ is the sensitivity of the scaled performance measure $P^0 = s_p^0 P$. Among all implementable actions $s_p\mathbf{y}$, $\mathbf{a}^0 = \mathbf{y}^0$ describes the one that is "closest" to the first-best action $\mathbf{a}^{FB} = \mathbf{d}$. The alignment between $\mathbf{y}$ and $\mathbf{d}$ is referred to as the *congruity* of the performance measure P.

A number of metrics for congruity have been suggested (Feltham and Xie (1994); Feltham and Wu (2000); Datar et al. (2001)). On inspection of (2), I believe it is most promising to refer to Baker (2000; 2002) and adopt the cosine of the angle between $\mathbf{d}$ and $\mathbf{y}$. Since the cosine can be written as

$$\cos(\mathbf{d},\mathbf{y}) = \frac{\mathbf{d}'\mathbf{y}}{\sqrt{(\mathbf{d}'\mathbf{d})}\,\sqrt{(\mathbf{y}'\mathbf{y})}},$$

this measure of congruity naturally relates the total surplus (2) to the first-best total surplus given by $\Pi^{FB} = \frac{1}{2}\mathbf{d}'\mathbf{d}$. This relation is stated in the following lemma:

Lemma 1 *The total surplus in a risk-neutral linear agency model with quadratic effort cost is given by*

$$\Pi^0 = (\cos(\mathbf{d},\mathbf{y}))^2\,\Pi^{FB}.$$

I denote the congruity ϕ of performance measure P with respect to the firm's objective V as $\phi(\mathbf{d},\mathbf{y}) = (\cos(\mathbf{d},\mathbf{y}))^2$. Squaring scales the cosine measure to the unit interval, and the second-best total profit $\Pi(s^0) = \phi\Pi^{FB}$ is a linear function of this measure.

3 Relational Contracts

3.1 Basic Idea

As long as the performance measures of a formal contract are not perfectly congruent with the firm's objectives, the question arises whether non-verifiable information can be used to

2 See Feltham and Xie (1994, 433).

improve the contract. Relational contracts are one of the instruments that can accomplish this improvement. These "informal agreements and unwritten codes of conduct" (Baker et al. (2002)) are not meant to be enforced by law, because they are based on outcomes that can only be observed ex post by the contracting parties. Relational contracts have substance only if it is in both parties' best interests to honor the agreement. In a one-time working relationship, this will never be the case, since the principal will always benefit from refusing a voluntary payment. Therefore, relational contracts are best analyzed in a multiperiod framework in which the parties repeatedly agree upon a contract. I use this approach to analyze how subjective rewards can be utilized to improve a formal contract.

This question has already been examined in studies such as Baker et al. (1994), who analyze how formal and relational contracts can be combined in an infinitely repeated agency relationship. These authors assume that any subjective assessment of firm value is complemented by a distorted verifiable performance measure, whose sensitivities are privately observed by the agent after contracting. Two payments are determined ex ante: a piece rate from the formal contract based on the verifiable measure, and a bonus from the relational contract based on a nonverifiable, subjective evaluation. Baker et al. show that the existence of a verifiable performance measure that is sufficiently close to perfect may rule out any relational agreement. The reason they give for this negative result is convincing: since the bonus is not enforceable, it must be in the principal's best interest to pay. This case applies only if the amount the principal could save by refusing the bonus is less than his future benefit from a continuing relational contract[3]. Such benefits depend on the principal's profit after a defection. If the fallback position is a formal contract – and I note that this assumption is critical to their negative result; to obtain comparable results, I too use this assumption in most of the following analysis – then the profit depends critically on the distortion of the verifiable measure. The less distorted the measure, the less the principal benefits from maintaining a relational agreement. But if the principal's fallback position is to cease production, Baker et al. (1994) find that formal and relational contract work as complements.

However, this focus on the principal's fallback position stresses only one side of the coin, because a less distorted verifiable performance measure may also reduce the payable bonus. Incentives already provided by a formal contract need not be sustained by the relational agreement. For example, if the verifiable performance measure is nearly perfect, then the subjective evaluation requires little fine-tuning and the payable bonus is likely to be small. To reduce the bonus to a minimum, a proper coordination of the two contracts is essential: all available information must be considered for the relational contract. The agreement considered by Baker et al. (1994) is rather coarse in this respect. The bonus is based exclusively on the realization of firm value, although the verifiable measure P could also be taken into account. A straightforward way to incorporate P into a relational contract

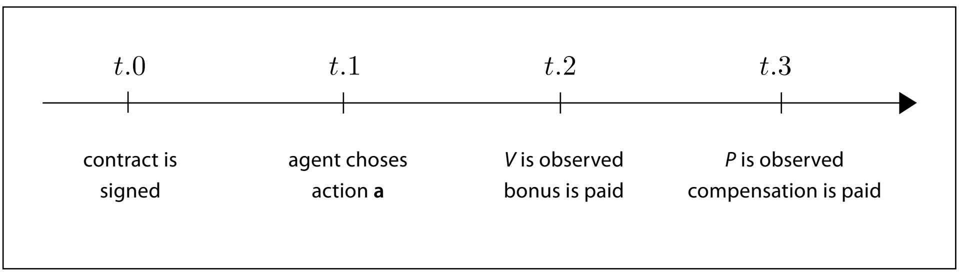
3 Baker et al. (1994) assume that both parties apply Grim-trigger strategies in the repeated game. In broad terms, this strategy can be described as follows: both parties use a cooperative strategy (work hard and pay the bonus) as long as no party defects, and procede in noncooperative mode as soon as one party breaks the agreement. Although this equilibrium is not a unique to the dynamic game, the assumption is without loss of generality for the obtainable payoffs (see Abreu (1988)).

would be to offer a bonus for reaching a certain target value of an aggregate measure derived from V and P. This approach is similar to that taken in Budde (2007), in which I consider a system $\mathbf{P} = (P_1, \ldots, P_m)$ of partly unverifiable measures. It emerges that if $\mathbf{P}$ is capable of mimicking the principal's objective V, then the optimal relational contract is built on an aggregate measure $V - P^0$[4]. The second-best contract s^0 is then offered as a formal contract. In this way, subjective rewards provide exactly the right incentive for making up the effort gap $\mathbf{d} - \mathbf{y}^0$ that is not provided in the formal contract. Furthermore, the relational bonus is kept to a minimum.

However, this result builds on the assumption that $\mathbf{P}$ can be assessed subjectively without noise. Only then can the agent be sure to receive the bonus if he acts according to the agreement. Under noisy performance measurement, the bonus is uncertain, and its required amount depends on both congruity and precision. Thus, although the same logic should apply, the results just mentioned cannot be directly transferred to the present setting.

But the same coordination can be achieved by a different agreement, one that does not require a precise assessment of the agent's impact on P. The key idea is to combine subjective rewards not with the realized amount of the agent's piece rate $s_P P$, but with its expected amount. This procedure is possible if the verifiable measure has not yet been realized when the bonus is paid. The principal will then trade off the bonus against the expected payment from the formal contract, which is not subject to exogenous factors. To allow for this consideration, I assume that the following timeline of events applies in each period t of the dynamic game:

Figure 1: Timeline of events



This time line is quite realistic. Financial measures such as accounting income are usually issued late because accounts are not settled until the end of the accounting period. In contrast, the subjective evaluation is not subject to such terms and may become available as soon as the agent has completed his tasks. Given this progression, the following agreement can reproduce the effects of a bonus based on $V - P^0$:

4 Although V and P may both be considered measures of financial success, one might wonder about the validity of an aggregate measure $V - P^0$ built from two vectors of different dimension. As can be seen from the share parameter (1), however, the formal contract already transforms P into a payment with the dimension of V.

1. On date $t.0$, the two parties sign the formal contract $(s_b,\ s_p^0)$. This contract is augmented by a clause stating that the principal may fulfil a predefined fraction of his obligations by paying an amount B by date $t.2$.

2. In addition to this formal contract, the parties informally agree that the principal should make use of this clause if V meets a certain target $\underline{V}$.

3. Defection is understood as

   (a) the agent not delivering $\underline{V}$ or

   (b) the principal not paying B although $\underline{V}$ has been achieved.

By stipulating an exchange of payment between the formal and relational contracts, the mandatory payment of the formal contract, which is legally due even if the principal decides to defect, can be held at a high level. This obligation enhances the credibility of the relational contract. For example, if the verifiable measure is almost perfectly congruent, then there will be almost no difference between the mandatory payment and the voluntary payment. Consequently, the principal bears very little incremental cost if he pays the bonus instead. Since only these incremental costs have to be traded off against the benefits of an ongoing relational contract, the latter is more likely to be credible[5].

The following analysis studies the benefits of such an agreement. As a first step, I analyze the benchmark case of a relational contract which is exclusively based on V. The main findings of the literature are corroborated in this example, thereby providing an intuitive explanation in terms of the congruity index defined in section 2. I then turn to the more elaborate multistage agreement, first considering the hypothetical case of a bonus based on $V - P^0$. This contract is quite similar to that studied in Budde (2007) and serves as a benchmark for the subsequent analysis in which I prove that the same result can be achieved with an opting-out clause in the formal contract. By comparing the results, it emerges that relational contracts are credible for a much wider range of parameters when an exchange of payments is applied.

3.2 A Bonus Offered in Addition to a Piece Rate

To study the effects of a relational contract, in the following analysis I assume that the stage game described in Section 2 is repeated ad infinitum. In this subsection, I assume that the principal takes into account unverifiable information by offering a hybrid contract of the form

$$S = s_b + s_p P + s_v V, \tag{3}$$

5 A similar approach is taken by Pearce and Stacchetti (1998), who consider the renegotiation of a formal contract in the stage game. In their model, however, the instrument is only used for risk sharing and consumption smoothing. The present model focuses on congruity aspects.

which includes the payments s_b and s_pP from a formal agreement and s_vV from a relational contract.

The agent's expected utility from this contract is $\mathrm{E}U(S, a) = s_b + s_p\mathbf{y}'\mathbf{a} + s_v\mathbf{d}'\mathbf{a} - C(\mathbf{a})$. If the agent trusts in the principal's promise to pay s_vV, then his effort will be

$$\mathbf{a}(s_p, s_v) = s_p\mathbf{y} + s_v\mathbf{d}. \tag{4}$$

Under the assumption that P is not perfectly congruent, the first-best effort allocation can be obtained by this contract only for the purely relational contract $s_p = 0$.

More generally, the optimal share parameters s_p and s_v of the two contracts can be derived analytically. Given the agent's action choice (4), the principal maximizes

$$\begin{aligned} \mathrm{E}U^P(\mathbf{a}^h) &= V(\mathbf{a}^h) - C(\mathbf{a}^h) - U^R \\ &= \mathbf{d}'(s_p\mathbf{y} + s_v\mathbf{d}) - (s_p\mathbf{y} + s_v\mathbf{d})'(s_p\mathbf{y} + s_v\mathbf{d})/2 - U^R. \end{aligned} \tag{5}$$

The optimal value of s_p, given the bonus s_v, is

$$s_p(s_v) = (1 - s_v)\,\frac{\mathbf{d}'\mathbf{y}}{\mathbf{y}'\mathbf{y}} = (1 - s_v)s_p^0\,. \tag{6}$$

Compared to a purely formal contract, explicit incentives are reduced to a fraction $1 - s_v$ not covered by the relational contract (Baker et al. (1994, 1139)). Substituting s_p into (4), I see that the agent's action $\mathbf{a}^h = s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0$ is a convex combination of the first-best action and the second-best action. From this, the principal's objective (5) after rearrangement becomes

$$\begin{aligned} \mathrm{E}U^P(\mathbf{a}^h) &= \Pi^0 + s_v(2 - s_v)\,[\Pi^{FB} - \Pi^0] - U^R \\ &= \Pi^{FB}[\phi + s_v(2 - s_v)\,(1 - \phi) - \hat{\phi}], \end{aligned} \tag{7}$$

where $\hat{\phi} = U^R/\Pi^{FB}$ is the minimal level of congruity for which a pure formal contract is valuable. For the derivation of (7), see Appendix B.

The principal's utility is maximized for $s_v = 1$. Hence, he will choose the highest share rate $s_v \in [0, 1]$ for which the subjective reward is credible. To meet this condition, the payable bonus

$$\begin{aligned} s_vV &= s_v\,\mathbf{d}'(s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0) = s_v(\mathbf{y}^0)'\mathbf{y}^0 + s_v^2(\mathbf{d}'\mathbf{d} - (\mathbf{y}^0)'\mathbf{y}^0) \\ &= 2s_v\,[\Pi^0 + s_v(\Pi^{FB} - \Pi^0)] = 2s_v\,\Pi^{FB}\,[\phi + s_v\,(1 - \phi)] \end{aligned} \tag{8}$$

must not exceed the net present value of the benefits generated by the hybrid contract. As mentioned above, these benefits depend on the principal's fallback position. The latter is either $\Pi^0 - U^R$ for a continued formal contract or zero for ceasing production, whichever is greater. Thus, the principal in general gains

$$\begin{aligned}\Delta U^P &= \mathrm{E}U^P(a^h) - \max\{\Pi^0 - U^R,0\}\\ &= s_v(2 - s_v)\,[\Pi^{FB} - \Pi^0] + \min\{\Pi^0 - U^R,0\}\\ &= \Pi^{FB}[s_v(2 - s_v)(1-\phi) + \min\{\phi - \hat{\phi},0\}] \qquad (9)\end{aligned}$$

from an ongoing relational contract in each period.

In calculating the optimal bonus parameter s_v I focus on the case of a valuable purely formal contract, which might cost the principal more when a congruent, verifiable performance measure is available. Then s_v can be derived from the credibility constraint

$$\sum_{t=1}^{\infty} \Delta U^P(1+r)^{-t} = \frac{\Delta U^P}{r} \geq s_v V,$$

which states that the net present value of the principal's periodic benefits has to cover the payable bonus. The optimal level of s_v is thus

$$s_v = \begin{cases} 1 & \text{for } \phi \leq 1 - 2r \\ 2\,\dfrac{1-(1+r)\phi}{(1-\phi)(1+2r)} & \text{for } 1 - 2r < \phi \leq 1/(1+r). \\ 0 & \text{for } \phi > 1/(1+r) \end{cases} \qquad (10)$$

For the derivation of (10), see Appendix B.

Inspection of (10) confirms the structure derived by Baker et al. (1994), namely that a purely relational contract is offered for $\phi < 1 - 2r$. Under this contract, the principal obtains the first-best solution. For $1 - 2r < \phi \leq 1/(1+r)$, both a formal and a relational contract are agreed upon with a decreasing proportion of subjective reward.

Most importantly, no relational contract is credible if $\phi > 1/(1+r)$. In this case the second-best solution is achieved under a purely formal contract. Of course, this negative result holds only if there is a valuable formal contract. If $\phi < \hat{\phi}$, then the principal will not offer a formal contract after a potential defection, and higher congruity cannot restrict the use of subjective rewards. It only reduces the payable bonus, and therefore is always beneficial to the principal. As Baker et al. (1994) show, a combination of formal and relational contracts may even be beneficial in situations in which none of the two contracts would be valuable on its own.

Beyond the type of contract applied, it is also of interest whether congruity diminishes the principal's utility. Such an impairment seems apparent, because the bonus rate in (10) decreases with ϕ for the interior solution. The conjecture can be proved by a formal analysis:

Proposition 1 *If a relational contract is offered in addition to a formal contract and the purely formal contract is valuable, then the principal's expected net utility* EU^P *is*

1. *constant in ϕ (and equal to its first-best value) for $\phi < 1 - 2r$,*
2. *decreasing in ϕ for $1 - 2r \leq \phi < 1/(1 + r)$, and*
3. *increasing in ϕ for $\phi > 1/(1 + r)$.*

3.3 A Bonus Offered in Exchange for a Piece Rate

The relational contract considered in section 3.2 does not make use of all available information. In general, P could be considered in determining the payable bonus.

To see clearly the effects of using all information, suppose for a moment that P can be observed without noise. In this case, a relational contract can be based on V and P in the same manner that I used V in the previous analysis. A payment $S = s_b + s_p P + s_v V + s_{pr} P$ is stipulated, resulting in an action $\mathbf{a}(s_p, s_v, s_{pr}) = (s_p + s_{pr})\mathbf{y} + s_v \mathbf{d}$. According to the analysis in section 3.2, the principal will choose s_p and s_{pr} such that $s_p + s_{pr} = (1 - s_v)s_p^0$, and the implemented action again will be a convex combination of $\mathbf{a}^{FB} = \mathbf{d}$ and $\mathbf{a}^0 = \mathbf{y}^0$. The principal can exploit the remaining degree of freedom in choosing s_p and s_{pr} to reduce the expected bonus $s_v V + s_{pr} P$ to a minimum. Lemma 2 gives the solution:

Lemma 2 *If P is subjectively assessed without noise, the optimal hybrid contract has the following properties:*

1. *The formal contract is identical to the purely formal contract s_p^0.*
2. *The relational contract is based on $V - P^0$.*

The key idea of the result, which is outlined in subsection 3.1, shows that the relational contract should build on the formal contract, rather than vice versa as (6) suggests. The principal's utility from this contract, given s_v, is identical to his utility in section 3.2. However, the required bonus may decrease dramatically. Its value is

$$s_v(\mathbf{d} - \mathbf{y}^0)'\mathbf{a} = 2s_v^2(\Pi^{FB} - \Pi^0) = 2s_v^2\Pi^{FB}\,(1 - \phi). \tag{11}$$

For the derivation of (11), see Appendix B.

The reason for this reduction is obvious: the bonus now refers only to the extra profit generated by the relational agreement. Since the action induced by the second-best purely formal contract is as close as possible to the first-best action, combining a relational agreement with this contract minimizes the bonus payment regardless of which action $s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0$ the principal implements. Compared to (8), the bonus is particularly reduced in situations in which the verifiable performance measure has high congruity. Consequently, a higher share rate s_v can be determined. Its optimal value is

$$s_v = \begin{cases} 1 & \text{for } r \leq \frac{1}{2} \\ \frac{2}{1+2r} & \text{for } r > \frac{1}{2}. \end{cases} \tag{12}$$

For the derivation of (12), see Appendix B.

The optimal bonus is independent of the congruity ϕ because the relational contract refers to only that part of the desired action that has not been covered by the formal contract. In this manner, the bonus is reduced by the same amount that the principal's fallback position is improved. With respect to the credibility of the voluntary payment, the two effects exactly balance.

This contract modification inverts the negative results of subsection 3.2. The first contrast can be seen in (12), in which there is now a credible relational contract for any values of the congruity ϕ and discount rate r. Of course, the applied bonus might be rather low. But since the marginal bonus required to induce a deviation from the second-best action $\mathbf{a}^0$ is zero, s_v will always be positive. The second contrast, which concerns the wealth effects of congruity, follows when I substitute (12) into the principal's profit (7). This transformation leads to the following lemma:

Lemma 3 *Suppose P is subjectively assessed without noise, and a purely formal contract is valuable. If both V and P are used in the relational contract, the principal's expected net utility is nondecreasing in the congruity ϕ of the verifiable performance measures.*

If the principal could observe P without noise, none of the previous negative results would remain valid under a contract that uses all available information. I use this finding as a reference point for the more realistic scenario in which P is a noisy measure of the agent's performance. In this case, a relational contract based on $V - P^0$ becomes critical; due to the distribution of ε, very low levels of P could occur. Under the performance measure $V - P^0$, this fact would entail an extremely high bonus. Any promised reward would be refused with some positive probability, and the relational agreement would collapse.

As outlined in subsection 3.1, the same effect can be achieved by basing the bonus on the *payment* $s_P P$ instead of directly referring to P. If this payment is done before the performance measure is realized, the principal will trade off the bonus against the expected payment from the formal contract. Unlike the realized amount P, for a given action this quantity is bounded.

The most obvious way to mimic the elaborate contract would be for the principal to offer a formal contract $s_b + s_p^0 P$ at date $t.0$, then compensate the agent on date $t.2$ for a fraction s_v of the variable payment $s_p^0 P$ by paying $s_v V$. Unfortunately, this agreement is not feasible: because V is not verifiable, the exchange cannot be substantiated in the formal contract. However, the formal agreement is crucial, because otherwise, to exchange the formal contract payment for a minimum bonus payment, the principal could claim a very low level of V. Therefore, the opting-out clause of the formal contract must determine a *fixed* amount B to be paid in exchange for a fraction s_v of the variable payment defined in the formal contract. Accordingly, the parties should (informally) agree upon a target $\underline{V}$ for which the bonus is due.

If they do so, implementation of a certain action $\mathbf{a}^h = s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0$ requires a target $\underline{V} = s_v\mathbf{d}'\mathbf{a}^h$ of V. The promised bonus must be high enough that the agent prefers $\mathbf{a}^h$ to $\mathbf{a}^0$, the action induced by the formal contract alone. To determine the required amount, compare the agent's utility under the required action $\mathbf{a}^h$,

$$\begin{aligned}\mathrm{E}U^A(\mathbf{a}^h) &= s_b + (1 - s_v)s_p^0\mathbf{y}'(s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0)\\ &\quad + B - (s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0)'(s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0)/2,\end{aligned}$$

to his expected utility under the optimal action in case that he waives the bonus,

$$\mathrm{E}U^A(\mathbf{a}^0) = s_b + (\mathbf{y}^0)'\mathbf{y}^0 - (\mathbf{y}^0)'\mathbf{y}^0/2.$$

By this comparison, the minimal bonus

$$\underline{B} \equiv s_v(\mathbf{y}^0)'\mathbf{y}^0 + \frac{s_v}{2}\left[\mathbf{d}'\mathbf{d} - (\mathbf{y}^0)'\mathbf{y}^0\right]$$

required to implement $\mathbf{a}^h$ can be determined. Although this term differs slightly from the amount (8) required without the opting-out clause, it appears to have the same undesirable properties for credibility: since the marginal bonus $\mathrm{d}\underline{B}/\mathrm{d}s_v$ required for divergence from the second-best action $\mathbf{a}^0$ is positive, there will always exist a discount rate $r < \infty$ for which no relational contract is credible. This fact holds regardless of the performance measure congruity ϕ. Thus, at first glance, there is no obvious improvement.

The only improvement lies in the fact that the bonus is now offered in exchange for part of the principal's formal obligations. From this procedure, it is not the absolute bonus that dominates the principal's considerations, but the incremental cost he incurs by paying the bonus. Under the opting-out clause, the principal must take into account both the bonus B and the expected savings $s_v(\mathbf{y}^0)'\mathbf{y}^0$ on formal obligations. Given these savings, the principal's extra cost from granting a bonus is

$$\underline{B} - s_v(\mathbf{y}^0)'\mathbf{y}^0 + \frac{s_v}{2}\left[\mathbf{d}'\mathbf{d} - (\mathbf{y}^0)'\mathbf{y}^0\right]. \tag{13}$$

This amount is half that due under a relational contract based on $V - P^0$. Therefore, the best relational contract is even more credible than in the previous scenario. The bonus rate is given by

$$s_v = \begin{cases} 1 & \text{for } r \leq 1 \\ \dfrac{2}{1 + r} & \text{for } r > 1. \end{cases} \tag{14}$$

For the derivation of (14), see Appendix B.
The main result of this paper can be derived from this equation.

Proposition 2 *If a relational contract is offered in exchange for part of a formal contract and the purely formal contract is valuable, then*

1. *a hybrid contract is credible for any discount rate r and any level ϕ of congruity, and*
2. *the principal's expected net utility increases with ϕ for any discount rate and any level of congruity.*

The principal benefits even more under an opting-out clause than he did under the contract based on $V - P^0$. This fact is due to the contract type required by the opting-out clause. While the contract based on $V - P^0$ is a linear function of the aggregate performance measure, a bonus-type contract is used under the opting-out clause. By these means the payment is halved, because only the cost of the additional effort $s_v(\mathbf{d} - \mathbf{y}^0)$ has to be compensated. In contrast, a linear contract generally pays twice the cost when the cost function is quadratic. Thus, the result of proposition 2 could also have been achieved by a bonus-type relational contract based on $V - P^0$.

However, halving the bonus (10) stipulated in the initial contract based on V would not change the essential results of subsection 3.2. This bonus not only covers the cost of the additional effort $s_v(\mathbf{d} - \mathbf{y}^0)$, but also has to actuate any effort $(1 - s_v)\mathbf{y}^0$ not driven by the (reduced) formal contract. Because of this dual role, the marginal bonus required for divergence from the action induced by the formal contract is still positive. Although the bonus is credible for a wider range of discount rates and congruity levels, there are still parameters r and ϕ for which there is no credible relational contract. In general, credibility can only be obtained by paying a bonus that matches the cost incurred by diverging from the optimal pure formal contract.

To complete my analysis of the hybrid contract with an opting-out clause, I examine the case $\phi < \hat{\phi}$ for which a purely formal contract is not valuable to the principal. In this case, his benefit (9) from an ongoing contract is $\Pi^{FB}[s_v(2 - s_v)(1 - \phi) - (\hat{\phi} - \phi)]$. Its value increases with the congruity ϕ. Since in addition to that the incremental cost (13) decreases with ϕ, higher levels of s_v can be stipulated for higher levels of congruity. The optimal share rate is

$$s_v = \begin{cases} \frac{1}{1+r} + \frac{\sqrt{(1-\phi)(1+r\phi-\hat{\phi}(1+r))}}{(1-\phi)(1+r)} & \text{for } \phi \geq \frac{(1+r)\hat{\phi}-1}{r} \\ 0 & \text{for } \phi < \frac{(1+r)\hat{\phi}-1}{r}. \end{cases} \tag{15}$$

For the derivation of (15), see Appendix B.

When the congruity exceeds a critical level, the hybrid contract becomes valuable. This contract must include a minimum level $\underline{s}_v = 1/(1 + r)$ of subjective rewards. The associated formal contract would actually not be beneficial on its own. For higher levels of congruity, the share rate increases further. As $\phi \to \hat{\phi}$, the share rate achieves its optimal level under a valuable formal contract.

In contrast to my previous findings, this result is well aligned with the second main finding of Baker et al. (1994), namely, that in the absence of a valuable formal contract, objective and subjective measures work as complements. If the verifiable performance measure becomes less distorted, then the bonus from the relational contract is increased.

Corollary 1 summarizes these effects with the findings of Proposition 2 and states the advantage of a more congruent verifiable performance measure as follows:

Corollary 1 *If the principal applies an opting-out clause to the hybrid contract, he always (weakly) benefits from a higher congruity of the verifiable performance measures.*

Regardless of his fallback position, the principal will always benefit from a less distorted performance measure. Thus, by considering the more elaborate contract, the paradox that arises from the parallel use of formal and relational contracts can be resolved.

3.4 Discussion

The opting-out clause suggested in this paper is closely related to renegotiation procedures considered in other studies. For example, Hermalin and Katz (1991) analyze a one-period agency in which the principal observes an unverifiable (perfect or imperfect) signal of the agent's action before the verifiable performance measure is realized. Based on this signal, the two parties renegotiate the initial contract. It emerges that if the principal has all the bargaining power (as in my model here) and observes the agent's action (a bit more than in my model), then the cost of implementing a certain action can be reduced to its first-best level. However, the set of actions that can be implemented is the same as that under the initial contract. It may even be reduced if the principal observes only a noisy signal of the agent's action.

Unlike my analysis in this paper, the main purpose of renegotiation in the model of Hermalin and Katz (1991) is to shield the agent from bearing the risk incorporated in the initial contract. Once the agent has chosen his action, there is no further need for incentive compensation. Contracting becomes merely a matter of efficient risk sharing, which, under the assumptions of a risk-neutral principal and risk-averse agent, is achieved by a fixed payment to the agent.

A similar exchange occurs in the model in this paper, but for a completely different reason. Here, the principal avoids a performance-based bonus payment to ensure that he keeps the agreement of the relational contract. Where a risk-averse agent in the one-shot model is protected against variations in his payment, in this paper's multiperiod model the principal is protected against his propensity to renege on his promise. This temptation arises in cases in which a high bonus payment is due.

Taking this rationale into account, the total benefits of the proposed contract can be divided into three effects. First, having a relational contract based on V enlarges the set of implementable actions. Second, consideration of P in this contract reduces the expected bonus payment. Third, the opting-out clause eliminates any uncertainty from the reward. Taken together, these effects bring about the closest alignment of interests that can credibly be supported by a relational contract.

A complete elimination of risk would not be possible if the subjective performance measure were also subject to noise. However, because the formal contract specifies a fixed bonus under the opting-out clause, the principal's propensity to break the agreement would not directly be affected, at least not if the noise terms of V and P are stochastically independent. (I note that if V and P were correlated, then a statistical inference could prompt the principal to refuse the bonus payment.) But a noisy observation of V would considerably change the agent's decision. When choosing his action, the agent would take into account the expected bonus payment. The required bonus – and thus its credibility – would critically depend on the precision of the performance measure (see Budde (2007)).

I do not examine risk-sharing issues in this present analysis. If the model were to consider a risk-averse agent, then consumption smoothing would become an issue in the multiperiod framework. Consumption smoothing is an issue in the model of Pearce and Stacchetti (1998), in which a formal contract is renegotiated in each period of an infinitely repeated game after the agent's action has been observed by the principal. However, unlike the present model, this unverifiable information cannot be observed until the verifiable outcome has been realized. Thus, the bonus payment serves as both a motivation device and as a means of consumption smoothing.

In the multitask model here, the introduction of risk aversion should also reinforce the main results of this paper. The principal's fallback position, a formal contract based on the noisy performance measure P, is impaired by risk aversion to a greater extent than a hybrid contract, which incorporates only part of the pure formal contract, and thus only part of the consequent risk premium. Thus, if a hybrid contract is credible under risk neutrality, it should also be credible under risk aversion.

From a technical viewpoint, the proposed opting-out clause may be interpreted as the inverse of a buyout agreement. Demski and Sappington (1991) consider a model in which the principal retains the right to sell his business to the agent at a predefined price. This option provides a strong incentive for the agent to choose the first-best effort. However, in equilibrium, the principal will never execute this option. It serves only as a threat point for the agent. Conversely, in the present setting, the principal holds an option to buy (a certain fraction of) the agent's title from the formal contract at a predefined price. Unlike the buyout agreement considered by Demski and Sappington (1991), this option will always be executed. In particular, according to (14), for interest rates up to 100% the formal contract will always be completely replaced by the bonus payment. Its role is mainly to determine the threat point for the agent, which in this case is a fallback to the formal contract.

Clearly, the procedure could be reversed; a fixed bonus might be exchanged for a variable payment in case the agent does not create the targeted value added $\underline{V}$. Although this procedure conforms analytically to the contract considered by Demski and Sappington

(1991), its interpretation as a buyout agreement is less obvious. The variable payment in my model does not coincide with firm value, so exercising the option cannot be understood as selling the firm to the agent.

On the contrary, the procedure proposed here may have a more natural interpretation. If P is the firm's accounting income, then the formal contract can be interpreted as the issuance of (short-dated) profit participation certificates to the agent. The corresponding relational contract would then consist of the agent's promise to deliver a certain target $\underline{V}$ of the principal's unverifiable objective, and the principal's agreement to buy a specified number of these certificates from the agent if the target is met.

Crucial for the use of such an agreement is that earnings have not been announced when the repurchase is due. Thus, contrary to a widely accepted opinion, the late issue of accounting information may prove advantageous in the present model. In this regard, the present paper is related to the literature on the timing of accounting information. These studies often focus on the agent's information (e.g., Demski and Sappington (1986); Arya et al. (2000); Christensen and Feltham (2001)) and the question of at what point of time this information should be communicated. In the model here, my main concern is the principal's decision of whether to keep the agreement or not. Therefore, my paper is more closely related to models of double moral hazard (Arya et al. (1997)), in which the principal takes a discretionary action that may be conditioned on some piece of information. Under both double moral hazard and relational contracting, the delay serves as a commitment device to prevent the principal from behaving opportunistically.

4 Conclusion

In this paper I show that in the proper combination, formal and relational contracts provide complementary incentives. The key feature of the proposed relational contract is that it makes use of both formal performance measures and unverifiable information. In this manner, the contract reduces the implicit incentives to that part of the desired action that cannot be induced by the formal contract alone. As a consequence, the payable bonus is reduced, which makes the relational contract credible for a much wider range of discount factors.

The contractual arrangement of an opting-out clause makes possible the use of all available information for the relational contract. When exercising this option, the principal reduces his obligations under the formal contract, and the relevant cost of the bonus payment is reduced. It is crucial that the principal be unaware of the agent's verifiable performance when he decides whether or not to pay the bonus. Consequently, the timeliness of any unverifiable information that could substantiate the relational contract becomes extremely important. The principal's ignorance of his hard obligations can only be exploited to improve the credibility of the relational agreement if this information becomes available at the foreseen point in time.

A practical implication of this result is that norms for the preparation of strategic performance information may become an important aspect of incentive provision. Such norms may improve the verifiability of leading indicator variables, resulting in a more congruent aggregate of verifiable measures. As this analysis shows, the agency will always benefit from this improvement. On the other hand, norms may prove essential to the remaining unverifiable measures if they fix dates by which the data has to be collected.

Appendix

A Proofs

Proof of Lemma 1: The relation directly follows from computing Π^0/Π^{FB}.

Proof of Proposition 1: Upon substitution of (10), the principal's profit (7) becomes

$$\mathrm{E}U^P = \begin{cases} \Pi^{FB} - U^R & \text{for } \phi \leq 1-2r \\ \Pi^{FB}\left[\phi + 4r\dfrac{(2-\phi)(1-\phi(1+r))}{(1+2r)^2(1-\phi)}\right] - U^R & \text{for } 1-2r < \phi \leq 1/(1+r) \\ \Pi(s^0) = \phi\Pi^{FB} - U^R & \text{for } \phi > 1/(1+r). \end{cases}$$

Differentiating with respect to ϕ yields

$$\frac{\partial}{\partial\phi}\,\mathrm{E}U^P = \begin{cases} 0 & \text{for } \phi \leq 1-2r \\ \Pi^{FB}\dfrac{(1-\phi)^2 - 4r^2}{(1+2r)^2(1-\phi)^2} & \text{for } 1-2r < \phi \leq 1/(1+r) \\ \Pi^{FB} & \text{for } \phi > 1/(1+r). \end{cases}$$

The signs of the first and third terms are obvious. The second term is negative for $\phi \geq 1-2r$, which is always true in the relevant range of ϕ. □

Proof of Lemma 2: The principal aims to minimize the expected bonus $s_v\mathbf{d}'\mathbf{a} + s_{pr}\mathbf{y}'\mathbf{a}$, which for $\mathbf{a} = (s_p + s_{pr})\mathbf{y} + s_v\mathbf{d}$ becomes

$$s_v\mathbf{d}'((s_p + s_{pr})\mathbf{y} + s_v\mathbf{d}) + s_{pr}\mathbf{y}'((s_p + s_{pr})\mathbf{y} + s_v\mathbf{d}).$$

Differentiating with respect to s_p yields the first-order condition

$$s_v\mathbf{d}'\mathbf{y} + s_{pr}\mathbf{y}'\mathbf{y} = 0.$$

The corresponding optimal level $s_{pr} = -s_v\mathbf{d}'\mathbf{d}/\mathbf{y}'\mathbf{y} = -s_v s_p^0$ proves part 1 of the lemma: $s_vV + s_{pr}P = s_vV - s_v s_p^0 P = s_v(V - P^0)$.

I derive part 2 of the proposition by substituting s_{pr} into the above condition $(s_p + s_{pr} = 1 - s_v)$. This substitution yields $s_p = (1 - s_v)s_p^0 - s_{pr} = s_p^0$. □

Proof of Lemma 3: Upon substitution of (12), the principal's profit (7) becomes

$$\mathrm{E}U^P = \begin{cases} \Pi^{FB} - U^R & \text{for } r \leq \frac{1}{2} \\ \Pi^{FB}[\phi + \dfrac{8r}{(1+2r)^2}(1-\phi) & \text{for } r > \frac{1}{2}. \end{cases} \tag{16}$$

Differentiating with respect to ϕ yields

$$\frac{\partial}{\partial\phi}\,\mathrm{E}U^P = \begin{cases} 0 & \text{for } r \leq \frac{1}{2} \\ \Pi^{FB}[1 - \dfrac{8r}{(1+2r)^2}] & \text{for } r > \frac{1}{2}. \end{cases}$$

The second term is strictly positive if $(1-2r)^2 > 0$, which is always true in the relevant range of r. □

Proof of Proposition 2:

1. From (14), it is obvious that s_v is strictly positive for all levels of r.

2. Upon substitution of s_v, the principal's expected utility (7) becomes

$$\mathrm{E}U^P = \begin{cases} \Pi^{FB} - U^R & \text{for } r \leq \frac{1}{2} \\ \Pi^{FB}[\phi + \dfrac{1+2r}{(1+2r)^2}(1-\phi)] & \text{for } r > \frac{1}{2}. \end{cases} \tag{17}$$

Differentiating with respect to ϕ yields

$$\frac{\partial}{\partial\phi}\,\mathrm{E}U^P = \begin{cases} 0 & \text{for } r \leq \frac{1}{2} \\ \Pi^{FB}[1 - \dfrac{1+2r}{(1+r)^2}] & \text{for } r > \frac{1}{2}. \end{cases}$$

The second term is non-negative for $r^2 \geq 0$, which is always the case. □

Proof of Corollary 1: For $\phi \geq \hat{\phi}$, this claim directly follows from proposition 2. For $\phi < \hat{\phi}$, an explicit computation of $\mathrm{E}U_P$ would be complicated. Instead, I consider the principal's problem of choosing s_v to maximize his expected profit (7) subject to the credibility constraint. This problem takes the form:

$$\max\ \mathrm{E}U^P = \Pi^{FB}[s_v(2 - s_v)(1-\phi) - (\hat{\phi} - \phi)].$$
$$\text{s.t. } r s_v^2\, \Pi^{FB}(1-\phi) \leq \Pi^{FB}[s_v(2 - s_v)(1-\phi) - (\hat{\phi} - \phi)].$$

By the envelope theorem, the impact of congruity is given by the partial derivative of the Lagrangian with respect to ϕ,

$$\frac{\mathrm{dE}U^P}{\mathrm{d}\phi} = \frac{\partial \mathcal{L}}{\partial \phi} = (1 - s_v)^2 \Pi^{FB} (1 + \lambda) + \lambda r \Pi^{FB} s_v^2.$$

This derivative is non-negative because $\lambda \geq 0$. □

B Derivations

Derivation of Equation (7): With $\mathbf{a}^h = s_v \mathbf{d} + (1 - s_v)\mathbf{y}^0$, the principal's expected utility (5) becomes

$$\begin{aligned} \mathrm{E}U^P(\mathbf{a}^h) &= \mathbf{d}'(s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0) \\ &\quad - \tfrac{1}{2}[(s_v\mathbf{d} + (1 - s_v)\,\mathbf{y}^0)'(s_v\mathbf{d} + (1 - s_v)\mathbf{y}^0)] - U^R \\ &= s_v\mathbf{d}'\mathbf{d} + (1 - s_v)\mathbf{d}'\mathbf{y}^0 \\ &\quad - \tfrac{1}{2}[s_v^2\mathbf{d}'\mathbf{d} + 2s_v(1 - s_v)\mathbf{d}\mathbf{y}^0 + (1 - s_v)^2\,(\mathbf{y}^0)'\mathbf{y}^0] - U^R. \end{aligned}$$

Taking into account the fact that $\mathbf{d}'\mathbf{y}^0 = (\mathbf{y}^0)'\mathbf{y}^0$, this expression is equal to

$$\begin{aligned} \mathrm{E}U^P(\mathbf{a}^h) &= s_v\mathbf{d}'\mathbf{d} + (1 - s_v)(\mathbf{y}^0)'\mathbf{y}^0 - \tfrac{1}{2}[\,(s_v^2\mathbf{d}'\mathbf{d} + (1 - s_v^2)(\mathbf{y}^0)'\mathbf{y}^0] - U^R \\ &= \frac{s_v(2 - s_v)}{2}\mathbf{d}'\mathbf{d} + \frac{(1 - s_v)^2}{2}\,(\mathbf{y}^0)'\mathbf{y}^0 - U^R \\ &= \frac{(\mathbf{y}^0)'\mathbf{y}^0}{2} + s_v(2 - s_v)\left[\frac{\mathbf{d}'\mathbf{d}}{2} - \frac{(\mathbf{y}^0)'\mathbf{y}^0}{2}\right] - U^R \\ &= \Pi^0 + s_v(2 - s_v)\,[\Pi^{FB} - \Pi^0] - U^R. \end{aligned}$$

Derivation of Equation (10): The credibility of the bonus payment requires $r s_v V \leq \Delta U^P$, i.e.,

$$2 r s_v \Pi^{FB}[\phi + s_v(1 - \phi)] \leq \Pi^{FB}\,[s_v(2 - s_v)(1 - \phi) + \min\{\phi - \hat{\phi},\, 0\}].$$

The solution to this quadratic inequality is

$$0 \leq s_v \leq 2\frac{(1 - \phi(1 + r))}{(1 - \phi)(1 - 2r)}. \tag{18}$$

No solution exists if the right-hand term is negative, which is the case for $\phi > 1/(1 + r)$. In this case, no subjective reward is promised and $s_v = 0$. If $\phi \leq 1 - 2r$, the right-hand term is larger than one. Then the credibility constraint is not binding, and the principal chooses the first-best share rate $s_v = 1$. In all other cases, s_v equals the upper bound in (18).

Derivation of Equality (11): The required bonus can be calculated as follows:

$$\begin{aligned} s_v(\mathbf{d}-\mathbf{y}^0)'\mathbf{a} &= s_v(\mathbf{d}-\mathbf{y}^0)'(s_v\mathbf{d} + (1-s_v)\mathbf{y}^0) \\ &= s_v(\mathbf{d}-\mathbf{y}^0)'(\mathbf{y}^0 + s_v(\mathbf{d}-\mathbf{y}^0)) \\ &= s_v^2(\mathbf{d}-\mathbf{y}^0)'(\mathbf{d}-\mathbf{y}^0) \\ &= s_v^2[\mathbf{d}'\mathbf{d} - (\mathbf{y}^0)'\mathbf{y}^0] \\ &= 2s_v^2(\Pi^{FB} - \Pi^0). \end{aligned}$$

Derivation of equation (12): The credibility of the bonus payment requires $rs_vV \leq \Delta U^P$, i.e.,

$$2rs_v^2\Pi^{FB}(1-\phi) \leq \Pi^{FB}[s_v(2-s_v)(1-\phi) + \min\{\phi - \hat{\phi}, 0\}].$$

The solution to this quadratic inequality is

$$0 \leq s_v \leq \frac{2}{1+2r}\,. \tag{19}$$

If $r \leq 1/2$, the right-hand term is larger than one. Then the credibility constraint is not binding, and the principal chooses the first-best share rate $s_v = 1$. In all other cases, s_v equals the upper bound in (19).

Derivation of equation (14): The credibility of the bonus payment requires $rs_vV \leq \Delta U^P$, i.e.,

$$rs_v^2\Pi^{FB}(1-\phi) \leq \Pi^{FB}[s_v(2-s_v)(1-\phi)].$$

The solution to this quadratic inequality is

$$0 \leq s_v \leq \frac{2}{1+r}\,. \tag{20}$$

If $r \leq 1$, the right-hand term is larger than one. Then the credibility constraint is not binding, and the principal chooses the first-best share rate $s_v = 1$. In all other cases, s_v equals the upper bound in (20).

Derivation of Equation (15): The credibility constraint is

$$rs_v^2\Pi^{FB}(1-\phi) \leq \Pi^{FB}[s_v(2-s_v)(1-\phi) - (\hat{\phi} - \phi)].$$

This inequality is fulfilled for

$$\frac{1}{1+r} - \frac{\sqrt{(1-\phi)(1+r\phi \ -\hat{\phi}(1+r))}}{(1-\phi)(1+r)} \leq s_v$$

$$\leq \ \frac{1}{1+r} + \frac{\sqrt{(1-\phi)(1+r\phi \ -\hat{\phi}(1+r))}}{(1-\phi)(1+r)} \, . \qquad (21)$$

Such s_v exist if the radicand in (21) is non-negative, which holds for $r \leq (1-\hat{\phi})/(\hat{\phi}-\phi)$ or $\phi \geq [(1+r)\hat{\phi}-1]/r$.

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