



The Impact of Capacity Costs on Bidding Strategies in Procurement Auctions

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Abstract. This paper analyzes the impact of capacity costs on bidding strategies of firms participating in procurement auctions. More efficient firms will invest in advance due to their high probability of winning the auction while less efficient bidders prefer to wait with their investments until the outcome of the auction is known. However, in equilibrium both types of firms include a coverage for their investment costs in their bids and therefore adopt a full cost pricing policy.

The relevance of fixed costs for pricing decisions has continuously drawn the attention of management accountants at least since the direct cost controversy of the sixties. Despite this long tradition of research, the issue seems still to be unresolved. While current textbooks (see e.g., Horngren, Foster, and Datar (1994), Hansen and Mowen (1994)) and journal articles (see Burgsthaler and Noreen (1997)) emphasize the importance of marginal cost information for pricing decisions, there is also some theoretical evidence that full cost pricing might be beneficial for profit maximizing firms.

In particular, Banker and Hughes (1994) have demonstrated the economic sufficiency of full cost measures for pricing and capacity decisions under uncertainty. They consider a single period model of a multi-product monopolist who simultaneously commits to product prices and initial capacity levels for support activities before demand is realized. After the demand uncertainty is resolved the firm meets the entire demand at the full cost price by purchases of additional support activities at a certain penalty cost if needed. The Banker and Hughes approach has been criticized for several reasons. Both Balakrishnan and Sivaramakrishnan (1996) and Balachandran, Balakrishnan and Sivaramakrishnan (1997) focus on the capacity planning aspect of the model and demonstrate that capacity planning rules based on full product costs are neither necessary nor sufficient for optimal capacity planning. Finally, Göx (1997) analyzes the capacity planning problem in a multiperiod setting and demonstrates that a firm may distort its capacity decision when it commits to full cost pricing instead of economizing on additional demand information that becomes available within the budgeting cycle.

Unlike the above mentioned critiques, the model presented in this paper provides support for the optimality of full cost pricing in a competitive environment. In particular, we consider the impact of capacity costs on the optimal bidding strategies of firms participating in a single procurement auction. First, we analyze the capacity planning problem and identify

two different types of investment strategies: Firms with relatively low costs decide to invest in advance of the bidding contest because of their high probability of winning the auction. In contrast to their low cost competitors less efficient firms prefer to wait with their capacity purchases until the outcome of the auction contest becomes known with certainty although they incur penalty costs if they do not invest ex ante. Despite these different investment strategies, both types of investors include a coverage for their capacity costs into their equilibrium bids. Bidders of the low cost type, however, are only able to recover their sunk investment ex ante, i.e., in expectation, while high cost type bidders are able to recover their capacity costs ex post, i.e., with certainty because these costs are only incurred in case of winning the auction contest.

Cohen and Loeb (1990) already consider the allocation of incremental fixed costs to a number of independent auctions and find that each participating firm allocates the total amount of its fixed costs ex ante. However, since the fixed costs in their model are only incurred when the firm wins at least one project, they are in principle avoidable and the allocation problem is a matter of indivisibility. Unlike Cohen and Loeb, the fixed costs in our model are caused by a voluntary commitment to capacity levels before the firms know with certainty if these capacities will ever be utilized. Once the investment is made, the costs are sunk and can not be recovered when the auction contest is lost.¹

The remainder of this article proceeds as follows: The first section introduces the model assumptions. In Section 2 the optimal solutions for the firm's capacity choices and bidding strategies are derived under quite general conditions. Section 3 gives an example for the special case of two bidders with uniformly distributed costs, and Section 4 concludes with a short summary.

1. Model

Consider a procurement auction with n risk-neutral firms, indexed by $i = 1, \dots, n$, which are supposed to submit simultaneous sealed bids for a single project. The project is awarded to the lowest bidder at the price offered in the bid. In case of commensurate bids, the project is assigned to each bidder with the same probability. Thus, the type of auction under consideration is a first-price sealed-bid auction, which is common practice in government procurement activities and often even required by law (McAfee and McMillan (1987)).

If firm i wins the project, it incurs a direct cost of production, denoted by $c_i \in C = [\underline{c}, \bar{c}] \subset \mathbf{R}^+$. The actual value of c_i is private information of firm i . All other firms and the bid taker share identical beliefs about c_i which can be characterized by the probability distribution function $F_i(c_i)$. To keep the analysis tractable, we restrict ourselves to the case of ex ante symmetric bidders with uncorrelated direct cost parameters. Thus, the c_i are i.i.d. random variables with cumulative distribution function $F_i(\cdot) = F(\cdot)$ for all i . In addition, we assume that $f(\cdot)$ is absolutely continuous with probability density function $f(\cdot) = F'(\cdot)$ over the support C . Up to this point, the setting is strategically equivalent to the well-known symmetric independent private values model of a first-price sealed-bid auction (Milgrom and Weber (1982)).

The distinguishing feature of our model arises from the additional capacity planning problem faced by the bidders. In particular, we assume that the winning firm does not only

employ direct inputs like raw materials or direct labor to realize the project, but also requires inputs which are usually supplied before the firm knows if it is awarded the contract. These inputs will henceforth be referred to as capacity. Once the firm has decided on the amount of capacity to be installed in advance of the bidding contest, it can purchase additional capacity units only if it is willing to pay θr per unit with $\theta > 1$, where r denotes the original acquisition cost of the resource. Thus, ex post provision of capacity implies a penalty $(\theta - 1) > 0$. A natural interpretation would be a labor contract including an agreement about overtime premia where r is the wage per hour and θr the payment per extra work hour. Another example is a long term contract for materials where the difference between the contracted price and the spot market price may be due to quantity discounts. These discounts typically arise when a firm is able to pool its input demands over a range of products or projects by early planning.²

Let $k_i \in [0, 1]$ be the fraction of the required capacity installed in advance of the bidding contest. In determining k_i , the firms participating in the auction have to trade off the potential cost savings from early investment in the case of winning the contest against the risk of sinking the investment costs when losing. The impact of this trade-off on the firm's bidding strategies is analyzed in the next section.

2. Equilibrium Bidding Strategies

All firms participating in the procurement auction face identical decision problems of submitting a sealed bid given the privately known value of their direct cost parameter c_i . Thus, we entirely focus on a symmetric Bayesian Nash equilibrium where all firms adopt the same bidding strategy $b_i = B(c_i)$, allowing us to restrict the analysis to the decision problem of a representative firm. Since the contract is awarded to the lowest bidder, the players' bidding strategies $B(c_i)$ must be a strictly increasing and continuous function of the direct cost parameter c_i (McAfee and McMillan (1987)).³ Moreover, for reasons that will become obvious during the analysis, we assume that $B(c_i)$ is piecewise differentiable. Accordingly, the probability that firm i wins the auction can be written as

$$\text{Prob}\{i \text{ wins}\} = [1 - F(B^{-1}(b_i))]^{n-1} \equiv H(B^{-1}(b_i)). \quad (1)$$

Accordingly, firm i 's expected profit if it provides a fraction k_i of the required project capacity in advance and bids b_i for the contract is given by

$$E[\Pi_i] = [b_i - c_i - \theta r(1 - k_i)]H(B^{-1}(b_i)) - rk_i. \quad (2)$$

The first term in (2) is the expected profit contribution of the project, whereas the capacity costs are given by the second term. Consider first the capacity choice of firm i : Differentiating the expected profit with respect to k_i yields

$$\frac{\partial E[\Pi_i]}{\partial k_i} = \theta r H(B^{-1}(b_i)) - r, \quad (3)$$

indicating that the firm's expected profit strictly increases (decreases) in k_i when the expected marginal penalty costs for additional capacity purchases exceed (fall short of) the marginal

cost of capacity installed in advance of the bidding contest. This observation implies the following bang-bang solution:

Lemma 1 *The optimal capacity choice of firm i is given by*

$$k^* = \begin{cases} 1 & \text{for } \underline{c} \leq c_i < \hat{c} \\ 0 & \text{for } \hat{c} \leq c_i \leq \bar{c} \end{cases} \quad \text{where } \hat{c} = H^{-1}(1/\theta). \quad (4)$$

Proof: Since expected profit is linear in k_i , the firm chooses the maximum capacity level $k = 1$ when (3) is positive and the minimum capacity $k_i = 0$ when (3) is negative. Taking into account that in equilibrium the condition $B^{-1}(b_i^*) = c_i$ must hold for all i , the critical value \hat{c} for the indifferent bidder is obtained by solving (3) for c_i . ■

From Lemma 1 we obtain the intuitively appealing result that the high cost firms prefer to wait until the acceptance of their bid because the threat of incurring sunk capacity costs outweighs the possible cost savings in the relatively unlikely case of acceptance. In contrast, low cost firms prefer to install the entire capacity in advance due to their high probability of winning, and a bidder with cost \hat{c} is indifferent between installing capacity in advance and waiting until the acceptance of his bid. The choice between the polar cases of waiting to invest and full ex ante investment may be regarded as a real option (Dixit and Pindyck (1994)). However, the value of waiting to invest is not determined by an exogenous source of uncertainty as in real options models but by the probability of winning the auction which is in turn determined by the bidder's direct cost parameter c_i in a unique way.

According to Lemma 1, we have to distinguish two types of capacity supply when analyzing the equilibrium bidding strategy $B(\cdot)$. From the firm's first-order condition

$$\frac{\partial E[\Pi_i(b_i, k_i^*)]}{\partial b_i} = \begin{cases} H(\cdot) + [b_i - c_i]H'(\cdot)(B^{-1})'(b_i) = 0 & \text{for } \underline{c} \leq c_i < \hat{c} \\ H(\cdot) + [b_i - c_i - \theta r]H'(\cdot)(B^{-1})'(b_i) & \text{for } \hat{c} \leq c_i \leq \bar{c} \end{cases}, \quad (5)$$

where $H(\cdot)$ and $H'(\cdot)$ denote $H(B^{-1}(b_i))$ and $H'(B^{-1}(b_i))$ respectively, we can derive the following identities:⁴

$$H(c_i)B'(c_i) + H'(c_i)B(c_i) = \begin{cases} c_i H'(c_i) & \text{for } \underline{c} \leq c_i < \hat{c} \\ (c_i + \theta r)H'(c_i) & \text{for } \hat{c} \leq c_i \leq \bar{c} \end{cases}, \quad (6)$$

requiring that the marginal expected revenue of the project on the left hand side of (6) must equal the marginal expected cost of the project, given by the right hand side of (6). The first equation in (6) could also be derived in a setting without additional cost of capacity supply. The distinguishing term $\theta r H'(c_i)$ in the second identity of (6) denotes the cost of ex post installed capacity. However, although the first order condition does not contain capacity costs if the firm provides full capacity in advance, capacity costs appear in all cost types' equilibrium bids:

Lemma 2 *The firm's equilibrium bidding strategy is given by*

$$B(c_i) = c_i + \frac{\int_{c_i}^{\bar{c}} H(z) dz}{H(c_i)} + \begin{cases} \frac{r}{H(c_i)} & \text{for } \underline{c} \leq c_i < \hat{c} \\ \theta r & \text{for } \hat{c} \leq c_i \leq \bar{c} \end{cases} \quad (7)$$

where \hat{c} is defined in (4)⁵.

Proof: See Appendix. ■

From (7), the optimal bid consists of three terms. The first term represents the actual direct project costs of bidder i , the second exhibits the firm's rent for its private information, and the third term allocates capacity costs to the bid. Since the sum of the first two terms in (7), $\beta(c_i) = c_i + \int_{c_i}^{\bar{c}} H(z) dz / H(c_i)$, would be the optimal bidding strategy in the corresponding standard model without capacity costs (see McAfee and McMillan (1987)), we can derive the following result about the impact of capacity costs on the firm's bidding strategy by comparing both strategies:

Proposition 1 *Capacity costs are incorporated in the firm's pricing decisions as follows:*

1. *If no capacity is provided in advance, incremental capacity costs θr are fully covered by the bid.*
2. *If capacity is provided in advance, capacity costs are assigned to the bid such that they are fully covered ex ante.*

Proof:

1. The coverage for capacity costs in (7) equals the incremental capacity costs θr .
2. The coverage for capacity costs in (7) equals $r/H(c_i)$. Since in equilibrium firm i 's probability of winning the contest is $H(c_i)$, the expected coverage of capacity cost is r , the historical capacity acquisition costs. ■

The first result in *Proposition 1* is a straightforward extension of the standard model without capacity costs where the direct production cost of firm i is substituted by the term $\tilde{c}_i = c_i + \theta r$. The second result, namely the question of why low cost firms allocate capacity costs to their bids, requires some additional considerations. Since high cost firms will wait to invest and include their "direct cost" \tilde{c}_i in their bid functions, low cost firms can also add a markup for capacity costs because an optimal bid function schedule $B(\cdot)$ must be continuous for a continuous type space. This argument is best understood by considering the critical firm of type \hat{c} . Without allocating capacity cost, its bid would be $\beta(\hat{c}) = \hat{c} + \int_{\hat{c}}^{\bar{c}} H(z) dz / H(\hat{c})$. But a competitor of type $(\hat{c} + \epsilon)$ bids $B(\hat{c} + \epsilon) = \theta r + (\hat{c} + \epsilon) + \int_{(\hat{c} + \epsilon)}^{\bar{c}} H(z) dz / H(\hat{c} + \epsilon)$ which equals $\beta(\hat{c}) + \theta r$ as ϵ approaches zero. Hence, the firm of type \hat{c} could increase its bid to $B(\hat{c})$ without decreasing its winning probability. Taking this boundary condition into account, we get the bid function $B(\hat{c})$ including capacity costs for all cost types (see proof of *Lemma 2*).

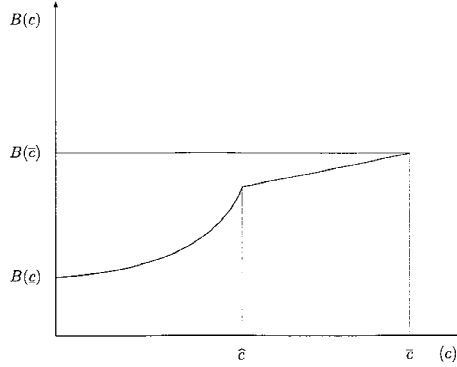


Figure 1. Optimal bidding strategies.

3. The Uniform Case

The following example illustrates the preceding results. Suppose that there are only $n = 2$ bidders with uniformly distributed costs on the $[0, 1]$ interval. The cumulative distribution function becomes $F(c_i) = c_i$, and firms i 's probability of winning the auction is $H(c_i) = 1 - c_i$. Substituting for $H(c_i)$ in (7) yields the following optimal bidding strategy:

$$b_i^* = c_i + \frac{1 - c_i}{2} + \begin{cases} \frac{r}{1 - c_i} & \text{for } 0 \leq c_i < \hat{c} \\ \theta r & \text{for } \hat{c} \leq c_i \leq 1 \end{cases}. \quad (8)$$

The value of the direct cost parameter for the critical bidder being indifferent between incurring sunk costs and waiting to invest is $\hat{c} = (\theta - 1)/\theta$. According to (8), each bidder demands a reimbursement for his direct costs, an information rent of $(1 - c_i)/2$, and an additional payment for his capacity costs. Obviously, the most efficient type is allowed to extract the highest possible information rent of $1/2$, whereas the information rent of higher cost types strictly decreases in c_i . In contrast, the capacity cost term strictly increases in c_i for cost types $c_i < \hat{c}$. This effect has a natural interpretation. Since the most efficient cost type wins the auction with probability one, it can be certain to recover its investment costs and hence bids $B(\underline{c}) = 1/2 + r$, i.e. it confines itself with a full cost reimbursement and receives the full information rent as its surplus. In contrast, a less efficient firm can only recover its investment outlays with a probability strictly less than one. Therefore, it will adjust its capacity cost term in order to cover the capacity costs in expectation. Hence, it demands strictly more than r . Finally, the cost types $c_i \geq \hat{c}$ are demanding a full cost reimbursement plus their information rent, which is zero for the least efficient type, i.e., $B(\bar{c}) = 1 + \theta r$. The results are summarized in Figure 1. Starting from the lowest possible bid $B(\bar{c})$, the equilibrium bid function strictly increases in c_i with an increasing rate⁶ up to the point where capacity precommitment is no longer optimal. Beyond the critical value \hat{c} , the bid function increases linearly in c_i until the maximum possible bid $B(\bar{c})$, namely the bid of the lease efficient type, is achieved.

4. Summary and Conclusions

This paper has examined the impact of capacity costs on competitive bidding strategies of firms participating in procurement auctions. First, two different types of investment strategies were identified: More efficient firms decide to invest in advance of the bidding contest due to their high probability of winning the auction. In contrast, less efficient firms with relatively low probabilities of winning the auction prefer to wait with their capacity purchases until the outcome of the auction game becomes known with certainty. For both types of investors the optimal bidding strategies include a coverage for their investment costs. While the high cost type's investment costs are avoidable and thus recovered with certainty if the contest is won, the low cost types are only able to recover their sunk investment costs in expectation. Nevertheless, full cost pricing is identified as a profit maximizing strategy for all cost types in the context of our model.

Appendix

Proof of Lemma 2: To derive the equilibrium bidding strategy we have to solve the piecewise defined differential equation given by (6). The equilibrium bid for cost types $c_i \geq \hat{c}$ is characterized by the identity

$$H(c_i)B'(c_i) + H'(c_i)B(c_i) = (c_i + \theta r)H'(c_i).$$

Integrating this ordinary differential equation yields

$$B(c_i)H(c_i) = - \int_{c_i}^{\bar{c}} (z + \theta r)H'(z)dz + \bar{K}. \quad (9)$$

To determine the constant \bar{K} , we look at the maximum cost firm. Substituting c_i by \bar{c} in (9) yields

$$\underbrace{B(\bar{c})}_{=0} \underbrace{H(\bar{c})}_{=0} = - \underbrace{\int_{\bar{c}}^{\bar{c}} [z + \theta r]H'(z)dz}_{=0} + \bar{K}.$$

Thus, $\bar{K} = 0$ and

$$\begin{aligned} B(c_i) &= \frac{- \int_{c_i}^{\bar{c}} zH'(z)dz - \theta r \int_{c_i}^{\bar{c}} H'(z)dz}{H(c_i)} \\ &= \theta r - \frac{\int_{c_i}^{\bar{c}} zH'(z)dz}{H(c_i)} \\ &= \theta r + c_i + \frac{\int_{c_i}^{\bar{c}} H(z)dz}{H(c_i)} \end{aligned} \quad (10)$$

for $c_i \geq \hat{c}$ where the first equality follows from $H(\bar{c}) = 0$ and the second step is obtained from integration by parts.

From (6), the equilibrium bidding strategy for cost types $c_i < \hat{c}$ is characterized by the identity

$$H(c_i)B'(c_i) + H'(c_i)B(c_i) = c_i H'(c_i)$$

as in the standard independent private values model without capacity costs. However, in contrast to the standard model this part of the bid function is only optimal for cost types $c_i < \hat{c}$. Thus, the general solution of the differential equation is given by

$$B(c_i)H(c_i) = - \int_{c_i}^{\hat{c}} zH'(z)dz + \underline{K}.$$

To determine the constant \underline{K} we look at the critical bidder who is indifferent between investing in advance and waiting. Since B is continuous, his bid must be the same under both investment strategies. Thus, substituting $c_i = \hat{c}$ yields

$$B(\hat{c})H(\hat{c}) = - \underbrace{\int_{\hat{c}}^{\hat{c}} zH'(z)dz}_{=0} + \underline{K}.$$

Hence, $\underline{K} = B(\hat{c})H(\hat{c})$ and we obtain the particular solution

$$B(c_i)H(c_i) = - \int_{c_i}^{\hat{c}} zH'(z)dz + B(\hat{c})H(\hat{c})$$

or

$$B(c_i)H(c_i) = c_i H(c_i) - \hat{c}H(\hat{c}) + \int_{c_i}^{\hat{c}} H(z)dz + B(\hat{c})H(\hat{c})$$

after an integration by parts. Utilizing (10) and substituting

$$B(\hat{c}) = \theta r + \hat{c} + \frac{\int_{\hat{c}}^{\bar{c}} H(z)dz}{H(\hat{c})}$$

yields

$$\begin{aligned} B(c_i)H(c_i) &= -\hat{c}H(\hat{c}) + c_i H(c_i) + \int_{c_i}^{\hat{c}} H(z)dz + \left(\theta r + \hat{c} + \frac{\int_{\hat{c}}^{\bar{c}} H(z)dz}{H(\hat{c})} \right) H(\hat{c}) \\ &= c_i H(c_i) + \theta r H(\hat{c}) + \int_{c_i}^{\bar{c}} H(z)dz. \end{aligned}$$

Finally, utilizing the fact that $H(\hat{c}) = 1/\theta$ yields the second part of the equilibrium bid function

$$B(c_i) = c_i + \frac{r}{H(c_i)} + \frac{\int_{c_i}^{\bar{c}} H(z)dz}{H(c_i)}$$

for cost types $c_i < \hat{c}$. ■

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Notes

1. To stress this point, consider the Cohen and Loeb setting with each firm participating only in one auction. Then the indivisibility problem vanishes and the incremental “fixed costs” become direct project costs because they are entirely avoidable when the firm fails to win the auction.
2. See Atkinson, Banker, Kaplan and Young (1997), Chapter 7 for further examples.
3. This standard result for independent private value auctions also holds for the extended setting considered here, because the optimal capacity is also a function of the cost type (Equation 4). Denote this function by $k^* := h(c_i)$, then the optimal bidding strategy of bidder i may equivalently be expressed as $b_i = \hat{B}(c_i, h(c_i)) := B(c_i)$.
4. This step follows from the condition for a symmetric equilibrium $B^{-1}(b_i) = c_i$ and the fact that $(B^{-1})'(b_i) = 1/B'(c_i)$ from the inverse function theorem.
5. An implicit assumption for the (social) optimality of (7) is that the value of the project to the bid taker exceeds the highest possible bid. Denote this value by v , then $v > \bar{c} + \theta r$ guarantees optimality because the least efficient type earns no information rent.
6. Note that b_i^* is strictly convex in c_i : $d^2 B_i(c_i)/dc_i^2 = 2r/(1 - c_i)^3 > 0$.

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