Limited Liability and the Risk-Incentive Relationship*

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Abstract

Several empirical findings have challenged the traditional view on the trade-off between risk and incentives. By combining risk aversion and limited liability in a standard principal-agent model, the empirical puzzle on the positive relationship between risk and incentives can be explained. Increasing risk leads to a less informative performance signal. Under limited liability, the principal may optimally react by increasing the weight on the signal and, hence, choosing higher-powered incentives.

Key Words: moral hazard, limited liability, risk-incentive relationship

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1 Introduction

In the last two decades, field studies as well as findings from laboratory experiments have challenged standard results in principal-agent theory and stimulated alternative models on incentives within behavioral economics.[1] Applying Fehr's and Schmidt's (1999) concept of inequity aversion, Demougin, Fluet and Helm (2006), Kragl and Schmid (2009) and Englmaier and Wambach (2010) discuss the optimal principal-agent contract for situations where agents also care for the incomes of others. Közsegi and Rabin (2006) consider the case of loss averse agents with endogenous reference points. Herweg, Müller and Weinschenk (forthcoming) show that in this case a bonus contract is typically the best instrument to deal with moral hazard problems. Benabou and Tirole (2006) and Sliwka (2007) address the problem that extrinsic rewards can crowd out agents' intrinsic motivation. Finally, Prendergast (2002a, 2002b) points to the puzzle that in practice we do not necessarily find a negative relation between risk and incentives, as suggested by principal-agent theory.

Our paper deals with the last puzzle raised by Prendergast (2002a, 2002b): In the standard principal-agent moral-hazard model with a risk neutral principal and a risk averse agent, the principal typically reacts to increased exogenous risk by reducing incentives.[2] If risk is low and, hence, the risk premium (or risk costs) from high-powered incentives is small the principal will optimally choose high incentives for the agent. However, if risk increases so that inducing incentives becomes rather expensive, the principal will prefer low-powered incentives. As pointed out by Prendergast, several empirical studies contradict the negative relationship between risk and incentives. In order to explain this puzzle, he adds the possibility of input monitoring, favoritism or unverifiable performance signals to the textbook model and shows for risk neutral agents that the negative relationship may disappear.

In the following, we will offer an explanation for the empirical puzzle without extending the principal-agent textbook model. Microeconomics text-

[1]For an overview, see Demougin, Fabel and Thomann (2009), Section 2.

[2]See, e.g., Laffont and Martimort (2002), Section 4.4.

books typically use only one contractual friction – either risk aversion or limited liability – in order to create a meaningful incentive problem.[3] In practice, however, agents are likely to be both risk averse and protected by limited liability. Therefore, we combine the two standard contracting problems in our model, which leads to an explanation for a positive relationship between risk and incentives that seems to be most natural. In our model, the agent earns a non-negative rent from limited liability. If risk increases and, as a result, the performance signal becomes less informative, it can pay for the principal to increase the weight on this signal by choosing higher-powered incentives. Since the agent's individual rationality constraint is non-binding at the optimum, the principal's additional incentive costs will not increase too steeply: They only increase in terms of expected wage payments whereas the progressively increasing effort costs simply reduce the agent's rent.

Our paper is organized as follows. The following section gives an overview of the related literature. In Section 3, we introduce the principal-agent model with risk aversion and limited liability. Section 4 analyzes the possibility of a positive risk-incentive relationship under the optimal contract. Sections 5 contains an illustrating example. Section 6 will conclude.

2 Related Literature

Prendergast (2002a) motivates the empirical puzzle on the risk-incentive relationship by referring to empirical studies on executive compensation, incentive contracts for sharecroppers and the impact of risk on a company's decision whether to franchise or to run own stores. The papers by Rao (1971) and Allen and Lueck (1992) seem to be most interesting as they find a significantly positive relation between exogenous risk and endogenously chosen incentives.

The empirical results of Rao (1971) show that, even in the beginning of principal-agent theory, data on incentive pay are not in line with the standard textbook result. Interestingly, Rao investigates sharecropping, the most

[3] Compare Sections 4.3 and 4.4 in Laffont and Martimort (2002).

often used example for a principal-agent relation in microeconomics theory. Rao shows that in India two different kinds of contracts coexist – crop-sharing arrangements that share the profit risk between tenants and landlords and fixed-cash rents agreements that shift the entire risk to the tenants. The empirical findings clearly point out that fixed-cash rents (crop-sharing contracts) are observable in situations with high (low) economic uncertainty, which is in sharp contrast with the optimal incentive contract under a risk averse agent and a risk neutral principal.

Allen and Lueck (1992) also empirically investigate share-cropping agreements, using data from landowner-farmer relationships in the United States. They observe the same two contract types as Rao: a crop-sharing contract that divides harvested crops and, hence, income risk and a fixed-rent contract that guarantees the landowner a fixed amount of cash and therefore leaves full risk for the farmer. While corn and sorghum belong to the most risky crops, wheat and soybeans are considerably less risky. Following principal-agent theory the production of wheat and soybeans should be governed by fixed-rent contracts. However, the data clearly contradict this hypothesis. Moreover, in counties where the yield of corn and wheat is more variable over time the use of crop-sharing contracts is decreasing.

Other empirical studies, in particular those on top management compensation (e.g., Garen 1994, Bushman, Indjejikian and Smith 1996, Ittner, Larcker and Rajan 1997) and franchising (e.g., Anderson and Schmittlein 1984, John and Weitz 1988) find neither a significantly positive nor a significantly negative risk-incentive relationship, which also casts doubts on the principal-agent textbook model.

As a direct consequence to the empirical studies, theorists have considered modifications of the standard principal-agent model in order to explain the empirical puzzle on the risk-incentive relationship. Zabojnik (1996) introduces a second stochastic term into the LEN model[4] that increases the agent's productivity of effort and is observed by the agent before he takes his

[4] This model builds on the work by Holmström and Milgrom (1987) and assumes **L**inear production functions, linear incentive contracts and a linear utility function of the principal, an **E**xponential utility function of the agent, and **N**ormally distributed noise.

action. If this multiplicative uncertainty dominates the original stochastic influence of the additive noise term, incentives may be increasing in risk.

In the paper by Prendergast (2002a), the principal can choose between input-based and output-based contracts given a risk neutral and unlimitedly liable agent. If uncertainty increases, the principal may prefer to switch from an input-based contract to an output-based one. Under an input-based contract, the principal pays for a certain effort level which can be perfectly verified. Hence, pay is exclusively used to compensate the agent for his disutility of effort, but not for generating incentives. Under an output-based contract, the principal optimally chooses a 100 percent piece rate ("selling the firm") and a negative fixed payment that extracts all rents. Thus, the switch from an input-based contract to an output-based one is accompanied by a switch from zero incentives to highest-powered incentives.

Prendergast (2002b) offers two other explanations for the risk-incentive puzzle which are both based on the existence of subjective performance evaluation, using unverifiable signals. The first explanation uses the observation that, in practice, supervisors' evaluations do not only reflect worker performances but also their personal preferences towards individual workers. Prendergast shows that contracting under such favoritism can lead to a positive risk-incentive relationship. The second approach builds on the fact that a principal can misrepresent subjective performance evaluation in order to save labor costs. In other words, if performance signals are unverifiable and contracts do not become self-enforcing due to repeated interaction it is always optimal for the principal to claim poor performance of the agent irrespective of his true performance. Again, this approach can imply a positive risk-incentive relation.

Another explanation for the empirical puzzle is offered by Raith (2003). Here, the positive relationship between risk and incentives comes as a by-product. Raith assumes that agents are risk averse so that incentives imply risk costs. However, since agents face a binding participation constraint and principals always realize zero profits due to competitive product markets, neither the agent nor the principal (but society) has to bear the risk costs from inducing incentives. Hence, less extreme competition may lead to different findings concerning the trade-off.

Finally, Wright (2004) and Serfes (2005) independently develop a matching approach that can explain the puzzle. Here, we have heterogeneous firms that use stochastic production technologies differing in risk. Moreover, also agents are heterogeneous being characterized by different degrees of risk aversion. Two effects are important. First, the more risky a production technology the smaller will be an agent's optimal incentives, which is just the standard trade-off in the textbook model. Second, less risk averse agents get higher incentives. Note that both effects may work into opposite directions. If less risk-averse agents are hired by more risky firms and the second effect dominates the first one, we will have a positive relation between risk and incentives.

Importantly, all theoretical models that offer an explanation for the empirical puzzle introduce additional assumptions to the textbook model. In the following, we will show that under pure textbook assumptions a positive risk-incentive relationship can be explained when combining the two standard contractual frictions, namely risk aversion and limited liability.

3 The Model

We consider a typical moral-hazard problem between a risk averse agent and a risk neutral principal which is based on the binary-signal model used by Demougin and Garvie (1991) and Demougin and Fluet (2001a). The agent chooses a non-negative effort a that is unobservable to the principal. The non-contractible value of this effort to the principal is described by the function $v(a)$ with $v'(a) > 0$ and $v''(a) < 0$. In choosing a, the agent incurs a private cost $c(a)$, which, together with his utility $u(w)$ from wealth w, describes his preferences by the utility function $U(w, a) = u(w) - c(a)$. We assume that $u(0) = 0$, $u' > 0$, $u'' \leq 0$, and $c'(a) > 0$, $c''(a) > 0, \forall a > 0$. To ensure an interior solution we assume $c(0) = 0$, $c'(0) = 0$ and $\lim_{a \to \infty} c'(a) = \infty$. The agent's reservation utility is normalized to $\bar{U} = 0$.

Principal and agent observe a contractible signal $s \in \left\{s^L, s^H\right\}$ on the agent's performance. The outcome $s = s^H$ is favorable information about the agent's effort choice in the sense of Milgrom (1981). Let the probability of this favorable outcome be $p(a)$ with $p'(a) > 0$ (implying the strict monotone likelihood ratio property) and $p''(a) < 0$ (convexity of the distribution function condition). Since the signal s is the only observable and verifiable information on the agent's performance, the principal offers a payment scheme $w(s)$ with $w\left(s^L\right) = w_L$ and $w\left(s^H\right) = w_H$. We assume that the agent is protected by limited liability in the sense of $w_L, w_H \geq 0$.

To make the analysis of the risk-incentive trade-off more precise, we first provide definitions of *more risky* (less informative) outcomes and *higher-powered* or stronger incentives. To that purpose, we compare different distributions on binary signals $s, \hat{s} \in \{s^L, s^H\}$ with probabilities $p(a)$ and $\hat{p}(a)$ for the favorable outcome s^H. Thus, the support of the outcome distribution does not change, but probabilities do. We define a performance signal $\hat{s}$ to be *more risky* (less informative with respect to the agent's action) than a signal s, if $\hat{s}$ is a garbling of s, i.e. if there exists a number $b \in (0, 1/2]$ such that

$$\hat{p}(a) = (1-b) \cdot p(a) + b \cdot (1 - p(a)) = b + (1-2b)p(a).$$

This is a special case of Blackwell informativeness, where the garbling is symmetric among realizations.[5] We will use this garbling in the following section. $b \in (0, 1/2]$ is without loss of generality. It only makes sure that the favorable outcome in s is also the favorable one in $\hat{s}$. Garbling can easily be interpreted in terms of risk in the binary model since the variance of signal s is $p(a)(1-p(a))(s^H - s^L)^2$, which is maximized for $p(a) = 1/2$. Obviously, $(1-b) \cdot p(a) + b \cdot (1-p(a))$ is closer to 1/2 than $p(a)$ for all $p(a) \in [0,1]$ and $b \in (0, 1/2]$.

Due to the garbling condition, our definition of a more risky performance signal is equivalent to the notion of "less information" used in information economics and agency theory. The garbling condition is therefore closely related to several concepts of informativeness familiar in economic theory. Since the likelihood ratio $\hat{p}'(a)/\hat{p}(a)$ is decreasing in b, garbling in our model yields

[5] Note that our findings will qualitatively hold for asymmetric garbling. Extending the analysis to asymmetric garbling can only increase the set of possible cases for which a positive risk-incentive relation holds.

a performance measure which is *less efficient* in the sense of Kim's (1995) Mean Preserving Spread (MPS) criterion at any level of effort. The same holds for Lehmann's (1988) concept of effectiveness, as applied to agency problems by Jewitt (1997). Other informativeness criteria are not straightforwardly applicable because of the binary structure of our model. These are the integral criterion of Demougin and Fluet (2001b), which is equivalent to the MPS criterion for continuously distributed performance measures, but refers to signal realizations with identical values of the distribution functions, and Holmström's (1979) informativeness criterion, which compares information systems that include one another and therefore could only be binary in trivial cases.

Finally, we specify the meaning of higher-powered incentives and a positive risk-incentive relationship. The payment scheme

$$\hat{w} = \begin{cases} \hat{w}_H & \text{if } \hat{s} = s^H \\ \hat{w}_L & \text{if } \hat{s} = s^L \end{cases}$$

based on the signal $\hat{s}$ is *higher-powered* than the scheme

$$w = \begin{cases} w_H & \text{if } s = s^H \\ w_L & \text{if } s = s^L \end{cases}$$

based on s if $\hat{w}_H - \hat{w}_L > w_H - w_L$. We will speak of a *positive risk-incentive relationship* if

$$\left. \frac{\partial \left(\hat{w}_H - \hat{w}_L \right)}{\partial b} \right|_{b=0} > 0,$$

that is if marginal garbling yields higher-powered incentives.

4 Risk and Incentives

In setting w_L and w_H, the principal aims at maximizing his profit net of expected wage payments, provided the agent accepts the contract and chooses the desired action, and the wages fulfill the limited-liability constraint:

$$\max_{w_L, w_H, a} v(a) - p(a)w_H - (1 - p(a))\, w_L$$

$$\text{subject to} \quad p(a)u(w_H) + (1 - p(a))u(w_L) - c(a) \geq 0, \tag{IR}$$

$$a \in \arg\max_{\alpha} \{p(\alpha)u(w_H) + (1 - p(\alpha))u(w_L) - c(\alpha)\}, \tag{IC}$$

$$w_H, w_L \geq 0. \tag{LL}$$

Since both the monotone likelihood ratio property and the convexity of the distribution function condition hold, the incentive compatibility constraint can be replaced by the first-order condition

$$p'(a)\,(u(w_H) - u(w_L)) = c'(a).$$

In the optimal solution, the individual rationality constraint (IR) will be non-binding and the limited-liability condition (LL) will be binding for w_L. To see this, note that the agent can always obtain a non-negative expected utility by accepting the contract and choosing zero effort. Hence, the agent will always earn a non-negative rent. Since w_L decreases incentives but increases labor costs, the principal will optimally choose $w_L^* = 0$. The optimization problem is then reduced to

$$\max_{w_H, a} \pi\,(w_H) = v(a) - p(a)w_H$$

$$\text{subject to} \quad p'(a)u(w_H) = c'(a).$$

Now we introduce the symmetric garbling from Section 2. After replacing $p(a)$ by $\hat{p}\,(a) = (1 - b)p(a) + b(1 - p(a))$ $(b \in (0, 1/2])$ and w by $\hat{w}$ the corresponding optimization problem can be written as

$$\max_{\hat{w}_H, a} \pi\,(\hat{w}_H) = v(a) - [b + (1 - 2b)\,p(a)]\,\hat{w}_H$$

$$\text{subject to} \quad (1 - 2b)\,p'(a)u(\hat{w}_H) = c'(a). \tag{IC'}$$

The incentive constraint (IC') shows that a more risky performance signal requires higher-powered incentives for implementing the same effort level a.

Lemma 1 *To implement a given action a at minimal cost, higher-powered incentives are necessary under a more risky performance measure.*

Proof. Obvious from (IC'). ■

The intuition is straightforward: if the outcome becomes more risky and, hence, the performance signal is less informative about the agent's effort choice, incentives will decline. To restore former incentives, the principal has to choose a higher weight for the performance signal, i.e. a higher value of $\hat{w}_H$.

It is not clear, however, whether the principal in general should react to increased risk by increasing incentives as well: a higher value of $\hat{w}_H$ also increases the principal's labor cost in case of a favorable performance signal and, hence, the agent's rent. The principal is therefore likely to reduce the implemented action in order to reduce the required wage payment. Which of the two countervailing effects dominates is not clear from the outset.

In case of a finite action space $A = \{a_1, \ldots, a_n\}$, it is easy to construct situations in which a higher wage spread will result, as the following example shows:

Example 2 *Let the agent's action space be $\{a_1, a_2\}$ with $v(a_1) = 0$ and $v(a_2) = 4$, the agent's utility be given by $U(w, a) = \sqrt{w} - c(a)$ with $c(a_1) = 0$ and $c(a_2) = 1$, and success probabilities be $p(a_1) = 0.25$ and $p(a_2) = 0.75$. Then, high effort a_2 is implemented by wages $w_L = 0$ and $w_H = 4$. The expected compensation cost is $E[w] = 0.25 \cdot 0 + 0.75 \cdot 4 = 3$, and the principal's net profit is $4 - 3 = 1$. The principal strictly prefers a_2 to a_1, since implementing a_1 yields a net profit of 0.*

Now consider a garbling of the form proposed in our definition of riskiness. Success probabilities become $\hat{p}(a_1) = 0.25 + 0.5b$ and $\hat{p}(a_2) = 0.75 - 0.5b$. High effort is implemented by wages $w_L = 0$ and $w_H = 1/(0.5 - b)^2$. As $1/(0.5 - b)^2 > 4$, $\forall b \in (0, 1/2)$, incentives become higher powered. The principal's net profit under a_2 is $4 - (0.75 - 0.5b)/(0.5 - b)^2$, which is larger than his zero profit under a_1 for $b < \left(7 - \sqrt{33}\right)/16$ or $b > \left(7 + \sqrt{33}\right)/16$.

For these levels of b, the principal still implements a_H after the garbling, and there is a positive risk-incentive relationship.

The positive finding of the example can be generalized: if under the original signal s the principal is not indifferent between the implemented action and another one, an incremental increase in risk will not result in a change of the desired action, and a positive risk-incentive relationship will apply:

Proposition 3 *If the principal strictly prefers the implemented action in a model with a finite action space, there is a positive risk-incentive relationship.*

Proof. Let A be ordered such that $c(a_j) > c(a_{j-1})$ for $j = 2, \ldots, n$. If both the monotone likelihood ratio (MLRP) and the convexity of the distribution function condition (CDFC) as defined by Grossman and Hart (1983) hold,[6] the incentive constraint (IC) for the implementation of a_j can be written as

$$(1-2b)\,[p(a_j) - p(a_{j-1})]u(\hat{w}_H) \geq c(a_j) - c(a_{j-1}), \qquad \text{(IC")}$$

which is fulfilled with equality in the second-best contract. Let a_s be the optimal action to be implemented by performance measure s, with a net profit of Π_s accruing to the principal. Moreover, let $\Pi_{-s} < \Pi_s$ denote the maximum net profit under the actions $a \in A \backslash a_s$. Now consider a garbling of s. From (IC"), the cost of inducing a_s is

$$\begin{aligned}\hat{W}(a_s, b) &= \hat{p}(a_s)u^{-1}\left(\frac{c(a_s) - c(a_{s-1})}{\hat{p}(a_s) - \hat{p}(a_{s-1})}\right) \\ &= [b + (1-2b)p(a_s)]u^{-1}\left(\frac{c(a_s) - c(a_{s-1})}{(1-2b)(p(a_s) - p(a_{s-1}))}\right).\end{aligned}$$

This function is continuous in b and identical to the cost of inducing a_s without garbling for $b = 0$. Therefore, there exists a critical value $\hat{b}$ up to which $\hat{\Pi}_s = v(a_s) - W(a_s, b)$ is greater than Π_{-s}. For values of b smaller than $\hat{b}$, action a_s will still be optimal under the garbled performance measure. The

[6] MLRP here is simply $p(a_j) \geq p(a_{j-1})$ for $j = 2, \ldots, n$, CDFC means that if $c(a_j) = \lambda c(a_i) + (1-\lambda)c(a_k)$ for some $\lambda \in [0,1]$, then $p(a_j) \geq \lambda p(a_i) + (1-\lambda)p(a_k)$.

positive risk-incentive relationship then follows from Lemma 1 and (IC"), respectively. ■

In case of a continuous action, the principal will most likely reduce the implemented action, and the existence of a positive risk-incentive trade-off will depend on how the primitives of the model drive the two countervailing effects identified above.

If the agent is risk neutral, the trade-off between the two effects is clear-cut if a high level of effort is implemented: The first-order effect of a higher required wage spread then outweighs the second-order effect of effort reduction. This result is stated more precisely in the following proposition.

Proposition 4 *If the agent is risk neutral, there is a positive risk-incentive relationship as long as the probability of a high payment exceeds 1/2.*

Proof. If the agent is risk neutral, the incentive compatibility constraint (IC') becomes $\hat{p}'(a)\hat{w}_H = c'(a)$ from which the required high-outcome wage $\hat{w}_H(a)$ (and thus, the wage spread) is given by

$$\hat{w}_H(a) = \frac{c'(a)}{\hat{p}'(a)}.$$

After a substitution of $\hat{w}_H = \hat{w}_H(a)$ in the objective function, the principal's optimization problem is

$$\max_a \pi(a) = v(a) - \hat{p}(a)\frac{c'(a)}{\hat{p}'(a)}.$$

The principal's choice of which action to implement is described by the first-order condition

$$\begin{aligned}\frac{\partial \pi(a)}{\partial a} &= v'(a) - \hat{p}'(a)\frac{c'(a)}{\hat{p}'(a)} - \hat{p}(a)\frac{\hat{p}'(a)c''(a) - \hat{p}''(a)c'(a)}{(\hat{p}'(a))^2} \\ &= v'(a) - c'(a) - \hat{p}(a)\frac{c''(a)}{\hat{p}'(a)} + \hat{p}(a)\frac{\hat{p}''(a)c'(a)}{(\hat{p}'(a))^2} = 0.\end{aligned}$$

Rearrangement with respect to $\hat{w}_H(a)$ yields

$$\hat{w}_H(a) = \frac{c'(a)}{\hat{p}'(a)} = \frac{c''(a)}{\hat{p}''(a)} - [v'(a) - c'(a)] \frac{\hat{p}'(a)}{\hat{p}(a)\hat{p}''(a)}.$$

To analyze the impact of garbling, next substitute for $\hat{p}(a) = b + (1-2b)p(a)$, $\hat{p}'(a) = (1-2b)p'(a)$ and $\hat{p}''(a) = (1-2b)p''(a)$ to get

$$\begin{aligned}\hat{w}_H(a) &= \frac{c''(a)}{(1-2b)p''(a)} - [v'(a) - c'(a)] \frac{(1-2b)p'(a)}{[b+(1-2b)p(a)](1-2b)p''(a)} \\ &= \frac{c''(a)}{(1-2b)p''(a)} - [v'(a) - c'(a)] \frac{p'(a)}{[b+(1-2b)p(a)]p''(a)}.\end{aligned}$$

The first term is increasing in b and the term in brackets is positive because the implemented second-best effort level is less than the first-best effort level, for which $v'(a) - c'(a) = 0$ would hold. Since $p'(a) > 0$ and $p''(a) < 0$ by assumption, $\hat{w}_H(a)$ is increasing in b if the term $b + (1-2b)p(a) = \hat{p}(a)$ in the denominator is decreasing in b, which holds for $p(a) > 1/2$. ■

If $p(a) > 1/2$, adding noise to the performance measure decreases the probability of a high payment w_H. But this is not the reason for the positive risk-incentive relationship because due to the fact that the likelihood ratio $p'(a)/p(a)$ also decreases, the required increase in w_H is such that the expected compensation cost to implement a given level of effort nevertheless increases. Rather, there is a positive relationship because for $p(a) < 1/2$, a lower effort level yields a higher variance of the signal's distribution (recall that the variance is maximized for $p(a) = 1/2$). As the proposition shows, this effect, in conjunction with the direct effect described in Lemma 1, overcompensates the effect of a lower wage spread, and it becomes beneficial for the principal to increase w_H if the environment becomes riskier. Less technically, the proposition therefore states that for a risk-neutral agent, there is a positive risk-incentive relationship if the agent's effort reduces risk.

If the agent is risk averse, the trade-off gets more subtle because in this case not only rents from limited liability, but also risk premia have to be taken into account. To analyze the trade-off by comparative statics, let $a^* = a^*(\hat{w}_H)$ denote the agent's incentive compatible effort choice implicitly described by (IC') with

$$\frac{\partial a^*}{\partial \hat{w}_H} = -\frac{(1-2b)\,p'(a^*)u'(\hat{w}_H)}{(1-2b)\,p''(a^*)u(\hat{w}_H) - c''(a^*)} > 0 \tag{1}$$

and

$$\frac{\partial a^*}{\partial b} = \frac{2p'(a^*)u(\hat{w}_H)}{(1-2b)\,p''(a^*)u(\hat{w}_H) - c''(a^*)} < 0. \tag{2}$$

Moreover, let $\hat{w}_H^*$ be the optimal high payment that maximizes $\pi\left(\hat{w}_H, a^*\left(\hat{w}_H\right)\right)$, being characterized by the first-order condition

$$v'(a^*)a^{*\prime}\left(\hat{w}_H^*\right) - \left[b + (1-2b)\,p(a^*)\right] - \left((1-2b)\,p'(a^*)a^{*\prime}\left(\hat{w}_H^*\right)\right)\hat{w}_H^* = 0. \tag{3}$$

Now we can analyze the possibility of a positive risk-incentive relationship. Implicit differentiation of (3) yields

$$\frac{\partial \hat{w}_H^*}{\partial b} = -\frac{d^2\pi\left(\hat{w}_H^*, a^*\left(\hat{w}_H^*\right)\right)/d\hat{w}_H^* db}{d^2\pi\left(\hat{w}_H^*, a^*\left(\hat{w}_H^*\right)\right)/d\hat{w}_H^{*2}}.$$

Since the second-order condition $d^2\pi\left(\hat{w}_H^*, a^*\left(\hat{w}_H^*\right)\right)/d\hat{w}_H^{*2} < 0$ is satisfied at the maximum we obtain

$$sign\left(\frac{\partial \hat{w}_H^*}{\partial b}\right) = sign\left(\frac{d^2\pi\left(\hat{w}_H^*, a^*\left(\hat{w}_H^*\right)\right)}{d\hat{w}_H^* db}\right).$$

The sign of this derivative depends on whether the marginal returns or the marginal costs from increasing incentives react more strongly to increased risk. Computing $d^2\pi\left(\hat{w}_H^*, a^*\left(\hat{w}_H^*\right)\right)/d\hat{w}_H^* db$ and evaluating at $b = 0$ gives the following result:

Proposition 5 *If*

$$\left(v''(a^*) - p''(a^*)\hat{w}_H^*\right)a^{*\prime}\left(\hat{w}_H^*\right)\left.\frac{\partial a^*}{\partial b}\right|_{b=0} + \left(v'(a^*) - p'(a^*)\hat{w}_H^*\right)\left.\frac{\partial a^{*\prime}\left(\hat{w}_H^*\right)}{\partial b}\right|_{b=0}$$
$$-\left(1 - 2p(a^*)\right) + \left(2a^{*\prime}\left(\hat{w}_H^*\right)\hat{w}_H^* - \left.\frac{\partial a^*}{\partial b}\right|_{b=0}\right)p'(a^*) > 0,$$

the optimal contract will exhibit a positive risk-incentive relationship.

The inequality of Proposition 5 consists of four expressions. The first one describes how marginal net profits react to an increase in risk and a subsequent adjustment of incentives. A risk increase leads to a reduction of effort for a given value of the high payment (i.e., $\partial a^*/\partial b|_{b=0} < 0$). Since $v\left(a\right)$ is concave, reducing a is favorable for creating additional incentives (i.e., for $a^{*\prime}\left(\hat{w}_H^*\right)$) because now incentives are induced at a higher productivity level. However, $p\left(a\right)$ is also concave so that increasing incentives at a lower effort level makes implementation costs increase more steeply as well. The interplay of these two effects determines the sign of the first expression.

The sign of the second expression in the inequality is identical to the sign of the mixed derivative $\left.\frac{\partial a^{*\prime}\left(\hat{w}_H^*\right)}{\partial b}\right|_{b=0}$. We know that marginal net profits from increased effort are positive since the first-order condition (3) can be rewritten as

$$v'(a^*) - p'(a^*)\hat{w}_H^* = \frac{p(a^*)}{a^{*\prime}\left(\hat{w}_H^*\right)}$$

for $b = 0$. However, the mixed derivative $\left.\frac{\partial a^{*\prime}\left(\hat{w}_H^*\right)}{\partial b}\right|_{b=0}$ may be either positive or negative. It measures how marginal incentives $a^{*\prime}\left(\hat{w}_H^*\right)$ react to increased risk. On the one hand, the agent will be discouraged if the performance signal becomes less informative. On the other hand, marginal incentives become more effective since the probability function $p\left(a\right)$ becomes steeper with decreased a due to concavity.

The third expression, $-\left(1 - 2p(a^*)\right)$, exactly measures the effect that we already know from the discussion of risk neutrality in Proposition 4. There, $p(a^*) > 1/2$ guarantees the existence of a positive risk-incentive relationship. The inequality points out that adding risk aversion automatically adds the traditional risk-incentive trade-off which makes the analysis more complicated.

The last expression in the inequality directly contrasts the positive incentive effect of an increased high payment (i.e., $a^{*\prime}\left(\hat{w}_H^*\right)$) with the negative discouragement effect of increased risk (i.e., $\partial a^*/\partial b|_{b=0}$). If the first effect dominates the second one so that effort reacts stronger to monetary incentives than to increased risk, the fourth expression in the inequality of Proposition 5 will be positive.

Note that the last two expressions in the inequality can be rewritten as

$$2a^{*\prime}\left(\hat{w}_H^*\right)\hat{w}_H^* p'(a^*) - \left[1 - 2p(a^*) + p'(a^*)\left.\frac{\partial a^*}{\partial b}\right|_{b=0}\right]. \tag{4}$$

The first term is unambiguously positive. The term in square brackets corresponds to the reaction of the agent's rent to increased incentives and risk. The rent is defined as

$$R(\hat{w}_H^*) := [b + (1-2b)p(a^*)]u(\hat{w}_H^*) - c(a^*)$$

with a^* depending on $\hat{w}_H^*$ and b according to equations (1) and (2). Applying the envelope theorem yields

$$\frac{dR}{d\hat{w}_H^*} = \frac{\partial R}{\partial \hat{w}_H^*} = [b + (1-2b)p(a^*)]u'(\hat{w}_H^*) > 0.$$

Hence, as it is well-known from the textbook model with limited liability, increasing incentives for a given risk will increase the agent's rent. The derivative with respect to b at $b = 0$ reads as

$$\left[1 - 2p(a^*) + p'(a^*)\left.\frac{\partial a^*}{\partial b}\right|_{b=0}\right]u'(\hat{w}_H).$$

The expression in square brackets is identical to the one in (4). Thus, if the increase of the rent due to higher-powered incentives decreases in risk, this will strongly favor a positive risk-incentive relationship.

To sum up, a higher risk has favorable effects on increasing incentives which may become more effective and less costly. If these positive effects dominate the negative ones in form of a higher wage payment $\hat{w}_H$ and a less informative performance signal, there will be a positive relationship between risk and incentives under limited liability. Note that the agent's limited liability is crucial for our findings as it makes the creation of incentives relatively cheap for the principal: The individual rationality constraint is non-binding at the optimum. Hence, increasing incentives via $\hat{w}_H$ only raises the principal's incentive costs in terms of expected money payment. However, the principal can disregard the steeply increasing effort costs $c(a)$, which only decrease the agent's rent.

5 An Example

Proposition 5 offers a sufficient condition for a positive risk-incentive relationship. In this section, we illustrate that a positive risk-incentive relationship may well result from a textbook-like specification of the model. To that purpose, we use a very simple parameterized version of our model with $p(a) = a^{\frac{1}{2}}$, $v(a) = 2a^{\frac{1}{2}}$, $u(\hat{w}_H) = 2(\hat{w}_H)^{\frac{1}{2}}$, and $c(a) = \frac{1}{2}a^2$.

The incentive constraint for this parameterized version is given by equation (IC'):

$$(1-2b)\, p'(a) u(\hat{w}_H) = c'(a) \Leftrightarrow (1-2b)^{\frac{1}{3}} (\hat{w}_H)^{\frac{1}{6}} = a^{\frac{1}{2}}.$$

We already now that the optimal low payment is zero since the agent is protected by limited liability and that we do not have to care for the individual rationality constraint, which is non-binding under the optimal contract. Hence, we only have to solve for the optimal value of the high payment, $\hat{w}_H^*$. Inserting the above incentive constraint into the principal's objective function

$$\pi(\hat{w}_H) = v(a) - [b + (1-2b)\, p(a)]\, \hat{w}_H$$

leads to expected net profits, which are described by the strictly concave function

$$\pi(\hat{w}_H) = 2(1-2b)^{\frac{1}{3}} (\hat{w}_H)^{\frac{1}{6}} - \left(b + (1-2b)^{\frac{4}{3}} (\hat{w}_H)^{\frac{1}{6}}\right) \hat{w}_H.$$

The first-order condition yields

$$\pi'(\hat{w}_H) = -\left(b + \frac{(7\hat{w}_H(1-2b) - 2)(1-2b)^{\frac{1}{3}}}{6\hat{w}_H^{\frac{5}{6}}}\right) = 0. \quad (5)$$

Using $b = 0$ and solving for $\hat{w}_H$ gives the optimal high payment $\hat{w}_H^* = \frac{2}{7}$.[7] In the general model of Section 4, we had to calculate $d^2\pi\left(\hat{w}_H^*, a^*\left(\hat{w}_H^*\right)\right)/d\hat{w}_H^* db$ and evaluate at $b = 0$. If the outcome is positive, we have a positive risk-incentive relationship. For the parameterized version of this section, there will be a positive risk-incentive relationship if

$$\left.\frac{\partial\pi'\left(\hat{w}_H^*\right)}{\partial b}\right|_{b=0} > 0.$$

Inserting $\hat{w}_H^* = \frac{2}{7}$ into $\pi'\left(\hat{w}_H\right)$ described by (5) and differentiating with respect to b at $b = 0$ leads to

$$\left.\frac{\partial\pi'\left(\hat{w}_H^*\right)}{\partial b}\right|_{b=0} = -1 - \left.\frac{\left(4 - 56\hat{w}_H^*\left(1-2b\right)\right)\left(1-2b\right)^{\frac{1}{3}}}{18\hat{w}_H^{*\frac{5}{6}}\left(1-2b\right)}\right|_{b=0} = 0.89365 > 0.$$

Hence, in the given parameterized setting implementation costs do not increase too steeply so that it pays for the principal to react to increased risk by choosing higher-powered incentives.

6 Conclusion

Given a risk averse agent, providing incentives is costly for the principal since incentive-compatible payment leads to a positive risk premium which usually increases in the magnitude of the exogenous risk. For this reason, the standard principal-agent moral-hazard model claims a negative relationship between risk and optimal incentives. Several empirical findings have challenged this traditional view. By combining risk aversion and limited liability – the two standard contractual problems given a verifiable performance signal – we obtain an explanation for a *positive* risk-incentive relationship without relying on additional assumptions from outside the textbook model. Empirical findings may therefore be well in line with the standard specification of the principal-agent moral hazard model.

[7] Note that (at $b = 0$) we have $p\left(a^*\right) = \left(a^*\right)^{\frac{1}{2}} = \left(\hat{w}_H^*\right)^{\frac{1}{6}} = 0.81156 < 1$.

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