Corrigendum to "Correlated information, mechanism design and informational rents" [J. Econ. Theory 123 (2005) 210–217]*

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Abstract

We amend an error in [S. Parreiras, Correlated information, mechanism design and informational rents, J. of Econ. Theory 123 (2005) 210–217]. Consequently, it is in general not possible to reinterpret a mechanism design model that violates the spanning condition of Crémer and McLean [Econometrica 56 (1988) 1247–1258] as one in which agents hold private information about the informativeness of their signals about other agents' types. Instead, such an interpretation is warranted only when the weights used to span an agent's set of beliefs stand in a singular relation with the prior type distribution that is known as an alternative characterization of Blackwell dominance.

Keywords: Mechanism design, surplus extraction, correlated information, Blackwell's theorem

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1 Introduction

Parreiras [4] studies a mechanism design problem where the agents' private information is correlated, but the set of agents' beliefs fails to satisfy Crémer and McLean's [2] spanning condition, which says that no agent type's belief is in the convex hull of the other types' beliefs. In particular, Parreiras considers a finite Bayesian game $\Gamma = (N, (C_i)_{i\in N}, (T_i)_{i\in N}, (p_i)_{i\in N},\ (u_i)_{i\in N})$, where N is the set of players, and for each $i \in N$, C_i is the set of possible actions, T_i is the set of types, p_i is player i's belief over the types of other players conditional on i's own type. He makes the intriguing claim that one may reinterpret any such game, in which beliefs violate the spanning condition, as a model in which an agent's private information consists of two parts: knowledge about his type and knowledge about the precision of his beliefs about the other agents' types. Thus, one may view agents as receiving two private signals. First, each agent i receives a binary signal θ with values in $\{\underline{\theta}, \overline{\theta}\}$ that tells him how informative his second private signal is about the types t_{-i} of other agents $-i$. Second, agent i receives a signal X^{θ} that, together with the first signal θ, reveals his type t_i and induces the belief p_i over the types of other players. The signal $X^{\overline{\theta}}$ is more informative than the signal $X^{\underline{\theta}}$ in the sense of Blackwell [1]. For this interpretation to be meaningful, the first signal θ should be stochastically independent of the types t_{-i} of other agents.[1]

We show that Parreiras' result rests on a hidden assumption which makes his offered interpretation applicable to only a very restricted class of models. We first illustrate in the simplest case where agent i has three types $T_i \equiv \{t_i^1, t_i^2, t_i^3\}$ that stochastic independence alone implies that Parreiras' interpretation is valid only for sets of beliefs $(p_i(\cdot|t_i))_{t_i}$, for which the spanning weights used to generate the dependent belief are uniquely pinned down by the (marginal) prior type distribution, $p(t_i)$. Second, we amend the gap in the proof of Parreiras' Proposition 2 and show that, in general, the result is only true when the spanning weights stand in a singular relation with the prior type distribution.

2 Three Types

First suppose that $T_i \equiv \{t_i^1, t_i^2, t_i^3\}$ and that the spanning condition on the beliefs p_i fails. This means that the belief of some agent type, say $p_i(\cdot|t_i^2)$, is a (strict) convex combination of the

[1] See Footnote 2 in Parreiras.

other two agent type beliefs, $p_i(\cdot|t_i^1)$ and $p_i(\cdot|t_i^3)$. Formally, there exists $\lambda \in (0,1)$ such that

$$p_i(t_{-i}|t_i^2) = \lambda p_i(t_{-i}|t_i^1) + (1-\lambda)p_i(t_{-i}|t_i^3). \tag{C1}$$

Following Parreiras, let $\mathcal{T}_i \equiv \{t_i^1, t_i^3\}$ denote the maximal subset of T_i so that the beliefs of any type $t_i \in \mathcal{T}_i$ about t_{-i} cannot be expressed as a convex combination of the other types. In Parreiras' the construction, agent i first privately observes either $\underline{\theta}$ or $\overline{\theta}$. The signal realization $\underline{\theta}$ reveals to agent i that $t_i \in T_i \backslash \mathcal{T}_i = \{t_i^2\}$, whereas signal realization $\overline{\theta}$ reveals to agent i that $t_i \in \mathcal{T}_i = \{t_i^1, t_i^3\}$.

Now, stochastic independence of θ and t_{-i} implies that the agent's beliefs over t_{-i} when the agent observes $\underline{\theta}$ equal his beliefs over t_{-i} when he observes $\overline{\theta}$. In particular, if the agent observes $\underline{\theta}$, he is sure that he is of type t_i^2. Hence, after observing $\underline{\theta}$, his belief about t_{-i} is

$$\begin{aligned} p_i(t_{-i}|\underline{\theta}) &= p_i(t_{-i}|t_i^2) \\ &= \lambda p_i(t_{-i}|t_i^1) + (1-\lambda)p_i(t_{-i}|t_i^3), \end{aligned} \tag{1}$$

where the second equality follows from (C1).

Instead, if agent i observes $\overline{\theta}$, he learns $t_i \in \mathcal{T}_i$ so that he updates the probability that he is of type $t_i \in \mathcal{T}_i$ from $p(t_i)$ to $p(t_i)/Pr(\mathcal{T}_i)$. Consequently his updated belief about t_{-i} is

$$p_i(t_{-i}|\overline{\theta}) = \frac{p(t_i^1)}{Pr(\mathcal{T}_i)} p_i(t_{-i}|t_i^1) + \frac{p(t_i^3)}{Pr(\mathcal{T}_i)} p_i(t_{-i}|t_i^3). \tag{2}$$

Comparison of (1) and (2) reveals that $p_i(t_{-i}|\underline{\theta})$ and $p_i(t_{-i}|\overline{\theta})$ coincide only if $\lambda = p(t_i^1)/Pr(\mathcal{T}_i)$. Hence, only in this case an interpretation in the spirit of Parreiras is possible. Otherwise, his Proposition 2 is violated, because stochastic independence fails.

3 General Case

Our second observation is that also in the general case with any number of types, Parreiras' construction implies a close relation between the prior $p(t_i)$ and the weights used to span interior beliefs in the convex hull of the set of the agent's beliefs. In the "only if"-part of the proof of Proposition 2, Parreiras defines $\mathcal{T}_i$ as the set of "extreme" types x of agent i whose beliefs $p_i(\cdot \mid x)$ are the extreme points of the convex hull of $\{p_i(\cdot \mid t_i) \mid t_i \in T_i\}$. Since the spanning condition is violated by assumption, the set of "interior" types $T_i \setminus \mathcal{T}_i$ is non–empty and for any

$y \in T_i \setminus \mathcal{T}_i$, there are weights $\lambda_{xy} \in [0, 1]$, $x \in \mathcal{T}_i$, $\sum_{x \in \mathcal{T}_i} \lambda_{xy} = 1$ so that

$$p_i(\cdot \mid y) = \sum_{x \in \mathcal{T}_i} \lambda_{xy} p_i(\cdot \mid x). \tag{C1'}$$

Parreiras then constructs a signal $X^{\bar{\theta}}$ with realizations x in $\mathcal{T}_i$, a signal $X^{\underline{\theta}}$ with realizations y in $T_i \setminus \mathcal{T}_i$, and garbling probabilities $B(x, y)$ that imply the Blackwell ranking between $X^{\bar{\theta}}$ and $X^{\underline{\theta}}$. Since the garbling probabilities add to one, we need

$$\sum_{y \in T_i \setminus \mathcal{T}_i} B_{(x,y)} = 1. \tag{C2'}$$

Parreiras' construction yields[2]

$$B_{(x,y)} = \lambda_{xy} \frac{p(y)}{p(x)} \frac{Pr(\mathcal{T}_i)}{(1 - Pr(\mathcal{T}_i))}.$$

This, together with (C2') reveals that the weights used to generate interior beliefs have to satisfy the condition

$$\frac{p(x)}{Pr(\mathcal{T}_i)} = \sum_{y \in T_i \setminus \mathcal{T}_i} \lambda_{xy} \frac{p(y)}{1 - Pr(\mathcal{T}_i)}. \tag{3}$$

The "only if"–part of Parreiras' Proposition 2 has, therefore, to be amended as follows.

Proposition 2 *(amended) If condition (C1') and, in addition, condition (3) hold, then it is possible to write the type set of agent i as $T_i = \{x_1, \ldots, x_n\} \times \{\underline{\theta}, \bar{\theta}\}$ and find signals $X^{\underline{\theta}}$ and $X^{\bar{\theta}}$ so that $X^{\bar{\theta}}$ Blackwell–dominates $X^{\underline{\theta}}$.*

Hence, for Parreiras' insight to hold, the additional condition (3) is needed. Because in the three types case the set $T_i \setminus \mathcal{T}_i$ is a singleton and, hence, $p(y) = 1 - Pr(\mathcal{T}_i)$, condition (3) boils down to the condition $\lambda_x = p(x)/Pr(\mathcal{T}_i)$ that we derived in the previous section.

To give an intuitive interpretation of condition (3), observe that $p(x)/Pr(\mathcal{T}_i)$ is the probability that agent i is of type x, conditional on being an extreme type. Similarly, $p(y)/(1 - Pr(\mathcal{T}_i))$ is the probability that agent i is of type y, conditional on being an interior type. Thus, condition (3) says that the conditional distribution of a type x conditional on being extreme corresponds to the distribution of the compound lottery which, first, plays out an interior type y and then, conditional on y, plays out x according to the "transition probabilities" λ_{xy}.

[2] Because, unfortunately, there are some typos in the expression of $B_{(x,y)}$ in Parreiras [4, p. 215], we provide an explicit derivation of $B_{(x,y)}$ in the appendix.

The additional condition (3) limits the applicability of the proposition and raises the question how restrictive the new condition (3) over and above condition (C1') actually is. As already shown for the three type case, we now argue that a mechanism design setup that satisfies (C1') generically fails to satisfy the new condition (3). To develop this argument, recall that the underlying primitive that determines whether (3) and (C1') are satisfied is the joint probability distribution on the type space $T_i \times T_{-i}$. Our claim is that if one considers a joint distribution which jointly satisfies (C1') and (3) then there always exists a perturbed probability distribution close to the original one which still satisfies condition (C1') yet violates (3). To see this, fix two extreme types $x', x'' \in \mathcal{T}_i$ and, starting from the joint distribution Pr, define for some $\delta \approx 0$ the perturbed probability as

$$\begin{aligned} Pr_\delta(x', t_{-i}) &= Pr(x', t_{-i}) \cdot (1+\delta), \\ Pr_\delta(x'', t_{-i}) &= Pr(x'', t_{-i}) \cdot \left(1 + \frac{p(x')}{p(x'')}\delta\right). \end{aligned}$$

for all t_{-i}, while leaving the probability of types (t_i, t_{-i}) for all $t_i \notin \{x', x''\}$ and t_{-i} unchanged. By construction, the perturbed probability distribution Pr_δ is well–defined for δ close enough to zero and converges to the original one as δ approaches zero. Because the probabilities of (x', t_{-i}) and (x'', t_{-i}) are scaled uniformly for all t_{-i}, the beliefs induced by x' and x'' are the same under the original and the perturbed probability distribution. By construction, the beliefs induced by $t_i \notin \{x', x''\}$ coincide trivially for the original and the perturbed probability distribution. As a result, the perturbation satisfies condition (C1') if the original distribution does so. However, the perturbed probability that x' respectively x'' occurs differs from the original probability, whereas the perturbation does not affect the probability of an interior type $y \in T_i \setminus \mathcal{T}_i$. Consequently, the perturbation only changes the left hand side of (3). Hence, if the original distribution satisfies (3), any perturbation Pr_δ violates it. This completes our argument that, given a mechanism design model for which the spanning condition is violated, condition (3) is singular in the sense that such a model rarely satisfies it.[3]

Finally, we can shed more light on Proposition 2 by stating conditions (C1') and (3) in terms of the signals $X^{\underline{\theta}}$ and $X^{\bar{\theta}}$. Let $p(\cdot \mid X^\theta)$ be the random variable given by the posterior belief over T_{-i} induced when observing X^θ. By construction of $X^{\bar{\theta}}$ and $X^{\underline{\theta}}$, (C1') and (3) are

[3]The set of probability measures that satisfy (C1') and violate (3) is dense and open within the set of probability measures that satisfy (C1').

equivalent to

$$
\begin{aligned}
p(\cdot \mid X^{\underline{\theta}} = y) &= \sum_{x\in\mathcal{T}_i} \lambda_{xy} p(\cdot \mid X^{\bar{\theta}} = x) \quad \forall y \in T_i \setminus \mathcal{T}_i, \\
Pr(X^{\bar{\theta}} = x) &= \sum_{y\in T_i\setminus\mathcal{T}_i} \lambda_{xy} Pr(X^{\underline{\theta}} = y) \quad \forall x \in \mathcal{T}_i.
\end{aligned}
$$

Observe that these two conditions mean that the posterior belief on T_{-i} induced by $X^{\bar{\theta}}$, understood as a random variable, is a mean preserving spread of the posterior belief on T_{-i} induced by $X^{\underline{\theta}}$. In fact, by condition (5) of Theorem 12.2.2 in Blackwell and Girshick [2, p. 328], this is *equivalent* to the Blackwell dominance of $X^{\bar{\theta}}$ over $X^{\underline{\theta}}$. Thus, Proposition 2 (amended) is essentially an expression of Blackwell and Girshick's fundamental equivalence.

References

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[2] D. Blackwell, M. A. Girshick, Theory of Games And Statistical Decisions, John Wiley and Sons, New York, 1954.

[3] J. Crémer, R. McLean, Full extraction of the surplus in bayesian, dominant strategy auctions, Econometrica 56 (1988), 1247-1258.

[4] S. Parreiras, Correlated information, mechanism design and informational rents, J. Econ. Theory 123 (2005), 210-217.

Appendix

Construction of garbling weights $B_{(x,y)}$ We follow Parreiras and define the signals $X^{\bar{\theta}}$ and $X^{\underline{\theta}}$ by setting

$$
Pr(X^{\bar{\theta}} = x, t_{-i}) = \begin{cases} 0 & if \quad x \notin \mathcal{T}_i \\ \frac{p(x,t_{-i})}{Pr(\mathcal{T}_i)} & if \quad x \in \mathcal{T}_i \end{cases}, \quad Pr(X^{\underline{\theta}} = y, t_{-i}) = \begin{cases} \frac{p(y,t_{-i})}{1-Pr(\mathcal{T}_i)} & if \quad y \notin \mathcal{T}_i \\ 0 & if \quad y \in \mathcal{T}_i \end{cases} \tag{4}
$$

It then follows

$$
\begin{aligned}
\frac{p(t_i = (y, \underline{\theta})|t_{-i})}{p(t_{i2} = \underline{\theta})} &= Pr(X^{\underline{\theta}} = y \mid t_{-i}) \\
&= \frac{Pr(X^{\underline{\theta}} = y, t_{-i})}{p(t_{-i})} \\
&= \frac{p(y, t_{-i})}{p(t_{-i})(1 - Pr(\mathcal{T}_i))} && \text{(by (4))} \\
&= \frac{p(y)}{p(t_{-i})(1 - Pr(\mathcal{T}_i))} p_i(t_{-i} \mid y) && \text{(by Bayes' rule)} \\
&= \sum_{x \in \mathcal{T}_i} \lambda_{xy} \frac{p(y)}{p(t_{-i})(1 - Pr(\mathcal{T}_i))} p_i(t_{-i} \mid x) && \text{(by (C1'))} \\
&= \sum_{x \in \mathcal{T}_i} \lambda_{xy} \frac{p(y)}{p(x)p(t_{-i})(1 - Pr(\mathcal{T}_i))} p(x, t_{-i}) && \text{(by Bayes' rule)} \\
&= \sum_{x \in \mathcal{T}_i} \lambda_{xy} \frac{p(y)Pr(\mathcal{T}_i)}{p(x)p(t_{-i})(1 - Pr(\mathcal{T}_i))} Pr(X^{\bar{\theta}} = x, t_{-i}) && \text{(by (4))} \\
&= \sum_{x \in \mathcal{T}_i} \lambda_{xy} \frac{p(y)Pr(\mathcal{T}_i)}{p(x)(1 - Pr(\mathcal{T}_i))} Pr(X^{\bar{\theta}} = x \mid t_{-i}) \\
&= \sum_{x \in suppX^{\bar{\theta}}} \lambda_{xy} \frac{p(y)Pr(\mathcal{T}_i)}{p(x)(1 - Pr(\mathcal{T}_i))} \frac{p(t_i = (x, \overline{\theta})|t_{-i})}{p(t_{i2} = \overline{\theta})}.
\end{aligned}
$$

Hence, Blackwell's garbling condition requires $B_{(x,y)} = \lambda_{xy} \frac{p(y)Pr(\mathcal{T}_i)}{p(x)(1-Pr(\mathcal{T}_i))}$.