A Stylized Model of the German UMTS Auction[1]

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Abstract

We analyze the recent German and Austrian UMTS license auctions via a model of an open, ascending, uniform-price auction with multi-unit-demand, incomplete information, bidder asymmetry, and market externalities. Our main result is that the German outcome, which involved ex-post spurious price increments, while absurd on first sight, can be characterized as an equilibrium outcome. The flexibility of the design probably allowed the government to extract a higher revenue, but it is not clear that this flexibility improved overall welfare.

JEL classification: D44, H21

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1. Introduction

This paper illustrates the possibility that rational bidders in an open ascending auction may suffer from ex-post regret because it turns out that they have paid significantly more than necessary. In the recent German UMTS spectrum auction (which had a flexible design allowing for up to 6 licenses), the two dominant incumbents Mannesmann Mobilfunk and Deutsche Telekom, attempted to push one of the potential entrants out of the market by continuing to bid on additional capacity that was not essential in order to obtain a license. The seventh bidder dropped out at a level of approximately Euro 5 bn per license but the final price was above Euro 8 bn, and the allocation of licenses was the same that would have been obtained if the winning bidders had reduced their demands at the lower level. Thus, the incumbents' predatory strategy was unsuccessful. Telekom officials later said: "The levels reached were insane." (Financial Times, 18.8.2000).

We offer here a formal analysis of the above phenomenon, based on a stylized model of the German UMTS auction which incorporates incomplete information, bidder asymmetry, and market externalities. Our central result is that, in the flexible design that was chosen by the German regulator[4], there exists an equilibrium where one of the incumbents tries to push the weakest potential entrant out of the market, but fails with positive probability. When preemption is unsuccessful, an allocation arises that could have been reached at a lower price level if the winning bidders had reduced their demand earlier. As a consequence, prices generated by the German design, conditional on the event that a six-player market occurs, can be higher than those obtained in a less flexible design in which

[4]In fact, Germany and Austria were the only European countries in which the number of licenses was determined endogenously.

the number of licenses is determined exogenously.

The so-called *exposure problem* in multi-object auctions with complementarities has received attention in the literature. For example, Cramton (1997) offers a simple example where, even under complete information, not allowing package bids[5] leads to inefficient non-participation. The example is as follows: there is one bidder with a car and a trailer, who values only two parking slots together at \$100, and another bidder with a car, who values just one slot at \$75. The only equilibrium in a simultaneous ascending auction without package bids is for the first bidder to drastically reduce demand by non-participation (since he needs to bid up to \$75 per slot in order to outbid the other player), and for the second bidder to place a minimum bid for one of the slots. In the presence of incomplete information, the first bidder will bid for the pair only if there is a sufficiently high probability that the second bidder has a low valuation. In contrast to most of the literature, the need to avoid exposure by reducing demand in our setting is a result of the various expected scenarios in the future UMTS market, which generate endogenous externalities[6] from additional entrants. The exposure problem can arise in auctions with externalities even with decreasing marginal valuations. As a consequence, compared to a defensive bidding strategy, a bidder can make losses with positive probability, and may regret his bidding strategy ex-post.

Regretful outcomes may occur for other reasons than from a flexible auction design. In a study for the Dutch Ministry of Finance, Bennett and Canoy (2000) identify a number of reasons for overbidding in spectrum auctions. Besides the non-equilibrium phenomenon of the "winner's curse", they refer to misaligned

[5] I.e., bids on a combination of items.

[6] Auction with externalities are studied in Jehiel, Moldovanu and Stacchetti (1996, 1999).

management incentives (such as fear of reputation loss or of a negative impact on analyst's evaluations), and to overly enthusiastic profitability perceptions. While we do not deny that these factors may have played a role in the European UMTS auctions, we want to clarify here that the auction format used in Germany and Austria included an additional risk that had to be faced even by perfectly rational bidders.

We see at least three reasons why policy makers should avoid designs that create an exposure problem: 1) Bidders' awareness of the involved risk may reduce the number of bidders, and thereby reduce competition. Prior to the German UMTS auction six eligible bidders withdrew from the contest, and the auction started with only seven bidders for a maximum of six licenses. Even worse, in the subsequent Austrian auction conducted in fall 2000 the outcome of the German mechanism was still fresh in memory. Only six bidders entered the auction that also used the German design. 2) The outcome of the auction (which may not be regret-free) necessarily leads to attempts to renegotiate the license fees or the license terms[7]. 3) Such a huge exposure as in the UMTS case leads to shareholder value destruction, and affects therefore the financial stability of the industry.

The auction formats used in the European UMTS auctions are currently and extensively studied, e.g., by Börgers and Dustmann (2001), Grimm, Riedel, and Wolfstetter (2001)[8], Jehiel and Moldovanu (2001a, c), Klemperer (2001), and

[7]After the auction, one of the winning bidders brought a suit against the government, accusing it of raising purposes on purpose (besides being the auctioneer, the government is also the majority owner of the largest incumbent). At present, the terms of the licenses are renegotiated, and it is very unlikely that 6 independent UMTS networks will exist.

[8]They consider a three-player four-stage game of incomplete information, in which the weakest entrant be one of two types. It is shown that under certain conditions, there is an equilibrium in this game in which one dominant incumbent tries predation, and another resigns. Grimm et al. (2001) argue that there is a free-rider problem between the two incumbents when a remaining frequency block can be purchased cheaply in the second stage of the auction. While

van Damme (2001). Engelbrecht-Wiggans and Kahn (1998) and Noussair (1995) analyze a more general uniform-price auction, a variant of which is used in this paper. Our analysis is also related to the strand of work following Gilbert and Newbery (1982), who stressed the natural asymmetry between incumbents and entrants in patent auctions.

The rest of the paper is structured as follows. In Section 2, we briefly describe the German design, and the realized outcomes in Austria and Germany. In Section 3 we describe the auction model. In Section 4, we derive an equilibrium, and discuss its features. In Section 5, we survey several efficiency considerations. Section 6 concludes. Appendix A provides details of the auction model, and appendix B contains technical proofs.

2. Brief description of the German and Austrian UMTS auctions

The auction had two stages, each conducted in an open, ascending, simultaneous multiple round format[9]. In the first stage, 12 identical frequency blocks of 2x5 MHz (Megahertz) each[10] were auctioned. Two blocks were needed to get a license, but bidders were allowed to buy at most three blocks. Assuming that all blocks get sold at this stage, the number of licenses could vary between four and six. At each round, a firm had to place bids on at least two and at most on three blocks. If in a certain round a firm placed bids on two blocks, it was constrained to place bids on only two blocks (or exit) in all later rounds.

some aspects of their analysis are related, it is not clear with discrete types why the incumbent can in fact end up in regret. After all, if the model is a proxy for an open ascending auction, then the incumbent will be able to observe whether the entrant has a high valuation already if the level of bids exceeds the entrant's low valuation.

[9]For a complete description of the design, see the official document by the Regulierungsbehörde für Telekommunikation und Post (2000).

[10]This spectrum was in the so-called "paired" band.

Only the bidders who obtained a license in the first stage were entitled to participate in the second stage, where five frequency blocks of 5 MHz each[11] were auctioned. A bidder was allowed to buy between zero and five such blocks. In addition, the second stage offered for sale the blocks in the paired band which were not sold in the first stage (if any). Each bidder was allowed to buy at most one such block.

There were 7 bidders in the German auction, after 6 other potential bidders withdrew from the auction. Table I summarizes a few facts about the bidding consortia. The first stage ended on August 17, 2000, after 3 weeks and 173 rounds of bidding, and resulted in 6 licenses being awarded. The licensed firms were E-Plus Hutchison, Group 3G, Mannesmann Mobilfunk, MobilCom Multimedia, T-Mobil, and VIAG Interkom. The revenue was approximately Euro 50 bn. Each licensed firm acquired 2 blocks of paired spectrum, paying approximately Euro 8.4 bn. An interesting phenomenon occurred after one of the potential entrants, Debitel, left the auction in round 125, after the price level had reached Euro 2.5 bn per block. Since 6 firms were left bidding for a maximum of 6 licenses, the auction could have stopped immediately. Instead, the remaining firms (and in particular the two large incumbents) continued bidding in order to acquire more capacity. But no other firm was willing to quit, and as a result, bidding stopped in round 173 after Deutsche Telekom and Mannesmann Mobilfunk decided to confine themselves with a small license. Compared to round 125, there was **no change** in the physical allocation, but, collectively, firms paid Euro 20 bn more[12].

- Table I here -

[11] This spectrum was in the so-called "unpaired" band.

[12] For more thorough discussion of the design and the possible outcomes, see Jehiel and Moldovanu (2001) or Grimm, Riedel, and Wolfstetter (2001). For an industry evaluation prior to the German auction, see Deutsche Bank (2000).

The Austrian auction opened on November 2, 2000, and followed the German design. The following companies participated: Connect, Hutchison 3G, Mannesmann 3G, max.mobil., Mobilkom, and 3G Mobile (Telefonica). The minimum bid for a frequency block in the paired band was Euro 50 m. As there were exactly 6 bidders for a maximum of 6 licenses, the license auction could have, in principle, ended immediately at the reserve price. Nevertheless, bidding continued for further 16 rounds, before stopping with 6 licensed firms, each paying on average about Euro 118 m per license.

3. The model

We focus on the first stage of the German design. We later discuss later why the second stage does not affect the arguments. We will also abstract from the fact that the German auction must be considered as a part of a global process, in which international telecom firms fight over the European market[13]. The entire European dimension is of major relevance, and more research on this topic is certainly desirable[14].

There are 12 frequency blocks and $n = 6$ bidders[15]. Each bidder i has valuations of the form $v_m^i(k)$, where m is the number of frequency blocks, and k is the number of players in the UMTS market. These valuations are assumed to be increasing in m and decreasing in k. We assume that bidders can be ordered according to their valuations, i.e., that

$$v_m^1(k) > v_m^2(k) > ... > v_m^6(k) > 0 \tag{1}$$

[13]van Damme (2001) suggests that the high prices in Germany resulted from a global struggle mainly between KPN and Telefonica.

[14]Many other aspects remain outside the formal model. E.g., bidders likely cared about their relative performance; budget constraints might have played an important role.

[15]In Germany, the actual auction started with seven bidders, but the "real action" started only when six bidders were left (while behavior till that stage is more or less standard).

for all $m \in \{2,3\}$ and all $k \in \{4,5,6\}$.

Bidder $i = 6$ has an ex-ante unknown valuation

$$v := v_2^6(6) \in [\underline{v}, \overline{v}] \tag{2}$$

which is private information to him. The distribution of v is assumed to have full support on $[\underline{v}, \overline{v}]$. The corresponding cumulative distribution function is denoted by $F(v)$, and is assumed to be differentiable.

To focus the analysis on the interesting case where the dominant incumbent fights the weakest entrant, we assume that it is ex-ante not clear whether the dominant incumbent's per-block valuation for a large license in a five player market is below or above the weakest entrant's per-block valuation of a small license in a six-player market, i.e., we assume

$$\frac{\underline{v}}{2} < \frac{v_3^1(5)}{3} < \frac{\overline{v}}{2}. \tag{3}$$

We also assume that for all bidders but bidder 1, the value of the third frequency block is not too large, i.e., that

$$\frac{v_3^2(4)}{3} < \frac{\underline{v}}{2}. \tag{4}$$

Some important consequences of the above conditions on the valuations are illustrated in Figure 1. This diagram will be especially useful in determining the price and allocation that will result from unilateral deviations of profit-maximizing firms. - Figure 1 here -

Bidders could bid for either two or three blocks, under the restriction that they may not increase the number of requested units during the auction ("activity

rule")[16]. To capture these strategic possibilities[17], we summarize i's strategy by a pair $b^i = (b_3^i, b_2^i)$, such that i bids up to b_3^i for three blocks, and up to b_2^i for two blocks, where

$$0 \leq b_3^i \leq b_2^i. \tag{5}$$

The frequency blocks are assigned to the 12 highest unit bids under the provision that no bidder obtains only one block[18]. The *final price* p^* is equal to the highest losing bid. The precise mechanics that determine price and allocation are explained in Appendix A.

4. Analysis

Proposition 1 (equilibrium). *The following strategy profile constitutes an equilibrium in the stylized UMTS auction*

$$b^{*,i} = \begin{cases} (\beta^*, \dfrac{v_2^1(6)}{2}) & i = 1 \\ (\dfrac{v_3^i(5)}{3}, \dfrac{v_2^i(6)}{2}) & i = 2, ..., 6 \end{cases} \tag{6}$$

where, depending on the exogenous parameters, the dominant incumbent chooses either $\beta^* = v_3^2(5)/3$ *("accommodate"), or* $\beta^* \in [\underline{v}/2, v_3^1(5)/3]$ *("fight").*

Proof. See Appendix B. □

The strategy profile described above allows for two basic ways in which the dominant incumbent (bidder 1) behaves in equilibrium. Either, he reduces demand to two units , or he tries to push the weakest bidder out of the market. Note that the equilibrium is not an artifact of our sealed-bid specification. Indeed,

[16]The introduction of a minimum bid is straightforward, and therefore omitted.

[17]It will become clear that the resulting restrictions are not severe because, given our informational assumptions, no firm observes valuable information before the end of the auction.

[18]In case of ties we assume a uniformly randomized tie-breaking rule.

because there is uncertainty about $v_2^6(6)$ only, no useful information is revealed until the end of the auction. More precisely, there is no value of conditioning one's bid on the observed equilibrium behavior of the weakest bidder because his only information-bearing action is exiting the auction. However, in equilibrium, this action does not occur unless it ends the game. Note also that the introduction of a second stage, as in the actual auction format, is not likely to affect the structure of the above equilibrium[19].

The payoff consequences of accommodating vs. fighting are as follows. If the dominant incumbent reduces his demand early, then all six bidders obtain two frequency blocks for a unit price of

$$p_0 := \frac{v_3^2(5)}{3}, \tag{7}$$

and bidder 1's expected payoff is correspondingly

$$\underline{U}_1 = v_2^1(6) - 2\frac{v_3^2(5)}{3}. \tag{8}$$

If, however, he fights the weakest entrant by bidding for three blocks up to $\beta \in [\underline{v}/2, v_3^1(5)/3]$, his expected payoff is

$$U_1(\beta) = \int_{\underline{v}}^{2\beta} \{v_3^1(5) - \frac{3v}{2}\} dF(v) + \int_{2\beta}^{\overline{v}} \{v_2^1(6) - 2\beta\} dF(v). \tag{9}$$

Indeed, if $v/2 < \beta$, the incumbent wins a large license in a five-player market, and pays $v/2$ per unit, at which the entrant gives up. On the other hand, if $v/2 > \beta$, the incumbent wins only a small license in a six-player market, and pays a per-unit price at which he gives up bidding for three blocks.

[19] Such a stage is strategically most relevant in the case where one block is not sold in the first period. In our equilibrium, this is the result of a successful preemption by the dominant incumbent. The left-over block is then auctioned among all winners of the first stage. Applying backward induction, rational bidders will have formed beliefs about the expected value in such a setting.

Proposition 2 (exposure). *The expected utility difference between fighting and accommodating the entrant is*

$$\begin{aligned} U_1(\beta) - \underline{U}_1 &\equiv \Delta U_1 = pr(\frac{v}{2} < \beta) \left\{ v_3^1(5) - v_2^1(6) - \frac{v_3^2(5)}{3} - 3\Delta^w \right\} \\ &+ \quad pr(\frac{v}{2} \geq \beta) \left\{ -2\Delta^l \right\}, \end{aligned} \tag{10}$$

where

$$\Delta^w \; := \; = E[\frac{v}{2} | \frac{v}{2} < \beta] - \frac{v_3^2(5)}{3} \tag{11}$$

$$\Delta^l \; := \; = \beta - \frac{v_3^2(5)}{3} \tag{12}$$

are the expected increments in the per-block prices in case the incumbent wins and loses the battle, respectively.

Proof. Immediate from (8) and (9). □

Equation (10) captures the dominant incumbent's dilemma. If the incumbent wins, i.e., if $v < \beta$, then he realizes scale effects and ensures oligopoly gains. He has to pay $\frac{v_3^2(5)}{3}$ for the additional block, and the price of preemption Δ^w for all three frequency block. This is a desirable outcome for the incumbent. However, if the entrant wins, the incumbent pays an additional increment Δ^l for each of the two frequency blocks he would have obtained anyway! This illustrates the exposure problem inherent in the German auction design. We now derive the optimal bidding strategy for bidder 1.

Proposition 3 (bid shading). *If the dominant incumbent fights the weakest entrant, then the optimal bid β^* is given by*

$$v_3^1(5) - v_2^1(6) = \beta^* + \frac{1 - F(2\beta^*)}{f(2\beta^*)}. \tag{13}$$

Moreover, to mitigate the exposure problem, the dominant incumbent always shades his bid for the third block, i.e., $\beta^* < v_3^1(5)/3$.

Proof. See the Appendix B. □

The above bid shading is an instance of a more general phenomenon of bid shading in uniform price auctions (see for example Ausubel and Cramton, 1998). The incumbent lowers his demand for the third block because a higher bid for that block increases the price for the first two blocks (see Proposition 2).

Proposition 3 suggests the following comparative statics result in a given setting in which the dominant incumbent fights the weakest entrant. Consider a more aggressive weakest entrant in the sense that the distribution function of his valuation is $G(v)$, where $G(v)$ dominates $F(v)$ in the hazard-rate order[20]. Then, from (13), the equilibrium bid of the dominant incumbent against such an entrant is lower, and he may even choose to accommodate the entrant without fighting. As a consequence, the entrant would like to appear strong, in order to de-motivate the incumbent, which leads to lower prices and a higher likelihood of winning for the entrant. On the other hand, the auctioneer has incentives to make entrants look weak, in order to encourage a battle.

The above expression for the optimal bid shows that the equilibrium bid depends only on the marginal valuation for a third block, and on the advantage of operating in a five-player market. As higher prices must be paid for all three blocks, this opens up the possibility of *regret*. We say that regret occurs whenever the

[20] I.e., for all v, we have

$$\frac{F'(v)}{1-F(v)} \leq \frac{G'(v)}{1-G(v)}.$$

This stochastic order implies the standard first-order stochastic dominance.

bidding strategies do not form an ex-post equilibrium, i.e., the dominant incumbent could have reached the same allocation at a lower payment (by not fighting the entrant), under the assumption that he knows the outcome of the auction.

Proposition 4 (regret). *There exist parameter values under which the dominant incumbent fights the weakest entrant in equilibrium, but fails with positive probability.*

Proof. For $\underline{v} \leq v \leq \overline{v}$, let $F(v) = 1 - (\frac{\overline{v} - v}{\overline{v} - \underline{v}})^4$. We determine equilibrium strategies under the condition that the incumbent participates with a demand of three blocks. From Proposition 1, we know that the weakest entrant bids up to $v/2$. Proposition 3 predicts that the fighting incumbent will bid up to

$$\beta^* = 2\{v_3^1(5) - v_2^1(6)\} - \frac{\overline{v}}{2}. \tag{14}$$

The dominant incumbent's utility can be higher from fighting, as the following argument shows. Choose $v_3^2(5)$ close to $\overline{v}$, and note that

$$\frac{\partial U_1}{\partial \beta}(\frac{v}{2}) = \frac{8}{\overline{v} - v}\{v_3^1(5) - v_2^1(6) - \beta\} - 2 \tag{15}$$

$$= 4\frac{\overline{v}/2 - \beta^*}{\overline{v} - v} > 0. \tag{16}$$

Thus, accommodating is always suboptimal. Note now that whenever $v/2 > \beta^*$, the entrant obtains a license, and the incumbent regrets his bidding strategy[21].

□

[21] This logic immediately extends to the more general case where there is a sufficiently high probability for a valuation v close to, but still above $\underline{v}$. Then demand reduction effects are not very strong, and bidder 1 actively bids for the third frequency block, at least when the entrant's minimum per-block valuation $\underline{v}/2$ is not much above the accomodating level p_0.

We have focused the analysis on a tractable case that resembles the outcome observed in Germany. We want now to briefly consider two variations:

1) Assume that bidder 2 has the same valuations as bidder 1. Then there are two equilibria. In one equilibrium, both dominant incumbents bid up to β^*, in a joint effort to push the weakest entrant out off the market. In the other, just one incumbent bids up to β^*, and the other is content with a small license. In our opinion, the second equilibrium is less likely than the first (see also our discussion of the second stage above.)

2) The four-player outcome is not unlikely from an ex-ante perspective. If both the additional value of a third frequency block, and of a four-player market are very high, then demand reduction effects are of minor importance, and a four-license outcome can occurred (see also Jehiel and Moldovanu, 2001a).

While our assumptions offer an equilibrium rationalization for the observed bidding behavior, Klemperer (2001) argues that the data reveals outright irrational players in the German UMTS auctions. It is useful to see how his argument is reflected in our model. Klemperer argues that the marginal probability of reaching the weakest bidder's valuation was increasing towards the end of the auction, i.e., $f'(v) > 0$ for high values of v. If this is the case, the second order condition for an interior solution β^* cannot be satisfied because, by Proposition 3,

$$\frac{\partial^2 U_1}{\partial \beta^2}(\beta^*) = 4\{v_3^1(5) - v_2^1(6) - \beta^*\}f'(2\beta^*) + 2f(2\beta^*\} > 0 \tag{17}$$

Thus, the incumbents's expected utility is convex in the bid, and he cannot rationally reduce demand. This is surely an intriguing hypothesis, but was this marginal probability really rising ? Klemperer's argument is based on one of the weak bidders announcement that it will bid up to the U.K. level. If Deutsche Telekom was already expecting the weakest bidder to leave then, it should become

increasingly confident that this was about to happen as the auction proceeds. However, it is not clear that 94% of the UK level (the level at which the incumbents gave up fighting) was "too low." Moreover, the entrant's message could have been purely strategic. As we have seen above, an entrant will be accommodated at lower prices if its competitors believe that its valuation is larger (e.g., in the hazard order). If, say, Deutsche Telekom expected the weakest bidder to leave at 90% the UK level, then the marginal probability could have been falling already.

Grimm et al. (2001) argue that there was a free-rider problem between the incumbents (the general phenomenon in an auction with incumbents and new entrants has been first identified by Jehiel and Moldovanu, 2001c). The idea is that it is sufficient that one incumbent bids for a large license in order to drive out entrant. If preemption is successful, then, assuming that marginal valuations are decreasing, it is clear that the incumbent with the small licence will win the left-over frequency block in the second stage. However, it is not clear that marginal valuations are really decreasing because it may be very attractive for, say, Mannesmann, to have four frequency blocks, while Deutsche Telekom has only two. Risking such an outcome might be infeasible for an incumbent, especially given the necessity of ex-post justification[22].

5. Revenue and welfare

It is useful to compare the German design with an alternative uniform-price design in which six small licenses (of fixed capacities) are auctioned. Assume that there are seven bidders and that market externalities are sufficiently strong,

[22] We are grateful to Eric van Damme for making this point.

i.e.,

$$\frac{v_3^2(5)}{3} > \frac{v_2^7(6)}{2}. \tag{18}$$

It is not difficult to check that bidders $i = 1, ..., 6$ obtain a license each, at a per-block price of $v_2^7(6)/2$. Under condition (18), the flexible design generates higher prices because it creates an artificial demand driven by the market externalities: incumbents fear loss of profits if more firms enter the market. Threats to induce bad outcomes if the bidders do not pay enough are indeed a prominent feature of revenue maximizing auctions in the presence of negative externalities (see Jehiel et al., 1996, 1999).[23] A higher revenue occurs then if the probability of a six-player outcome is sufficiently high (i.e., demand by bidder 6 is sufficiently strong), or if the price paid for the left-over frequency packages in the second stage (if any) is not too low (i.e., the marginal valuations for blocks in a given market structure are not declining too fast).

The officially declared auction goal has been to maximize welfare rather than revenue. Is the flexible design suitable for attaining this goal? There are at least two reasons to believe that this need not be the case.

Value maximization is not a guarantee for efficiency. From the point of view of value maximization for the involved firms, a design which allows for a variable number of small and large licenses is better than those designs where the number of licenses and their capacity ware fixed ex-ante. While this argument is correct in principle, its implementation in the German and Austrian design mixed flexibility in that dimension with flexibility concerning the number of firms. Since overall industry profits may fall in the number of firms, while consumer surplus may

[23] Threats to induce bad outcomes if the bidders do not pay enough are a prominent feature of revenue maximizing auctions in the presence of negative externalities (see Jehiel et al., 1996, 1999).

increase, it is obvious that letting the firms themselves decide how many of them will be able to operate in the market need not lead to overall efficiency.

Bids do not represent true valuations. The complex design allows for many different equilibria (even for the same set of exogenous parameters), and therefore focused the bidders' attention on strategic issues. There are two reasons why bids can be distorted in the German design: 1) By inducing bidding on several blocks rather than on whole licenses, the design artificially created an auction with multi-unit demand. As shown above, there are then incentives to shade the bid for the third frequency block (see Ausubel and Cramton, 1998, for the general phenomenon in uniform price auctions with multi-unit demand). On the other hand, there was an artificial increase in demand because bids for third blocks (which were also motivated by a preemption goal) competed with "ordinary" bids for small licenses. By allowing 4 to 6 firms, a "threat" to sell to newcomers was in effect operative, and, in principle, avoidable for a high enough price. It is plausible that demand increase played the main role after Debitel's exit, while demand reduction finally took place when prices reached a very high level.

6. Concluding remarks

This paper offers a stylized model of the German UMTS auction, which allows to study the interplay between incomplete information and market externalities in a setting with asymmetric bidders. We describe an equilibrium in which the participating firms reduce demand one by one until, with positive probability, the six-license outcome is reached. The observed behavior is thus compatible with equilibrium considerations. We have also argued that, by endogenizing the market structure, the auction design led to an artificial increase in demand, which may lead to a higher revenue than a less flexible design. However, it is not clear

that the design is conducive towards efficiency.

Appendix A. Details of the auction model

This appendix describes in formal terms the determination of the final price and allocation in our auction model. The demand of bidder i at price p is given by

$$D^i(p) = \begin{cases} 3 & \text{if } p \leq b_3^i \\ 2 & \text{if } b_3^i < p \leq b_2^i \\ 0 & \text{if } p > b_2^i \end{cases}. \tag{19}$$

Aggregate demand is then

$$D(p) = \sum_{i=1}^{n} D^i(p). \tag{20}$$

Define the *robust* demand of bidder i by

$$D_+^i(p) = \lim_{\substack{\varepsilon \to 0 \\ \varepsilon > 0}} D^i(p + \varepsilon), \tag{21}$$

This captures the willingness to increase a bid above a standing price level p.[24] Aggregate robust demand is then given by

$$D_+(p) = \sum_{i=1}^{n} D_+^i(p). \tag{22}$$

The ascending auction format determines the smallest price which yields an aggregate robust demand of at most 12, i.e.,

$$p^* := \min\{p | p \geq 0 \text{ and } D_+(p) \leq 12\}. \tag{23}$$

Lemma 1. *The price p^* is well-defined. Moreover, if $n \geq 5$, then $p^*$is uniquely characterized by the property*

$$D(p^*) > 12 \geq D_+(p^*). \tag{24}$$

[24]Note that the limit exists because $D^i(p)$ is piece-wise constant.

Proof. The set $M := \{p|p \geq 0 \text{ and } D_+(p) \leq 12\}$ is non-empty since demand vanishes for high prices. As $D_+(p)$ is by definition semi-continuous from the right, a minimum always exists and p^* is well-defined. Next we show that p^* satisfies property (24). By definition, $12 \geq D_+(p^*)$. Assume by contradiction that $D(p^*) \leq 12$. Then, because $n \geq 5$, we have $D(0) > 12$, so that $p^* > 0$. Since $D(p)$ is piece-wise constant and semi-continuous from the left, there is a $\delta > 0$ such that $p^* - \delta > 0$ and $D(p^* - \delta) = D_+(p^* - \delta) \leq 12$, which is a contradiction to the definition of p^*. Assume now that $p' \geq 0$ satisfies $D(p') > 12 \geq D_+(p')$. Then, $p' \in M$. If $p' = 0$, then clearly $p^* = 0$. If $p' > 0$, then $D(p' - \delta) = D(p') > 12$ for all sufficiently small $\delta > 0$. In particular, $D_+(p' - \delta) > 12$ for any small $\delta > 0$, so that $p' = \min\{p|p \in M\}$. □

The allocation rule is specified as follows. If $D_+(p^*) = 12$, then assign $D^i_+(p^*)$ blocks to bidder i. If $D_+(p^*) < 12$, then the allocation is uniformly[25] random on assignments[26] $(D^1, ..., D^n)$ satisfying

$$D^i_+(p^*) \leq D^i \leq D^i(p^*), \tag{25}$$

and

$$\sum_{i=1}^{n} D^i = 12. \tag{26}$$

Appendix B. Proofs.

Proof of Proposition 1. We repeatedly use our assumptions, which are graphically summarized in Figure 1. The reader is encouraged to refer also to Figures 2 through 4 in order to keep oversight of the argument.

[25] This assumption is not critical. Any other random allocation in which each feasible assignment appears with strictly positive probability will lead to the same results.

[26] By Lemma 1, there is always at least one such assignment.

We first show that bidder 1, i.e., the dominant incumbent, chooses a bid

$$b_3^1 \in \{v_3^2(5)/3\} \cup [\underline{v}/2, v_3^1(5)/3]. \tag{27}$$

We then specify β^* within this set. We finish the proof by showing that none of the other bidders $i = 2, ..., 6$ has an incentive to deviate[27].

We denote by $D^{-i}(p)$ for the aggregate demand of bidders $j \neq i$ at price p. - Figure 2 here -

Claim 1. *The dominant incumbent chooses* $b_2^1 \geq v_3^2(5)/3$. Consider Figure 2, which exhibits the set of feasible strategies for bidder 1. Assume that $b_2^1 < v_3^2(5)/3$. Then, from Figure 1, we have $D^{-1}(b_2^1) \geq 11$, so that bidder 1 cannot obtain a license, yielding zero utility. However, accommodating by bidding $b^1 = (v_3^2(5)/3, v_2^1(6)/2)$ yields a small license in a six-player market at a price $v_3^2(5)/3 < v_2^1(6)/2$, and gives a strictly positive utility.

Claim 2. *The dominant incumbent chooses* $b_3^1 \geq v_3^2(5)/3$. Assume that $b_3^1 < v_3^2(5)/3$. Then, using claim 1, and with a view on Figure 1, we have $D_+(v_3^2(5)/3) = 12 < 13 = D(v_3^2(5)/3)$. By Lemma 1, the incumbent gets a small license at a per-unit price of $v_3^2(5)/3$. Thus, bidder 1 is as well-off by setting $b_3^1 = v_3^2(5)/3$.

Claim 3. *The dominant incumbent does not choose* $b_3^1 \in (v_3^2(5)/3, \underline{v}/2)$. Assume $b_3^1 \in (v_3^2(5)/3, \underline{v}/2)$. Then, $D_+(b_3^1) < 13 = D(b_3^1)$, and from Lemma 1, bidder 1 would obtain a small license at the per-unit price $b_3^1 > v_3^2(5)/3$ (this is clear if $b_2^1 > b_3^1$, and follows from the tie breaking rule if $b_2^1 \geq b_3^1$). Hence, he could decrease the final price by marginally lowering b_3^1 .

Claim 4. *The dominant incumbent chooses* $b_3^1 \leq v_3^1(5)/3$. Assume that $b_3^1 >$

[27] To simplify the exposition in case of indifference we argue as if bidders preferred to use the proposed strategies. Of course, this does not affect the formal argument.

$v_3^1(5)/3$. Then the expected payoff for bidder 1 is given by

$$U_1(b_3^1) = \int_{\underline{v}}^{2b_3^1} \{v_3^1(5) - \frac{3v}{2}\} dF(v) + \int_{2b_3^1}^{\overline{v}} \{v_2^1(6) - 2b_3^1\} dF(v) \tag{28}$$

Lowering b_3^1 while keeping b_2^1 constant is feasible, and the marginal benefit (taking the derivative from the left) is

$$\frac{dU_1(b_3^1)}{db_3^1} = -2\{v_3^1(5) - v_2^1(6) - b_3^1\} f(2b_3^1) + 2\{1 - F(2b_3^1)\}. \tag{29}$$

From assumptions (1) and (3), we have

$$\frac{v_3^1(5)}{3} < \frac{\overline{v}}{2} < \frac{v_2^1(6)}{2}, \tag{30}$$

Hence, $b_3^1 > v_3^1(5)/3$ yields

$$v_3^1(5) - v_2^1(6) - b_3^1 < 0. \tag{31}$$

Using this in (29) shows that lowering b_3^1 strictly increases expected payoffs when $b_3^1 \leq \overline{v}/2$, and does not affect expected payoffs when $b_3^1 > \overline{v}/2$.

Claim 5. *The dominant incumbent chooses* $b_2^1 = v_2^1(6)/2$. From claims 1 to 4, we know that either $b_3^1 = v_3^2(5)/3$ or $b_3^1 \in [\underline{v}/2, v_3^1(5)/3]$. In the first case, $D(v_3^2(5)/3) = 14$. If in this case bidder 1 chooses $b_2^1 = v_3^2(5)/3$, then $D_+(v_3^2(5)/3) = 10$, and therefore (by tie breaking), bidder 2 obtains a large license with positive probability, while bidder 1 has payoff zero. However, bidding any $b_2^1 > v_3^2(5)/3$, yields $D_+(v_3^2(5)/3) = 12$, and bidder 1 obtains a small license with certainty at a per-unit price of $v_3^2(5)/3 < v_2^1(6)/2$. Therefore, $b_2^1 = v_2^1(6)/2$ is optimal in this case. In the case $b_3^1 \in [\underline{v}/2, v_3^1(5)/3]$, expected payoff is given by (28), and hence it is independent of b_2^1. In particular, $b_2^1 = v_2^1(6)/2$ is a best response.

We have seen from claims 1 to 4 that (27) holds. Specify now β^* as a payoff-maximizing choice of b_3^1 given that $b_2^1 = v_2^1(6)/2$. Such an optimal choice does

always exist since b_3^1 is chosen from the compact interval $[v_3^1(5)/3, v_3^2(5)/3]$, and since the expected utility function is continuous on this interval [28]. Using also claim 5, it is clear now that the dominant incumbent has no incentive to deviate. We proceed by checking that deviations are not profitable for the remaining bidders.

- Figure 3 here -

Claim 6. *Bidders $i = 2, ..., 5$ choose $b_3^i \leq \overline{v}/2$.* If $b_3^i > \overline{v}/2$, then bidder i will outbid bidder 6 and obtain a large license with certainty. The resulting market will have four players if $v/2 < \beta^*$, and five players if $v/2 > \beta^*$ (ignoring the possibility of a tie that occurs with probability zero). Payoffs will be $v_3^i(4) - \frac{3}{2}v$ and $v_3^i(5) - v_3^1(5)$, respectively. Both are strictly negative because of assumptions (1) and (4).

Claim 7. *Bidders $i = 2, ..., 5$ choose (b_3^i, b_2^i) such that $b_3^i < \underline{v}/2$.* Assume $b_3^i \geq \underline{v}/2$. By claim 6, we know that then $b_3^i \in [\underline{v}/2, \overline{v}/2]$. Consider first the case that bidder i bids higher than bidder 1 for the third frequency block, i.e., that $b_3^i \geq \beta^*$. The argument is then similar to that used in claim 4. There is a fight between bidders i and 6, and expected payoffs for bidder i are given by

$$U_i(b_3^i) = \int_{\underline{v}}^{2b_3^i} \{v_3^i(5) - \frac{3v}{2}\} dF(v) + \int_{2b_3^i}^{2b_2^i} \{v_2^i(6) - 2b_3^i\} dF(v), \tag{32}$$

Lowering b_3^i while keeping b_2^i constant is feasible, and the marginal benefit is

$$\frac{dU_i(b_3^i)}{db_3^i} = -2\{v_3^i(5) - v_2^i(6) - b_3^i\} f(2b_3^i) + 2\{1 - F(2b_3^i)\}. \tag{33}$$

From assumptions (1) and (3), we have

$$\frac{v_3^i(5)}{3} < \frac{v_3^1(5)}{3} < \frac{\overline{v}}{2} < \frac{v_2^i(6)}{2}, \tag{34}$$

[28] Recall that the distribution of v has no atoms.

By $b_3^i \geq \underline{v}/2$, we obtain:

$$v_3^i(5) - v_2^i(6) - b_3^i < 0. \tag{35}$$

Hence, bidder i cannot be using an optimal strategy. Consider now the case where $\beta^* > b_3^i$. Then there is a battle between the dominant incumbent, bidder i, and bidder 6. Expected payoffs for bidder i are given by

$$\begin{aligned} U_i(b_3^i) &= \int_{\underline{v}}^{2b_3^i} \{v_3^i(5) - \frac{3v}{2}\} dF(v) + & (36) \\ &+ \int_{2\gamma}^{2\min\{\beta^*, b_2^i\}} \{v_2^i(5) - v\} dF(v) + & (37) \\ &+ \int_{2\min\{\beta^*, b_2^i{}'\}}^{\min\{\overline{v}, 2b_2^i\}} \{v_2^i(6) - 2\beta^*\} dF(v). & (38) \end{aligned}$$

Lowering b_3^i marginally yields (ceteris paribus)

$$\frac{dU_i(b_3^i)}{db_3^i} = -2\{v_3^i(5) - v_2^i(5) - b_3^i\} f(2b_3^i) > 0$$

where the inequality holds because (35) and $v_2^i(5) > v_2^i(6)$. Thus, again, bidder i can do better by lowering his bid for the third block.

Claim 8. *If bidders $i = 2, ..., 5$ choose (b_3^i, b_2^i) such that $b_3^i < \underline{v}/2$, then they set $b_2^i = v_2^i(6)/2$ and $b_3^i = v_3^i(5)/3$.* Assume that $b_3^i < \underline{v}/2$. Then, since $D^{-i}(\underline{v}/2) \geq 10$, bidder i cannot obtain a large license. Expected payoffs for bidder i are given by

$$U_i(b_2^i) = \int_{\underline{v}}^{2b_2^i} \{v_2^i(5) - v\} dF(v) \tag{39}$$

if $b_2^i < \beta^*$, and otherwise by

$$U_i(b_2^i) = \int_{\underline{v}}^{2\beta^*} \{v_2^i(5) - v\} dF(v) + \int_{2\beta^*}^{\min\{2b_2^i, \overline{v}\}} \{v_2^i(6) - v\} dF(v). \tag{40}$$

Since by assumption (1), we have $v_2^i(6)/2 > \overline{v}/2$, the integrands are positive, and bidder i may optimally choose $b_2^i = v_2^i(6)/2$. As the bid for the third block does not affect the outcome, bidder i may choose $b_3^i = v_3^i(5)/3$.

Claim 9. *Bidder* 6 *chooses* $(b_3^6, b_2^6) = (v_3^6(5)/3, v/2)$. This case is easier to deal with as bidder 6 has complete information about valuations. Note first that $b_3^6 \leq v_2^5(6)/2$ because otherwise bidder 6 wins a large license at a per-unit price $v_2^5(6)/2$, so that his payoff is - Figure 4 here -

$$v_3^6(5) - \frac{3}{2}v_2^5(6) < 0, \tag{41}$$

The inequality follows from (1) and (4) via

$$\frac{v_3^6(5)}{3} < \frac{v_3^1(5)}{3} < \frac{v_3^1(4)}{3} < \frac{\underline{v}}{2} < \frac{v_2^5(6)}{2}. \tag{42}$$

Figure 4 exhibits the payoffs resulting from individual bidding strategies. Assume first that $v/2 < \beta^*$. Then the candidate equilibrium strategy yields a payoff of zero, while any deviation yields a negative payoff with certainty. Assume now that $v/2 \geq \beta^*$. Then, again, a view on Figure 4 clarifies that the candidate equilibrium strategy is optimal. □

Proof of Proposition 3. By Proposition 1, if the dominant incumbent fights, then $\beta^* \in [\underline{v}/2, v_3^1(5)/3]$. Note first that bidder 1 never sets $b_3^1 = \underline{v}/2$ because the distribution of v has no atom at $\underline{v}/2$, and hence he may lower his bid for the third block without any risk, thereby lowering the final price. Thus, $b_3^1 > \underline{v}/2$. Differentiation in (9) gives

$$\frac{\partial U_1}{\partial \beta} = 2\{v_3^1(5) - v_2^1(6) - \beta\}f(2\beta) - 2\{1 - F(2\beta)\}. \tag{43}$$

In particular,

$$\frac{\partial U_1}{\partial \beta}(\frac{v_3^1(5)}{3}) = 2\{\frac{2v_3^1(5)}{3} - v_2^1(6)\}f(\frac{2v_3^1(5)}{3}) - 2\{1 - F(\frac{2v_3^1(5)}{3})\} < 0 \tag{44}$$

We have used

$$\frac{2v_3^1(5)}{3} < \overline{v} < v_2^1(6) \tag{45}$$

which follows by assumptions (3)and (1). Thus, the first-order boundary condition is not satisfied, and $\beta^* < v_3^1(5)/3$, which proves that a fighting bidder 1 must shade his bid in equilibrium. The necessary first-order condition (13) follows from (43). $\square$

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Consortium	Members	Ticker	Share	Funding Cost 05/26/00	Termination	Net Debt 2000e	Market cap May 2000	Turnover 99	EBITDA 99	Vicinity of business
DeTe Mobil GmbH	Deutsche Telekom	DTE GR	100,00%	55	May/20/08	43.5	212	35.5	15.3	high
Mannesmann Mobilfunk GmbH	Vodafone AirTouch	VOD LN	100,00%	69	May/27/09	41.2	195	27	6.3	high
E-Plus Mobilfunk GmbH	KPN	KPN SA	77,50%	67	Nov/05/08	6.5	65	8.1	3.36	high
	BellSouth	BLS US	22,50%	40	Sep/23/08	17.5	90	26,4	11.6	
Viag Interkom GmbH	E.ON=Viag/Veba	EOA GR	45,00%	45	estimate	5.16	28.4	49	4.09	high
	British Telecom	BT/A LN	45,00%	44	May/23/07	10	122	28.5	6.58	
	Telenor	TELNOR Corp	10,00%	41	Mar/08/06	2	NA	4	1	
debitel Multimedia GmbH	Swisscom	SCMN SW	74,00%	30	estimate	5.2	30	7.44	2.8	low
	Debis	DCX GR	10,00%	35	estimate					
	Metro	MEO GR	10,00%	70	estimate	4.08	11.6	43.8	1.95	
MobilCom Multimedia GmbH	MobilCom	MOB GR	100,00%	60	estimate	0.04	3.74 - 13.1	1.8	0.163	intermediate
WorldCom Wireless Deutschland	MCI WorldCom	WCOM US	100,00%	35	Aug/15/06	16.1	127.5	38.7	12.8	low
Comment: FT bought a 28.5% share of Mobilcom for Euro 3.74 bn, which renders a market cap of Euro 13.1 bn										

Table I: Bidding consortia in Germany

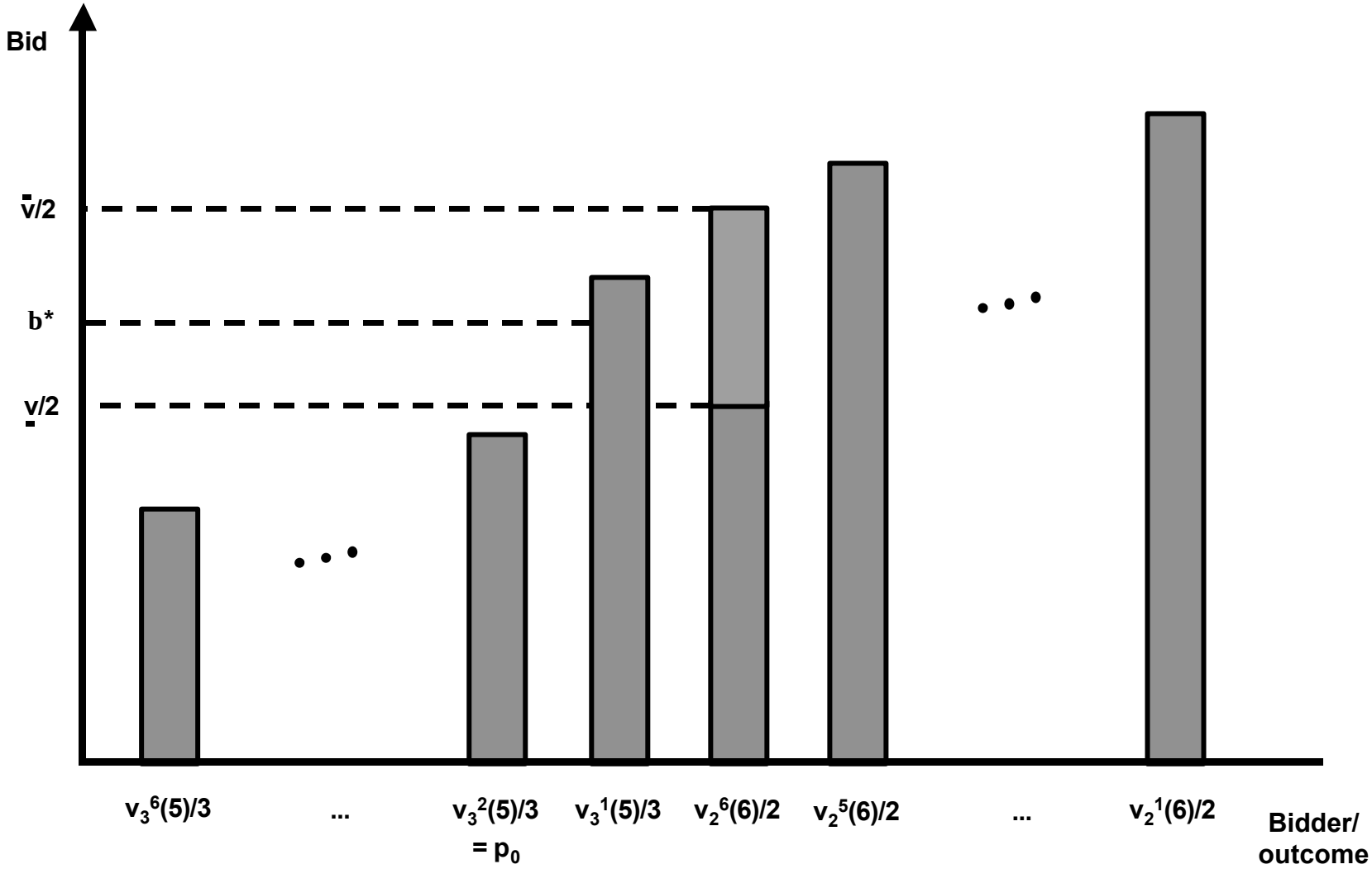

Figure 1: Equilibrium bids in the UMTS auction

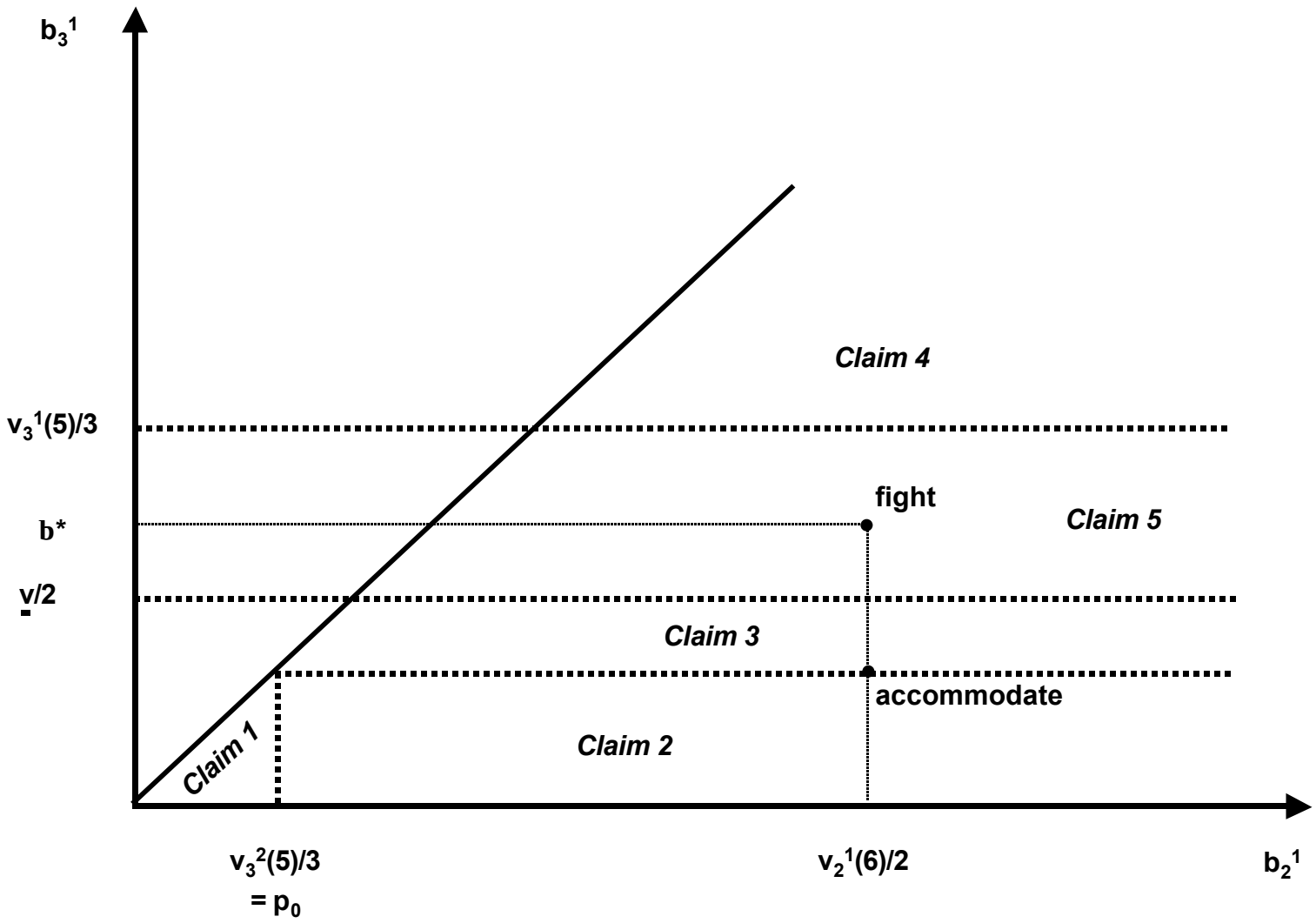

Figure 2: Equilibrium analysis for the dominant incumbent

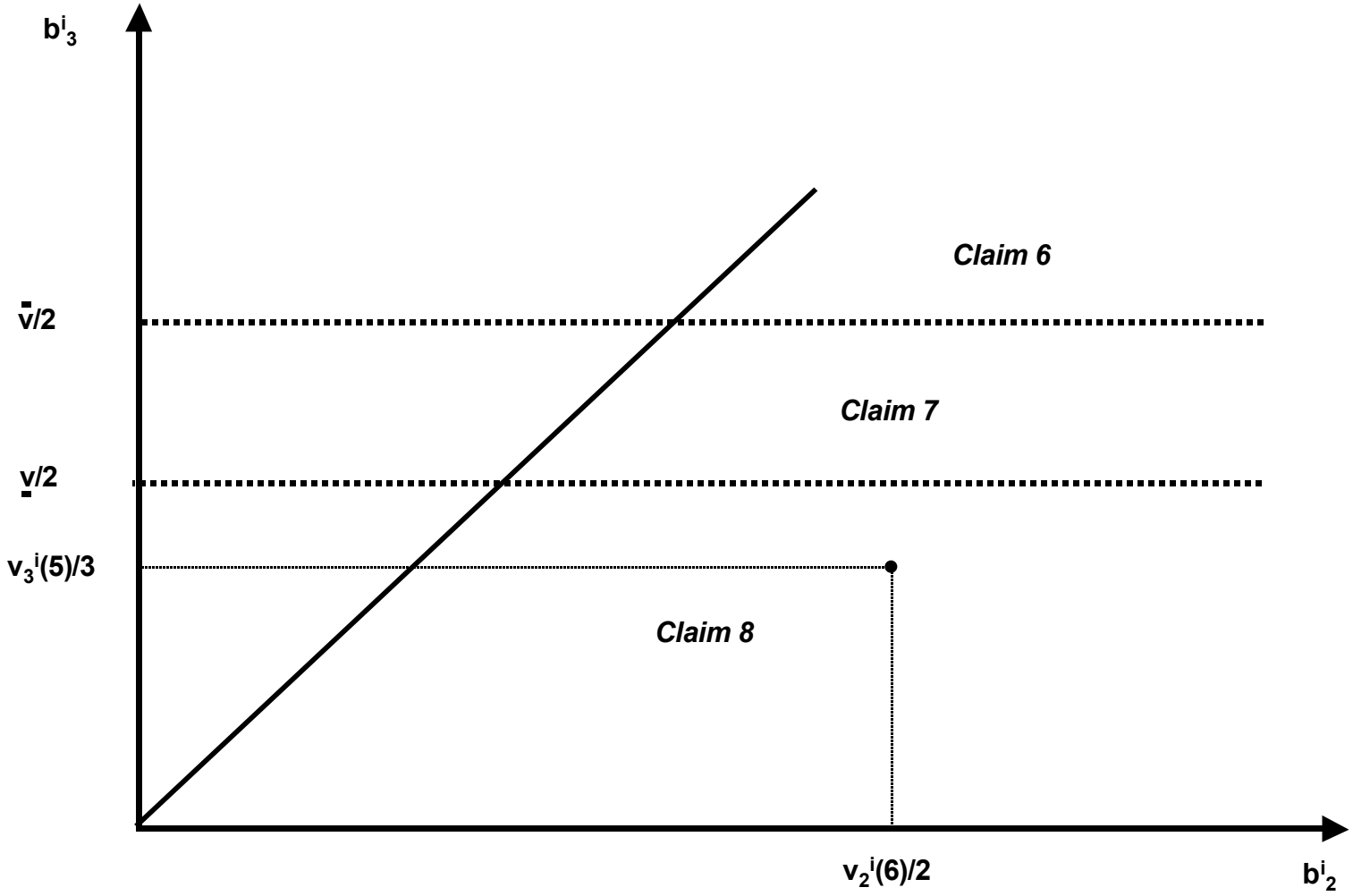

Figure 3: Equilibrium analysis for bidders i = 2, ..., 5

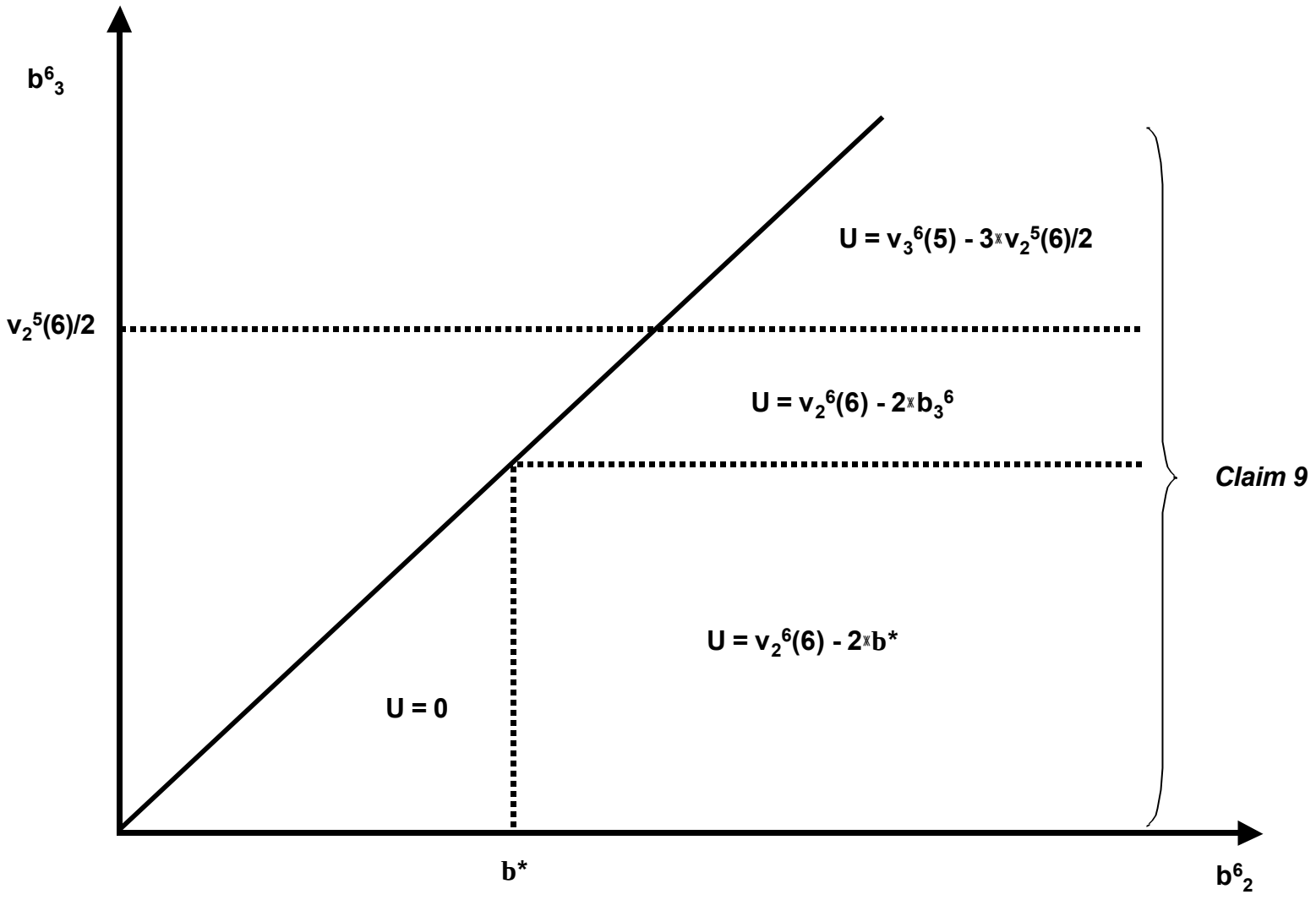

Figure 4: Equilibrium analysis for bidder i = 6