An experimental analysis of auctions with interdependent valuations[*]

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Abstract

We study experiments in an auction setting with interdependent valuation. Groups of three players receive private signals and then bid for a single, indivisible item. Valuations for the item differ within groups and depend asymmetrically on a bidder's own and other bidders' signals. Theoretically, the English auction yields efficient allocations, while other standard auction formats fail to do so.

Consistent with equilibrium predictions, we find that an English auction yields significantly more efficiency than a second-price sealed-bid auction.

We also study the seller's expected revenue and the bidders expected profits, and find that the experimental results are close to the theoretical predictions. (JEL C92, D44)

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1 Introduction

Consider an auction for a license to operate third-generation mobile telephony in a certain geographic area, and assume that one telecom firm conducts a survey of residential customers in order to forecast future demand. It is obvious that the survey's outcome will be valuable (e.g., affect valuations for the license) for other competitors as well. It is also likely that competitors that differ in their business plans will attach different weights to this information. For example a firm which plans to focus on businesses customers will attach another weight than a firm focused on residential customers. Even if several firms conduct such surveys, the weights attached to the gained information may be different. For example, a potential new entrant may highly value the information available to an incumbent, but not vice-versa. What rules should we choose for the auction in order to ensure that the license is sold to the firm that values it most? Are theoretically derived rules indeed likely to yield efficiency in practice? What are the connections between efficiency and revenue?

The answers to the above questions are by no means trivial, and the difficulties are due to the combined presence of both *asymmetric and interdependent* valuations. The term "interdependent" refers here to the fact that the valuation of a particular agent depends also on information available to other agents. In the *purely common value* case (where the object has a true value which is the same for all agents, but agents get different signals about it) efficiency is trivial, and is attained by all standard auctions (Dutch, English, first-price sealed-bid, second-price sealed-bid). In a *symmetric private value* case (where all agents share the same valuation function, but an agent's realized valuation depends solely on a signal available to that agent) all standard auctions are efficient and revenue equivalent if signals are interdependent[1]. No matter how signals are generated, and no

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[1] The English or second-price auction are efficient even if bidders with private values are asymmetric, but this does not hold for the Dutch or first-price sealed-bid format (see Vickrey 1961). In a symmetric interdependent values setting Milgrom and Weber (1982) showed that the English auction achieves a

matter how many objects are auctioned, an efficient allocation can be attained in the private values case by using a so-called Clarke-Groves-Vickrey mechanism. Following the theoretical developments, most of the experimental literature on auctions and bidding[2] considered situations where values are either private or purely common.

But, as our opening example suggests, many interesting, real-life applications will involve asymmetric interdependent values. Recognising this fact, a growing recent literature is concerned with the study of efficient allocation procedures in such settings[3]. Maskin (1992) pointed out that a *single-crossing* condition, which requires that a bidder's signal must have a higher impact on that bidder's value than on the opponents' values, is sufficient to ensure that the English auction is efficient in a framework with two asymmetric bidders having interdependent valuations for one object. By extending this single-crossing condition, Krishna (2000) identified two classes of settings where the English auction continues to be efficient even if there are more than two bidders. But, in general, the English auction with more players may fail to have an efficient equilibrium (see Perry and Reny (1999a) who also construct an alternative procedure).

In this paper we focus on an experimental setting with asymmetric, interdependent valuations: There is one object for sale, and there are three bidders (imagine them sitting at a round table). Each bidder receives a signal, and her valuation for the object is equal to her signal plus a constant weight multiplied by the (unobserved) signal of that bidder's right neighbour. Relating the asymmetry to a physical position (left or right) is a simple experimental device with which we approximate the asymmetries typically found in practice (see example above). The "symmetry in the asymmetry" used here is only a simplification, adopted in order to create a manageable biding environment (which

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higher revenue than all other formats.

[2]See Kagel (1995) for an excellent survey of this literature.

[3]See Maskin (1992), Maskin (2000), Dasgupta and Maskin (2000), Eso and Maskin (1999), Jehiel and Moldovanu (1998), Krishna (2000), Fieseler, Kittsteiner, and Moldovanu (2000), Perry and Reny (1999a), Perry and Reny (1999b).

is already complex since bidders may need to use sophisticated inference and bidding methods). We, therefore, chose to focus in a first study on one of the simplest possible settings that still allow our type of analysis. The "cyclical crossing" setting identified by Krishna (2000) is in fact a generalisation of the present one, and we can be sure that the English auction in our setting has an efficient equilibrium. The second-price sealed-bid auction, however, is not necessarily efficient since (due to the asymmetry) the agent with the highest signal may not have the highest value for the object. This shows that the task of aggregating the private information in order to award the object to the agent with the highest valuation is not easy. Besides forming estimates about valuations, our agents have to solve a non-trivial bidding problem where the "winner's curse" phenomenon plays a role.

The English auction achieves efficiency because relevant private information is gradually revealed during the auction process. In contrast, in a sealed-bid (or Dutch) auction a bidder must bid without any specific information about the realisations of competitors' signals (which affect that bidder's value). Finally, since the English auction is efficient while the other auction formats are not, the standard auctions are not necessarily revenue equivalent in our framework.

Given the above remarks, it is clear that the asymmetric interdependent valuations setting provides an excellent framework to experimentally test the efficiency and revenue properties of standard auctions.

We are not aware of any controlled laboratory experiment that studies auctions with asymmetric interdependent valuations. In the past experimentalists have concentrated on two extreme cases: Auctions with private valuations or auctions with purely common valuations. Let us briefly review these in order:

In the case of private values most experiments studied the case where signals are symmetrically distributed. Theoretically, the standard auctions formats should generate the same expected revenue and efficient allocation. In the laboratory, however, experimental-

ists found different revenues or efficiency properties of the different auction formats (see for example Coppinger, Smith, and Titus 1980, Cox, Roberson, and Smith 1982, Kagel, Harstad, and Levin 1987, Harstad 1990, Kagel and Levin 1993) which allowed them to study risk aversion, learning and understanding of participants in the auction situation. Most authors found bids to be higher in first-price sealed-bid auctions than in Dutch auctions, and also higher in second-price sealed-bid auctions than in English auctions.[4] Next, experimentalists have studied modifications of this framework where the standard auctions have theoretically different properties. The following two examples should illustrate this approach: With affiliated private values a first-price sealed-bid auction should theoretically generate less revenue than an English or second-price auction. This finding could not be replicated in the laboratory (Kagel, Harstad, and Levin 1987). In the case of multi-unit multi-value auctions a Vickrey auction should theoretically generate more efficiency and more revenue than a uniform price sealed-bid auction. In the laboratory the Vickrey auction, implemented in the ascending form proposed by Ausubel (1997), is indeed more efficient but raises, in contrast to the theory, less revenue than a uniform price sealed-bid (Kagel and Levin 2001).

In the purely common value case a bidder's expected profit should theoretically be larger with a first-price sealed-bid auction than with an English auction. In the laboratory, however, the reverse is true (Kagel and Levin 1992). The reason is the winner's curse (Bazerman and Samuelson 1983, Kagel and Levin 1986), which is much stronger in the first-price sealed-bid auction than in the English auction. Revealing relevant information publicly should theoretically increase the seller's revenue (Milgrom and Weber 1982). Experimentally this is the case for situations with a small number of bidders, but for situations with a large number of bidders introducing public information reduces revenue

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[4]Cox, Roberson, and Smith (1982) find bids in the second-price sealed-bid auction to be lower than in the English auction which is sometimes explained by the restrictive range of bids subjects could choose from in the experiment.

(Kagel and Levin 1986). The reason for this finding is, again, the winner's curse which is particularly strong in situations with a large number of bidders.

In the current paper we study the case of asymmetric interdependent valuations. There, again, we have a theoretical prediction: The English auction should generate efficient allocations in situations where the second-price sealed-bid auction does not. But it is not ex-ante clear whether this theoretical property carries over to the laboratory since theoretical bidding functions in the English auction are more complex than those in the second-price sealed-bid auction since bids must be revised as information is gradually revealed. For our analysis this means that we will concentrate on a specific difference between these two auction formats: the greater potential of the English auction to generate efficient allocations, and the more complex bidding functions in the English auction.

The paper is organised as follows: In section 2 we describe the experimental setup. In section 3 we compute equilibria for an English auction and for a second-price sealed-bid auction. We show that the English auction yields efficient allocations, while the second-price sealed-bid auction yields efficient allocations only if the agent with the highest signal has also the highest valuation for the object. Finally, we compute ex-ante expected revenues for the seller, and ex-ante expected profits for the bidders. We find that the seller's expected revenue is the same in the English auction as in the second-price sealed-bid auction. The bidders expect higher profits in the English auction. Hence, the loss due to the inefficiency of the second-price sealed-bid auction is fully borne by the bidders.

In section 4 we describe the experimental results and compare them to the theoretical predictions. In section 4.1 we compare the bids in the first stage of the English auction with the bids in the second-price sealed-bid auction (since these bids are based on the same information, i.e., on initial beliefs about competitors' signals). The experimental results agree very well with the theoretical predictions. In particular, we find that agents with higher signals bid more (note that this monotonicity is crucial for correct inferences during the second stage of the English auction). In section 4.2 we describe how the

experimental second-stage bids in the English auction depend on the bidders' own signals and on the bid of the first dropper. The left bidder's behaviour and the comparative sensitivities among left and right bidders are as predicted by theory. But right bidders (who have a quite complex, indirect inference problem) are not as sensitive to their own and to the first dropper's signal as in equilibrium. Such a deviation from equilibrium behaviour, however, has no substantial influence on efficiency and payoffs. In section 4.4 we compare the efficiency attained in the experiment by the two types of auctions. For 'simple' realisations of signals, where the bidder with the highest signal has also the highest value, both auction types achieve similar, high, measures of efficiency. In contrast, for 'hard' realisations of signals, where the above property does not hold, the English auction achieves significantly higher measures of efficiency. These findings agree well with the theoretical predictions. They are also consistent with the right bidder's deviation from equilibrium behaviour. In section 4.5 we describe the experimental results concerning expected revenues for the seller. While the experimental seller's revenues are higher than the theoretically predicted ones (which can be attributed to a small amount of over-bidding), we find that there is no significant difference among the two types of auctions. Again, this last finding agrees very well with the theoretical prediction. Finally, in section 4.6 we look at the bidders' expected profits in the experiment, and we find, as predicted by theory, that bidders are significantly better-off in the English auction.

Several concluding comments are gathered in section 5.

2   The Experimental Setup

The setup is as follows:

Three bidders, $i = 1, 2, 3$, bid for one unit of an indivisible object. Each bidder receives a private signal s_i. From the point of view of bidder i , bidder $(i + 1)$ modulo three is the bidder to the 'right' of i, and bidder $(i - 1)$ modulo three is the bidder to the left of

bidder i. Information about other bidders is revealed during the experiment only with respect to these relative positions, i.e. participants are informed about the bids or profit of their left or right neighbour in the auction. The do not get to know which person in the room this left or right neighbour is.

If bidder i successfully bids for the object and pays a price p then her payoff is given by $s_i + \alpha \cdot s_{i+1} - p$ where s_i is bidder i's private signal, s_{i+1} is the right neighbour's signal, and the weight α is a parameter that is varied during the experiment (see appendix A). The signal s_i is a uniformly distributed integer from $[0, 100]$, independent of s_{i+1} and s_{i-1}. Note that $\alpha = 0$ yields the independent private values case.

We compare two auction formats: An English auction and a second-price sealed-bid auction.

In the English auction we use an ascending clock design (see Kagel, Harstad, and Levin 1987, p. 1280). There are three clocks on each computer screen, one for each bidder. Clocks simultaneously start at a bid of -10^5 and synchronously move upwards every 2 seconds in equal steps ranging from 2 to 5 units of currency. Each bidder may stop her clock at any time by pushing a button. If a bidder stops her clock, then, at the next price increase, the other bidders observe that the respective clock has been stopped. When a unique clock is left active, the remaining bidder obtains the object at the price shown by the clock of the agent that stopped last. After each auction, the position of the winner, all signals, bids, and profits are communicated to the subjects. Information about past auctions within the same round is also visible on the screen.

To match the design of the English auction, we decided to have an ascending clock[6] design for the second-price sealed-bid auction as well: Clocks start at the same price and increase bids at the same speed until they are stopped by their owner. Bidders see only

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[5]We started with a negative bid to give bidders some 'wake up time' at the beginning of each auction.

[6]Within a multi-unit context Kagel and Levin (2001, p. 451) find experimental bids in an ascending clock second-price sealed-bid auction to be "essentially the same" as those in a traditional second-price sealed-bid auction.

their own clock and whether it is stopped or not — individual bids are 'sealed'. The auction ends when two clocks are stopped. The remaining bidder obtains the item at the price shown by the clock of the agent that stopped last, and the winner's identity, all signals, bids, and profits are communicated to the subjects. Information about past auctions within the same round is also visible on the screen.

We conducted 6 different experiments. In each experiment we had about 15 participants. These participants were randomly divided into groups of three to play rounds of 8 to 10 auctions. Within rounds, the parameters (the α and the auction format) were constant and known to participants. After each round, participants were again randomly divided into new groups. Changes in the parameters where announced publicly to the participants[7]. The first two rounds of each experiment were practice rounds that did not count for subjects' payoffs, and were not used for the analysis of the data. During the following eight rounds subjects were payed according to their success.

3   Equilibrium Predictions

In this section we compute symmetric equilibria for both auction formats. For simplicity of notation, we assume that signals are distributed uniformly between 0 and 1, and not, as in the experiment, between 0 and 100.

3.1   English auction

3.1.1   Bids in the English auction

In the English auction with 3 bidders we distinguish 2 stages: a first stage where all bidders are still active in the auction, and a second stage where only two bidders are left. The bidding strategy during the first stage may only depend on a bidder's own signal s. During the second stage, a bidding strategy may further depend on the price

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[7]See appendix A for a list of the parameters.

$\hat{b}_0$ where the first bidder dropped out, and on the first dropper's position (i.e. whether the bidder is to the left or to the right of the first dropper). Since only two bidders are left, the second stage ends before bidders change their decision at what price to drop out. Hence, the second stage is equivalent to a second-price sealed-bid auction. In a symmetric equilibrium, strategies are described by a triple $(b_0(s), b_L(s, \hat{b}_0), b_R(s, \hat{b}_0))$ where $b_0(\cdot)$ describes the initial bidding function, provided that no other bidder has left the auction. $\hat{b}_0$ is the price where the first bidder dropped out. $b_L(\cdot)$ describes the second stage bidding function of a bidder to the left of the first dropper. $b_R(\cdot)$ describes the second-stage bidding function of a bidder to the right of the first dropper. First, it is straightforward to show the following that:

Proposition 1 *For $\alpha > 1$ the English auction has no pure symmetric equilibrium where agents use strictly increasing bidding functions.*

The proof is given in appendix B.1. When calculating the equilibrium bids for the English auction in the following we will restrict ourselves to the case $\alpha \leq 1$.

Proposition 2 *Consider the bidding strategy $(b_0(s), b_L(s, \hat{b}_0), b_R(s, \hat{b}_0))$ defined by*

$$b_0(s) = s \cdot (1 + \alpha), \tag{1}$$

$$b_L(s, \hat{b}_0) = s_L + \frac{\alpha}{1 + \alpha}\hat{b}_0, \tag{2}$$

$$b_R(s_R, \hat{b}_0) = \frac{1}{1 - \alpha}s_R - \frac{\alpha^2}{1 - \alpha^2}\hat{b}_0. \tag{3}$$

Then in the English auction the strategy profile where each bidder bids according to $(b_0(s), b_L(s, \hat{b}_0), b_R(s, \hat{b}_0))$ is a Nash equilibrium. If $\alpha = 1$ it is optimal for the right bidder to always bid more than the left bidder.

Proof:

Bids in the first stage The bid $b_0(\cdot)$ determines a lower boundary for $b_L(\cdot)$ and $b_R(\cdot)$. We first assume that this lower boundary is not binding, and then check that the assumption is fulfilled in the computed strategies.

Assume that a bidder receives signal s and initially bids up to B, while the other bidders (with signals s_L and s_R) bid according to $b_0(\cdot)$, which is assumed to be strictly monotonically increasing. Denote by b_0^{-1} the inverse of $b_0(\cdot)$. Note that our bidder wins the auction with the initial bid B if and only if $s_L = s_R < b_0^{-1}(B)$. Her expected profit is given by

$$U_0(B) = \int_0^{b_0^{-1}(B)} (s + \alpha \cdot s_L - b_0(s_L)) \ ds_L \tag{4}$$

The first derivative is

$$\frac{\partial U_0}{\partial B} = (s + \alpha \cdot b_0^{-1}(B) - B)b_0^{-1\prime}(b_0(s)) \tag{5}$$

which is zero for $B = s \cdot (1 + \alpha)$. The second derivative $\partial^2 U_0 / \partial B^2$ is $-1/(1 + \alpha)^2 < 0$. Hence we have found a maximum, and the candidate equilibrium bidding function is

$$b_0(s) = s \cdot (1 + \alpha) \tag{6}$$

The left bidder Given that $b_0(\cdot)$ is strictly monotonically increasing, the first dropper's signal s_0 can be perfectly inferred from her bid $\hat{b}_0$. Hence, it is possible to write strategies during the second stage as functions of own signals and the first dropper's signal. After the first dropper has left the auction, the left bidder can infer her valuation for the good which is $s_L + s_0 \cdot \alpha$. Since the second stage is equivalent to a second-price sealed-bid auction, the left bidder, who now knows her valuation, has a dominant action: remain in the auction till the price exceeds valuation. Hence, the candidate equilibrium bidding function for the left bidder is

$$b_L(s_L, s_0) = s_L + s_0 \cdot \alpha \tag{7}$$

With the help of equation 1, the equilibrium bidding function can be expressed as a function of the own signal and the observed first bid:

$$b_{\mathrm{L}} = s_{\mathrm{L}} + \frac{\alpha}{1+\alpha}\hat{b}_0 \qquad (8)$$

Note that $\forall s_0 < s_{\mathrm{L}} : b_{\mathrm{L}}(s_{\mathrm{L}}, s_0) > b_0(s_{\mathrm{L}})$. Hence, for all possible signals, the candidate equilibrium bid in the first stage does not restrict the second-stage bid of the left bidder.

The right bidder   Let B be the bid of the right bidder, and let $b_{\mathrm{L}}(s_{\mathrm{L}}, s_0)$ be the bidding function of the left bidder which is strictly monotonic increasing in s_{L}. As long as $0 < s_{\mathrm{L}} < 1 + \alpha s_0$, the inverse with respect to s_{L} exists and will be called $b_{\mathrm{L}}^{-1}(s_{\mathrm{L}}, s_0)$. The right bidder will obtain the object as long as the signal s_{L} of the left bidder is lower than $b_{\mathrm{L}}^{-1}(B, s_0)$. The expected profit of the right bidder is

$$U_{\mathrm{R}}(B) = \int_0^{b_{\mathrm{L}}^{-1}(B,s_0)} \left(s_{\mathrm{R}} + \alpha \cdot s_{\mathrm{L}} - b_{\mathrm{L}}(s_{\mathrm{L}}, s_0)\right) ds_{\mathrm{L}} . \qquad (9)$$

Using equation 2 we calculate the derivative

$$\frac{\partial U_{\mathrm{R}}}{\partial B} = s_{\mathrm{R}} - B \cdot (1-\alpha) - s_0 \cdot \alpha^2 . \qquad (10)$$

Since $\partial^2 U_{\mathrm{R}}/\partial B^2 = \alpha - 1$, and by assumption $\alpha < 1$, the second order condition is always fulfilled[8].

Solving the first order condition, $\partial U_{\mathrm{R}}/\partial B = 0$, yields $B = (s_{\mathrm{R}} - s_0 \cdot \alpha^2)/(1-\alpha)$. We should note that this expression may be larger than the highest equilibrium bid of the left bidder $1 + \alpha s_0$, in which case the inverse bidding function $b_{\mathrm{L}}^{-1}(s_{\mathrm{L}}, s_0)$ is not defined. However, also in this case $(s_{\mathrm{R}} - s_0 \cdot \alpha^2)/(1-\alpha)$ is still a best response.[9]   Hence, the

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[8]See footnote 9 for the case $\alpha = 1$

[9]To see that, we solve $(s_{\mathrm{R}} - s_0 \cdot \alpha^2)/(1-\alpha) = 1 + \alpha s_0$ to obtain the critical value $s_{\mathrm{R}}^* = 1 - (1 - s_0)\alpha$. Whenever $s_{\mathrm{R}} > s_{\mathrm{R}}^*$ the right bidder always obtains the object. In this case the profit of the right bidder $(s_{\mathrm{R}} + \alpha s_{\mathrm{L}} - (s_{\mathrm{L}} + \alpha s_0))$ is at least $(s_{\mathrm{L}} - 1)(1 - \alpha)$ which is always positive, hence it is optimal for the right bidder to make such a high bid. (A bid of only $1 + \alpha s_0$ would be sufficient, of course). With a similar argument one finds that, for $\alpha = 1$, the right bidder also always wants to obtain the object.

candidate equilibrium bidding function for the right bidder is given by

$$b_R(s_R, s_0) = \frac{s_R - s_0 \cdot \alpha^2}{1 - \alpha} \tag{11}$$

If $\alpha \geq (1 - s_R)/(1 - s_0)$ the right bidder wants to obtain the object in any case. This can be achieved by making the bid stated in equation 3, or by any other bid larger than $1 + \alpha s_0$. With the help of equation 1, the equilibrium bidding function can be expressed as a function of own signal and the observed first bid.

$$b_R = \frac{1}{1 - \alpha} s_R - \frac{\alpha^2}{1 - \alpha^2} \hat{b}_0 \tag{12}$$

If $\alpha \geq (1 - s_R)/(1 - (b_0/(1 + \alpha)))$ the right bidder wants in equilibrium to obtain the object in any case which can be achieved by bidding according to equation 12 or submitting any other bid larger than $1 + \alpha b_0/(1 + \alpha)$.

It is interesting to note that the right bidder's bid $b_R(s_R, s_0)$ is decreasing in the first dropper's signal s_0. The intuition is as follows: The higher the price reached in the first stage, the lower the expected profit of the right bidder. Of relevance for the right bidder's profit is the left bidder's signal. That bidder's high bid may be motivated only by the presumably high signal of the first dropper (which is relevant for the left bidder's profit).

Note that $\forall s_0 < s_L : b_L(s_L, s_0) > b_0(s_L)$. Hence, for all possible signals, the candidate equilibrium bid in the first stage does not restrict the second-stage bid of the left bidder.

One can easily check that the above determined strategies form an equilibrium, no matter what the signals' distribution functions are. In fact, the displayed profile constitute an *ex-post* equilibrium[10].

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[10] An ex-post equilibrium is a strategy profile with the property that, for each vector signals, the joint action specified by the strategies constitutes a Nash equilibrium even when the vector of signals is common knowledge. An ex-post equilibrium, while not necessarily in dominant strategies, remains an equilibrium for any specification of signals' distributions.

3.1.2 Efficiency, Revenue and Profits in the English auction

We can now formulate the following proposition:

Proposition 3 *Assume that $0 \leq \alpha \leq 1$. Then for any realisation of signals, the English auction yields an efficient allocation.*

The proof is given in appendix B.2.

Note that for weights $\alpha > 1$ the efficient allocation rule is not monotonically increasing in signals, i.e., increasing the signal of a certain bidder may cause the object to be efficiently allocated to another bidder. As a consequence, the efficient allocation rule cannot be implemented. There exists no mechanism such that, in equilibrium, the object is always efficiently allocated.

In appendix B.3 we prove the following:

Proposition 4 *The seller's expected revenue in the English auction is*

$$R_{\mathrm{e}} = \frac{1}{8}(4 + 3\alpha) \tag{13}$$

The ex-ante (i.e., before signals are revealed) sum of expected profit for the three bidders is given by:

$$G_{\mathrm{e}} = \frac{2 + \alpha^2}{8} \tag{14}$$

3.2 The second-price sealed-bid auction

3.2.1 Bids in the second-price sealed-bid auction

In a second-price sealed-bid auction a bidding strategy can only depend on an agent's own signal. We consider symmetric equilibria. In appendix B.4 we prove the following:

Proposition 5 *The symmetric equilibrium bidding function in the second-price sealed-bid auction is given by*

$$b_{\mathrm{S}}(s) = s \cdot \left(1 + \frac{3}{4}\alpha\right) . \tag{15}$$

3.2.2 Efficiency, Revenue and Profits in the second-price sealed-bid auction

In contrast to the English auction, the allocation in the second-price sealed-bid auction is not always efficient, even for $\alpha < 1$. For illustration, consider an example where $\alpha = 1/2$, and where $(s_1, s_2, s_3) = (24, 0, 16)$. Valuations are give by $(v_1, v_2, v_3) = (24, 8, 28)$, and the efficient allocation is to give the object to bidder 3. Indeed, in the equilibrium of the English auction bidder 2 drops at a price of zero, bidder 1 drops at a price of 24, and bidder 3 obtains the object (she would stay in the auction till a price of 36). In contrast, in the second-price sealed-bid auction the ordering of equilibrium bids follows the ordering of signals: $(b_1^S, b_2^S, b_3^S) = (33, 0, 22)$. Bidder 1 obtains the object, which is not efficient.

In appendix B.5 we prove the following proposition:

Proposition 6 *The seller's expected revenue in the second-price sealed-bid auction is*

$$R_{\mathrm{s}} = \frac{1}{8}(4 + 3\alpha) \tag{16}$$

The ex-ante (i.e., before signals are revealed) sum of expected profits for the three bidders is given by:

$$G_{\mathrm{s}} = \frac{1}{4} \tag{17}$$

While the seller's expected revenues are the same in the two bidding formats, the bidders' expected profits differ. The efficiency loss occurring in the second-price sealed-bid auction is fully borne by the bidders.

4 Experimental Results

4.1 Initial bids

4.1.1 Raw data

In order to study the relation between initial bids and signals we show in figure 1 bids that are normalised to compensate for different weights α: The left graph shows $\hat{b}_0/(1+\alpha)$ for

the English auction, and the right graph shows $\hat{b}_S/(1+\frac{3}{4}\alpha)$ for the second-price sealed-bid auction. Each dot represents one initial bid. Only auctions with $\alpha < 1$ are shown.

[Figure 1 about here.]

In equilibrium the normalised bids must lie on the diagonal line. In the experiment bids are scattered around the diagonal line and obviously increase with signals. In the English auction other bidders can indeed infer from a high bid of the first dropper that this person has a high signal.

We observe also some overbidding in both auction types when signals are low[11].

4.1.2 Estimating the bidding function

In the second-price sealed-bid auction we observe bids for the first two droppers. The winning bid is only known to be higher than the two observed bids. In the English auction we observe the initial bid $\hat{b}_0$ for a unique bidder (the first dropper). For the remaining two bidders we only know that their unobserved initial bids $\hat{b}_{0,L}$ and $\hat{b}_{0,R}$ must have been larger then $\hat{b}_0$.

To estimate bidding functions we therefore use censored-normal regressions (Tobin 1958, Amemiya 1973, Amemiya 1984). In the English auction one realisation of the initial bid is known and the other two are right censored. In the second-price sealed-bid auction, two realisations are known, and the remaining one is right-censored. Calling the

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[11]In experiments with private value situations (where the winner's curse does not play a role) bids in the English auction quickly convergence to the equilibrium prediction, while in the second-price sealed-bid auction bids are higher than the equilibrium prediction(Kagel, Harstad, and Levin 1987, Kagel and Levin 1993). This bias disappears only when subjects gain experience (Harstad 1990). Over-bidding by low signal bidders in English auctions with pure common values is reported in Kagel and Levin (1992). In our experiment we find, on average, a significant amount of overbidding in all stages of the English auction and some, but not significant overbidding in the second-price sealed-bid auction. For both auction types the amount of overbidding does not change significantly with time. In section 4.4 we will relate overbidding to efficiency.

lowest bid $\hat{b}_0$ and the second-lowest bid $\hat{b}''$, bids enter the censored-normal regression as shown in the following table:

		first bidder	second bidder	winner
English auction	b_0	$= \hat{b}_0$	$\geq \hat{b}_0$	$\geq \hat{b}_0$
Second-price sealed-bid auction	b_0	$= \hat{b}_0$	$= \hat{b}''$	$\geq \hat{b}''$

To compensate for the impact of different αs we estimate $b_0 = \beta(1 + \alpha)s$ for the English auction and $b_0 = \beta(1 + \frac{3}{4}\alpha)s$ for the second-price sealed-bid auction.

When calculating levels of standard deviations and levels of significance we have to take into account that observations within any of our six experiments may be correlated. We can, however, assume that covariances of observations from different experiments are zero. Covariances of observations from the same experiment are replaced by the appropriate product of the residuals (Rogers 1993). We will use this approach throughout the paper to calculate standard errors.

There is only a small and not significant amount of overbidding in the second-price sealed-bid auction. We relate this finding to the fact that over-bidding in the second-price sealed-bid auction has an immediate effect whereas overbidding in the first stage of the English auction can most of the time be corrected in the second stage.

The following table shows the result of estimating a linear bidding function for all values of $\alpha \leq 1$.

	β	robust σ_β	$\chi^2(\beta = 1)$	$P > \chi^2$	95% conf. interval		n
English auction	1.173023	.0448517	14.88	0.0001	1.085115	1.260931	3420
Second-price sealed-bid auction	1.011867	.0148595	0.64	0.4245	.9827429	1.040991	1599

While in equilibrium β should be one for both auction formats, we find a significant amount of overbidding for the English auction[12].

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[12]We test for $\beta = 1$. The result of the appropriate χ^2 test is shown in the table.

4.2 Bids in the second stage of the English auction

Following the equilibrium bidding functions given in equations 2 and 12 we explain bids in the second stage as a linear function of the first bid, the second dropper's own signal, and a constant.

We will first estimate a simple bidding function, assuming that all bidders use the same function. They may systematically deviate from the equilibrium bidding function, but for different αs they deviate in the same way. This estimate qualitatively confirms the above equilibrium predictions for the left bidder, but not for the right bidder. To investigate whether all or only a few right bidders deviate we allow in a second step for different bidding functions for different bidders. To verify that the normalisations regarding α that we had to make in the first and second step were appropriate we allow in a third step for different deviations from equilibrium bids for different αs.

As in the estimation of the initial bid, we do not observe all realisations of the dependent variable. Hence, we again use the censored-normal regression approach. Calling the lowest bid $\hat{b}_0$ and the second-lowest bid $\hat{b}''$, bids enter the censored-normal regression as shown in the following table:

	first bidder	second bidder		winner	
		left of 1st	right of 1st	left of 1st	right of 1st
b_L	$\geq \hat{b}_0$	$= \hat{b}''$	$\geq \hat{b}_0$	$\geq \hat{b}''$	$\geq \hat{b}_0$
b_R	$\geq \hat{b}_0$	$\geq \hat{b}_0$	$= \hat{b}''$	$\geq \hat{b}_0$	$\geq \hat{b}''$

We normalise coefficients to disentangle the influence of α from the other effects and estimate bidding functions for rounds with $\alpha < 1$[13]

$$b_L \quad = \quad \beta_1^L s_L + \beta_2^L \frac{\alpha}{1+\alpha} b_0 + 100 \cdot (1+\alpha)\beta_0^L \tag{18}$$

---

[13]When estimating b_R we use only observations where $\alpha \leq (1 - s_R)/(1 - (b_0/(1+\alpha)))$ since only there we have a point equilibrium prediction (see the discussion of equation 12 above). However, including observations with $\alpha > (1 - s_R)/(1 - (b_0/(1+\alpha)))$ yields very similar results.

$$b_{\mathrm{R}} = \beta_1^R \frac{1}{1-\alpha} s_{\mathrm{R}} + \beta_2^R \frac{-\alpha^2}{1-\alpha^2} b_0 + 100 \cdot (1+\alpha)\beta_0^R \qquad (19)$$

The normalisation of the coefficients that describe the linear influence of own signal and of the first dropper's bid follows the equilibrium prediction (see equations 2 and 12) such that in equilibrium the coefficients are $\beta_1 = \beta_2 = 1$. Normalising the constant part of the estimation cannot be based on the equilibrium bidding strategies, since these do not include a constant. As we will see in figure 3 below, a constant that increases with α can explain a substantial part of the actual bidding behaviour. Hence, we normalise the constant to be $100 \cdot (1+\alpha)$, which is the maximal valuation of an object.

In a first step we estimate[14] equation 18 for all observations with $\alpha < 1$.

$n = 4314$	β	robust σ	z	$P > |z|$	95% conf. interval	
β_1^L	.663243	.028028	23.66	0.000	.6083091	.7181769
β_2^L	1.070481	.0588699	18.18	0.000	.955098	1.185864
β_1^L	.1572744	.010776	14.59	0.000	.1361539	.1783949

As it should be, the coefficient β_2^L is not significantly different from one ($\chi^2(1) = 1.43$), but β_1^L is significantly smaller than one ($\chi^2(1) = 144.36$). Doing the same exercise for equation 19 and all observations with $\alpha < \min\{1, (1 - s_{\mathrm{R}})/(1 - b_0/(1+\alpha))\}$ shows that the right bidder deviates much more from equilibrium.

$n = 4176$	β	robust σ	z	$P > |z|$	95% conf. interval	
β_1^R	.1833808	.0366421	5.00	0.000	.1115636	.2551981
β_2^R	.1475404	.0374883	3.94	0.000	.0740647	.2210161
β_1^R	.6122541	.0322076	19.01	0.000	.5491284	.6753798

Both β_1^L and β_2^L are significantly smaller than one ($\chi^2(1) = 496.68$ and $\chi^2(1) = 517.08$ respectively). Comparing coefficients from the two estimations shows that β_1^R is significantly smaller than β_1^L ($\chi^2(1) = 95.32$) and β_2^R is significantly smaller than β_2^L ($\chi^2(1) = 278.55$).

[14]Again we use a censored regression as described above and adjust standard errors for correlations within experiments, see section 4.1.2.

To investigate whether all participants deviate from equilibrium in the same way when they are in the position of the right bidder or whether only some participants make huge mistakes in this situation while other are still close to equilibrium we have to estimate equations 18 and 19 for each individual separately. The result is shown in figure 2.

[Figure 2 about here.]

The figure confirms that most left bidders (shown as 'o' in the graphs) are indeed relatively close to the equilibrium behaviour (point 'A' in both graphs). Left bidders are a little less sensitive to their own signal, which is compensated by an increased sensitivity to the first dropper's signal and a small constant part. Most of the right bidders (shown as '+'), however, are far away from the equilibrium prediction. They are closer to point 'B' in the graph, i.e. they do not react much to the signal of the first dropper and also react much too little to their own signal. This is compensated by a substantial constant part of the bidding function.[15] But right bidders are not completely insensitive to their own signal.

Figure 3 shows estimates of sensitivities to signals and to bids following equations 20 and 21.

[Figure 3 about here.]

$$b_{\mathrm{L}} = \beta_1 s_{\mathrm{L}} + \beta_2 b_0 + \beta_0 \tag{20}$$

$$b_{\mathrm{R}} = \beta_1 s_{\mathrm{R}} + \beta_2 b_0 + \beta_0 \tag{21}$$

The figure qualitatively confirms the above findings. Right bidders are indeed more sensitive to their own signal than left bidders. For both, however, sensitivity to their own signal (β_1) is smaller than in equilibrium. Right bidders are also, as predicted, less sensitive to the first dropper's bid than left bidders, however, in equilibrium sensitivity to

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[15]Kagel, Levin, and Richard (1996) report experimental results about information processing in English pure common value auctions. They find that the signal of the first dropper is correctly inferred, but that bidders follow a simple strategy where bids are based on an average of own signal and the first dropper's signal

the first dropper's bid (β_2) should be even smaller. Further, the normalisation of the constant term that we have chosen in the estimation of equations 18 and 19 seems to be justified. The estimates for β_0 are increasing in α and almost parallel to the dotted lines ($\beta_0 \cdot (1 + \alpha)$) which describe the normalisation chosen for β_0 in equations 18 and 19.

4.3 A second reference case: The naive right bidder

Given that the right bidder in the English auction strongly deviates from the equilibrium recommendation, we consider, in addition to the equilibrium, a second reference case: The first dropper and the bidder left to the first dropper follow their equilibrium bidding functions. The bidder right to the first dropper, however, bids according to

$$\tilde{b}_{\mathrm{R}} = (1 + \alpha).$$
(22)

This bidding strategy corresponds to point 'B' in figure 2. We will call this second reference case the 'case of a naive right bidder'. Notice that, given a naive right bidder the equilibrium strategies in the first stage and of the left bidder are still best replies.

4.4 Efficiency

In this section we study the efficiency properties of the English auction and the second-price sealed-bid auction. Even in the case of a naive right bidder the English auction is more efficient than the equilibrium allocation for sufficiently large α.[16]

For the analysis of the experimental results we measure efficiency in two different ways.

[Figure 4 about here.]

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[16]It is straightforward to show that for $\alpha \leq 1$ the second-price sealed-bid auction with equilibrium bids yields an efficient allocation in $1 - \alpha/2$ of all cases. The English auction yields always an efficient allocation in equilibrium, and yields an efficient allocation in $(1 + \alpha)/2$ of all cases with a naive right bidder.

The upper part of figure 4 shows the relative frequency of efficient allocations. On the left we show the results for all auctions. As in equilibrium, efficiency is higher in the English auction for $\alpha < 1$.[17] The middle and right part of figure 4 distinguish between 'simple' and 'hard' cases in an attempt to better understand where the additional efficiency in the English auction is gained. We define 'simple' cases to be realisations of signals where the bidder with the highest signal has also the highest valuation. 'Hard' cases are realisations of signals where the bidder with the highest valuation is not the bidder with the highest signal.

In 'simple' cases monotonicity of bids alone is sufficient for efficiency, and both auction formats are theoretically efficient for $\alpha < 1$. This seems to be supported by our data.[18]

In 'hard' cases the English auction theoretically achieves full efficiency (as long as $\alpha < 1$) while the second-price sealed-bid auction is never efficient. While in our experiment the English auction does not reach full efficiency, the relative frequency of efficient allocations is considerably higher than in the second-price sealed-bid auction.[19] To conclude, the

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[17]We make a probit estimate of the linear model $\eta = (\beta_s d_s + \beta_e (1 - d_s))\alpha + c$ where $\eta = 1$ if the allocation is efficient and 0 otherwise, and where $d_s = 1$ for the second-price sealed-bid auction case and 0 otherwise. In equilibrium we have $\beta_e = 0, \beta_s = -1/2$. Indeed the coefficient β_e is significantly larger than β_s. ($\chi^2(1) = 14.09, P_{>\chi^2} = 0.0002$) The test is based on a robust estimation that takes into account correlations of observations within experiments.

We can also test without using a linear approach. To do that we must concentrate on the case where $\alpha = 0.5$ since only this case has been analysed in all experiments both in the second-price sealed-bid auction and the English auction. In all six experiments the performance was better under the English auction. A one-sided binomial-test finds this to be significant ($P = 0.015625$).

[18]A comparison of the linear models as described in the previous footnote does not find the coefficients to be significantly different ($\chi^2(1) = 0.47, P_{>F} = 0.49$). The test is based on a robust estimation that takes into account correlations of observations within experiments. Also, a binomial test run for $\alpha = 0.5$ does not find a significant difference ($P = 0.109$)

[19]An F-test can be used to show that the average efficiency is significantly higher under the English auction ($F(1,5) = 119.83, P_{>F} = 0.0001$). The test is based on a robust estimation that takes into account correlations of observations within experiments. Also, a binomial test run for $\alpha = 0.5$ does find

English auction is more efficient then the second-price sealed-bid auction in hard cases (where it is supposed to be more efficient) and approximately as efficient as the second-price sealed-bid auction in simple cases (where it is supposed to be equally efficient).

Figure 4 also shows that efficiency decreases in α, i.e., the more complex the situation becomes, the harder it is for participants to find the efficient allocation. Moreover, both auction formats yield more efficiency in simple cases than in hard cases. Measuring the relative frequency of efficient allocations does not allow to distinguish between missing the efficient allocation by a substantial amount or only slightly. A second approach is shown in the lower part of figure 4. Let v_1, v_2, v_3 be the valuations of the three players. Let v^* be the winner's valuation, let $v_{rand} := (v_1 + v_2 + v_3)/3$ be the average value, and let $v_{max} := \max_i v_i$ be the maximal value. Then $(v^* - v_{rand})/(v_{max} - v_{rand})$ measures the degree of efficiency. Note that both measures are equal to 1 if allocations are always efficient (e.g., for $\alpha < 1$ in the equilibrium of the English auction). This measure of efficiency confirms the results obtained above.

We found that the additional efficiency of the English auction is gained where it is supposed to be gained, namely in what we call 'hard' cases. However, in theses cases the English auction does not reach the full (equilibrium) efficiency. We relate this failure to the behaviour of the right bidder.

[Figure 5 about here.]

To do that, we calculate the amount of overbidding, i.e. the difference between the actual highest bid in the English auction and the equilibrium value of this bid.[20] Averages for our six experiments are shown in figure 5. We find that inefficient allocations are the result of substantial underbidding of the right bidder and only moderate overbidding of the left bidder. As we see from figure 5, the mistake of the right bidder is larger than the mistake of the left bidder, both in the case of efficient and in the case of inefficient

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a significant difference ($P = 0.015625$)

[20] In cases where the right bidder has an interval of equilibrium bids we take the smallest difference.

allocations. To confirm this finding we calculate mean squared distances between actual bids and equilibrium bids. These distances are significantly higher for the right bidder than for the left bidder[21].

To summarise this section, we have found that the efficiency properties of the two auction schemes are in line with equilibrium predictions. At first glance this may be surprising since at least one of the bidders in the English auction, the right bidder, does not seem to follow the equilibrium recommendation. However, even with an extremely 'naive' right bidder, the English auction still has superior efficiency properties (see footnote 16) as long as $\alpha > 1/2$. Having said that, the next step is to find out who bears the efficiency loss in the second-price sealed-bid auction — the seller or the bidders.

4.5 The seller's expected revenue

From equations 13 and 16 we know that the equilibrium expected seller's revenue in the English auction is the same as in the second-price sealed-bid auction, namely $(4+3\alpha)/8$[22].

[Figure 6 about here.]

This property can also be found in our experimental data. The left part of figure 6 shows the seller's expected revenues. These revenues are very similar for both types of auctions. To confirm that, we estimate the following robust regression (allowing for correlated observations within experiments):

$$R = 100 \cdot \begin{cases} \beta_e \cdot \frac{1}{8}(4 + 3\alpha) & \text{English auction} \\ \beta_s \cdot \frac{1}{8}(4 + 3\alpha) & \text{second-price sealed-bid auction} \end{cases} \tag{23}$$

β_e measures sensitivity to α in the English auction, β_s measures sensitivity to α in the second-price sealed-bid auction. All coefficients should be 1 in equilibrium.

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[21]In all six experiments the mean squared distance is higher for the right bidder. We should reject the hypothesis that samples are drawn from the same distribution ($P = 0.031250$).

[22]In the case of the naive right bidder revenue is slightly higher, namely $(5 + 2\alpha)/8$

The results of a robust regression (allowing for correlated observations within each of our six experiments) shown in table 1 are in line with the equilibrium prediction. In particular we find that β_e and β_s are not significantly different[23].

4.6 The bidders' expected profit

As we have seen above in equations 14 and 17 bidders should be better off in the equilibrium of the English auction (where they obtain $(2+\alpha^2)/8$) than in the equilibrium of the second-price sealed-bid auction (where they only obtain $1/4$). Even with naive bidding expected profit $(3\alpha/8)$ is larger in the English auction than in the second-price sealed-bid auction as long as $\alpha > 2/3$. Naive bidding has only a relatively small cost which is for, sufficiently large α, compensated by the efficiency gains of the English auction.

The superiority of the English auction also holds in the experiment: The right part of figure 6 shows that, for each α, the bidders' profit is higher in the English auction than in the second-price sealed-bid auction.

To confirm that, we estimate the following robust regression (allowing for correlated observations within experiments):

$$G = 100 \cdot \begin{cases} \beta_E \cdot \frac{2+\alpha^2}{8} & + & c \cdot \frac{1}{4} & \text{English auction} \\ \beta_S \cdot \frac{2+\alpha^2}{8} & + & c \cdot \frac{1}{4} & \text{second-price sealed-bid auction} \end{cases} \tag{24}$$

---

[23]An F-test shows that $F(1,5) = 0.52, P_{>F} = 0.5033$. Notice that the F-test is based on the robust regression that takes into account correlations of observations within experiments.

A more conservative binomial test comes to a similar result. Estimates done for each experiment separately find $\beta_e > \beta_s$ in two cases and $\beta_e < \beta_s$ in four cases. We can not reject the hypothesis that β_e and β_s are drawn from the same distribution ($P = 0.6875$)

Results of the estimation are shown in table 2. By equations 14 and 17 we should expect $\beta_E = 1$, $\beta_S = 0$, and $c = 1$. Indeed β_S is significantly smaller than β_E[24] — the bidders' profit is higher under the English auction than under the second-price sealed-bid auction. However, all coefficients are smaller than the equilibrium prediction. Again we attribute this finding to some over-bidding that results in smaller profits for bidders[25].

5 Conclusion

We have experimentally compared an English auction with a second-price sealed-bid auction in a setting where bidders' valuations are asymmetric and interdependent. In our setting, the logic governing equilibrium behaviour is relatively complex. Nevertheless, we generally find that the experimental results are well aligned with theoretical predictions. In the English auction, we find that participants do not always correctly use the information revealed during the bidding process if the inference problem is too complex (i.e., for the right bidders). Still, bidders' information processing is sufficient in order to achieve significantly more efficiency in the English auction. The additional efficiency of the English auction is obtained only in 'hard' cases, i.e. in cases where the English auction is theoretically efficient while the second-price sealed-bid auction is not. In 'simple' cases where monotonicity is sufficient for efficiency and where both auction types are theoretically efficient, we find that both auction types are equally efficient in the experiment, and

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[24]Testing $\beta_E = \beta_S$ finds them significantly different ($F(1,5) = 7.08, P_{>F} = 0.0448$). The F-test is based on the robust regression that takes into account correlations of observations within experiments.

A more conservative binomial test comes to a similar result. Estimates done for each experiment separately find $\beta_E > \beta_S$ in all six cases. We should reject the hypothesis that β_E and β_S are drawn from the same distribution ($P = 0.015625$).

[25]Note that $1 - \beta_E$ can be taken as a measure for the winner's curse in the English auction while $-\beta_S$ corresponds to this measure for the second-price sealed-bid auction. As we can see in table 2, bidders in the English auction suffer from a larger winner's curse which is, however, more than compensated by the greater theoretical efficiency of the English auction.

that the measures of efficiency are indeed quite high.

We also find that the seller's expected revenue is close to the theoretically predicted value and, as predicted by theory, this revenue is not affected by the type of the auction.

Finally, we find that bidders are better off in the English auction than in the second-price sealed-bid auction. This finding agrees well with the theoretical observation that the efficiency loss in the second-price sealed-bid auction is fully borne by the bidders.

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A List of experiments

	Experiment	# Participants	# Auctions	α	sealed-bid	Euro/Taler
1.	1	15	10	.5	0	.05
2.	1	15	10	.5	1	.05
3.	1	15	10	.5	0	.05
4.	1	15	10	.9	0	.05
5.	1	15	10	.9	1	.05
6.	1	15	10	.9	0	.05
7.	1	15	10	1.5	0	.05
8.	2	12	8	.5	0	.0375
9.	2	12	8	.5	1	.0375
10.	2	12	8	.5	0	.0375
11.	2	12	8	.8	0	.0375
12.	2	12	8	.8	1	.0375
13.	2	12	8	.8	0	.0375
14.	2	12	8	1.2	0	.0375
15.	2	12	8	1.2	1	.0375
16.	2	12	8	1.2	0	.0375
17.	2	12	8	.5	0	.0375
18.	3	18	8	.5	0	.0425
19.	3	18	8	.5	1	.0425
20.	3	18	8	.5	0	.0425
21.	3	18	8	.9	0	.0425
22.	3	18	9	.9	1	.0425
23.	3	18	8	.9	0	.0425
24.	3	18	8	2	0	.0425
25.	4	15	8	.5	0	.0425
26.	4	15	8	.5	1	.0425
27.	4	15	8	.5	0	.0425
28.	4	15	8	.9	0	.0425
29.	4	15	8	.9	1	.0425
30.	4	15	8	.9	0	.0425
31.	4	15	8	1.7	0	.0425
32.	4	15	4	1.7	1	.0425
33.	5	18	8	.5	0	.0425
34.	5	18	8	.5	1	.0425
35.	5	18	8	.5	0	.0425
36.	5	18	8	.7	0	.0425
37.	5	18	8	.7	1	.0425
38.	5	18	8	.7	0	.0425
39.	5	18	8	1	0	.0425
40.	6	18	8	.5	0	.0425
41.	6	18	8	.5	1	.0425
42.	6	18	8	1.1	0	.0425
43.	6	18	8	1.1	0	.0425
44.	6	18	8	1.1	1	.0425
45.	6	18	8	.3	1	.0425
46.	6	18	8	.3	0	.0425
47.	6	18	8	.3	0	.0425
48.	6	18	8	.5	0	.0425

B Proofs

B.1 Proof of proposition 1

The proof follows by contradiction. Assume there were a symmetric and strictly increasing equilibrium bidding function $b_0(s)$ for the first stage. Assume also that bidders 1 and 3 follow this function and bid $b_0(s_1)$ and $b_0(s_3)$, respectively. We now show that bidder 2 can always improve by deviating and not bidding $b_0(s_2)$. To do that we first determine bidder 2's profit with a bid B. Denoting $S := b_0^{-1}(B)$ we distinguish 3 cases:

If $S < \min(s_1, s_3)$ then bidder 2 obtains zero.

If $s_3 < \min(S, s_1)$, i.e. bidder 3 drops first, then it is a dominant strategy for 2 to bid 2's true valuation, $s_2 + \alpha s_3$. Bidder 1, however, has a valuation of $s_1 + \alpha s_2$, which is always larger than $s_2 + \alpha s_3$ and waits for bidder 2 to leave the auction. Therefore bidder 2's profit is zero in this case.

If $s_1 < \min(S, s_3)$, i.e. bidder 1 drops first with $b_0(s_1) = s_1 \cdot (1 + \alpha)$, then bidder 3's dominant strategy is to bid the true valuation $s_3 + \alpha s_1$ which is larger than $b_0(s_1)$. Notice that, in this case, given the bidding function of bidder 3, bidder 2's value is increasing in bidder 3's bid. Hence, bidder 2 should either wait for bidder 3 or leave the auction immediately. If bidder 2 waits her profit is $s_2 + \alpha s_3 - (s_3 + \alpha s_1)$, which is positive in expectation:

$$
\begin{aligned}
g &= \int_0^S \left(\int_{s_1}^1 (s_2 + \alpha s_3 - (s_3 + \alpha s_1)) \, ds_3 \right) ds_1 \\
&= \frac{1}{6} S \left(3(\alpha - 1) - 3S\alpha + S^2(1 + \alpha) - 3(S - 2)s_2 \right) \quad (25)
\end{aligned}
$$

The first derivative of g at $S = s_2$ is positive:

$$
\left. \frac{dg}{dS} \right|_{S=s_2} = \frac{1}{2}(\alpha - 1)(s_2 - 1)^2 \quad (26)
$$

Hence, bidder 2 can always increase her profit by bidding slightly more than $b(s_2)$. Thus, bidding $b(s_2)$ can not be part of an equilibrium. $\qquad\square$

B.2 Proof of proposition 3

We first show that it is never efficient to allocate the good to the first dropper (who has the lowest signal). It is sufficient to show that $s_0 + \alpha \cdot s_R \leq s_R + \alpha \cdot s_L$. Rearranging yields $s_0 \leq (1 - \alpha)s_R + \alpha \cdot s_L$, which follows immediately from $s_0 \leq \min\{s_R, s_L\}$.

We now show that when the right bidder bids more than the left then it is indeed efficient to allocate the object to the right bidder, and vice versa. What we have to show is the following:

$$\underbrace{s_L + \alpha \cdot s_0}_{\text{left bid}} \lesseqgtr \underbrace{\frac{s_R - s_0 \cdot \alpha^2}{1 - \alpha}}_{\text{right bid}} \quad \Rightarrow \quad \underbrace{s_L + \alpha \cdot s_0}_{\text{left value}} \lesseqgtr \underbrace{s_R + s_L \cdot \alpha}_{\text{right value}} \tag{27}$$

Multiplying the left inequality with $1 - \alpha$ and adding $\alpha s_L - \alpha^2 \cdot s_0$ on both sides yields the inequality on the right hand. $\qquad \square$

B.3 Proof of proposition 4

Assume without loss of generality that bidder 2 determines the price. This means that either 1 or 3 have the lowest signal.

- If 1 has the lowest signal, then 2 determines the price only if 3 wins. Player 3 can only win if 3's signal is larger than the critical signal s_3^c which is defined by the condition $b_R(s_2, s_1) = b_L(s_3, s_1)$. This yields

$$s_3^c = \frac{s_2 - \alpha \cdot s_1}{1 - \alpha} \tag{28}$$

However, if 1 has the lowest signal and $s_3^c > 1$ then 3 can never win. Hence, player 2 will not determine the price iff $s_2 > s_2^c$ where s_2^c is defined as follows:

$$s_2^c = 1 - \alpha + \alpha \cdot s_1 \tag{29}$$

- If 3 has the lowest signal, then 1 will win if he has a signal higher than the critical signal s_1^c which is defined through $b_{\mathrm{L}}(s_2, s_3) = b_{\mathrm{R}}(s_1, s_3)$. Solving for s_1^c yields the following:

$$s_1^c = (1 - \alpha)s_2 + \alpha \cdot s_3 \tag{30}$$

Then the seller's expected revenue in the English auction is

$$
\begin{aligned}
R_{\mathrm{e}} \;=\; & 3\left(\int_0^1 \int_{s_3}^1 \int_{s_1^c}^1 b_{\mathrm{L}}(s_2, s_3)\, ds_1\, ds_2\, ds_3 + \right. \\
& \left. \int_0^1 \int_{s_1}^{s_2^c} \int_{s_3^c}^1 b_{\mathrm{R}}(s_2, s_1)\, ds_3\, ds_2\, ds_1 \right) = \frac{1}{8}(4 + 3\alpha)
\end{aligned} \tag{31}
$$

Similarly, the ex-ante (i.e., before signals are revealed) sum of expected profits for the three bidders is given by:

$$
\begin{aligned}
G_{\mathrm{e}} \;=\; & 3 \cdot \left(\int_0^1 \int_{s_3}^1 \int_{s_1^c}^1 s_1 + \alpha \cdot s_2 - b_{\mathrm{L}}(s_2, s_3)\, ds_1\, ds_2\, ds_3 + \right. \\
& \left. \int_0^1 \int_{s_1}^{s_2^c} \int_{s_3^c}^1 s_3 + \alpha \cdot s_1 - b_{\mathrm{R}}(s_2, s_1)\, ds_3\, ds_2\, ds_1 \right) = \frac{2 + \alpha^2}{8}
\end{aligned} \tag{32}
$$

$\square$

B.4   Proof of proposition 5

Take one of the bidders, and assume that her two neighbours, L and R, bid according to $b_{\mathrm{S}}(\cdot)$, which is strictly monotonically increasing and has inverse $b_{\mathrm{S}}^{-1}(B)$. Assume that our bidder bids B. Then she will obtain the object as long as $\max(s_{\mathrm{L}}, s_{\mathrm{R}}) < b_{\mathrm{S}}^{-1}(B)$. The value of the object is always $s + \alpha \cdot s_{\mathrm{R}}$ (where s is the own signal and s_{R} the right neighbour's signal).

This bidder's expected profit is given by

$$U(B) = \int_0^{b_\mathrm{S}^{-1}(B)} \left(\int_0^{s_\mathrm{R}} (s + \alpha \cdot s_\mathrm{R} - b_\mathrm{S}(s_\mathrm{R}))\ ds_\mathrm{L} + \int_{s_\mathrm{R}}^{b_\mathrm{S}^{-1}(B)} (s + \alpha \cdot s_\mathrm{R} - b_\mathrm{S}(s_\mathrm{L}))\ ds_\mathrm{L} \right) ds_\mathrm{R}$$

$$(33)$$

The derivative with respect to B is

$$\frac{\partial U}{\partial B} = \frac{1}{2} b_\mathrm{S}^{-1}(B) \left(3\alpha \cdot b_\mathrm{S}^{-1}(B) + 4(s - B) \right) \frac{\partial b_\mathrm{S}^{-1}(B)}{\partial B} \tag{34}$$

The first order condition $\partial U / \partial B = 0$ yields $B = s \cdot (4 + 3\alpha)/4$. Hence, the equilibrium candidate bidding function is

$$b_\mathrm{S}(s) = s \cdot \left(1 + \frac{3}{4}\alpha \right) \tag{35}$$

The second order condition is

$$\frac{\partial^2 U}{\partial B^2} = -\frac{32s}{(4 + 3\alpha)^2} < 0 \tag{36}$$

which is always fulfilled.

It is straightforward to show that the above computed strategies form a symmetric equilibrium for the assumed uniform distribution of signals. $\qquad\square$

B.5   Proof of proposition 6

To calculate the seller's expected revenue we assume without loss of generality that bidder 2 determines prices. Then either bidder 1 has the lowest signal and bidder 3 the highest signal, or vice versa. The seller's expected revenue is

$$R_\mathrm{s} = 3 \left(\int_0^1 \int_{s_1}^1 \int_{s_2}^1 b(s_2)\ ds_3\ ds_2\ ds_1 + \int_0^1 \int_{s_3}^1 \int_{s_2}^1 b(s_2)\ ds_1\ ds_2\ ds_3 \right) = \frac{1}{8}(4 + 3\alpha) \tag{37}$$

Note that the seller expects the same revenue as in the English auction! Since the respective allocation functions are not the same, the equality of expected revenues is not a

corollary of the revenue equivalence theorem, but rather a coincidence that occurs for the specific parameters used here.

Similarly, the ex-ante sum of the three bidder's expected profits is given by:

$$
\begin{aligned}
G_{\mathrm{s}} &= 3 \cdot \left(\int_0^1 \int_{s_1}^1 \int_{s_2}^1 s_3 + \alpha \cdot s_1 - b_{\mathrm{S}}(s_2)\, ds_3\, ds_2\, ds_1 + \right. \\
&\qquad \left. \int_0^1 \int_{s_3}^1 \int_{s_2}^1 s_1 + \alpha \cdot s_2 - b_{\mathrm{S}}(s_2)\, ds_1\, ds_2\, ds_3 \right) = \frac{1}{4}
\end{aligned}
\tag{38}
$$

$\square$

C   Remarks regarding the within subject design

In our experiment subjects are participating in several auctions where they see different signals, competitors' bids, αs, and different auction formats. When calculating standard deviations and the subsequent tests above we allowed for correlations of observations within experiments. We assumed, however, that such a correlation only affects the noise term of our model, and does not yield biased estimates of our coefficients. In this section we investigate this hypothesis further. As an example we take the bids in the second stage of the English auction. We show that, if there are at all effects from one auction to the next or one round to the next, these effects are small. Similarly to the estimation of equations 18 and 19 we estimate

$$
\begin{aligned}
b_{\mathrm{L}} &= \beta_1^L s_{\mathrm{L}} + \left(\beta_2^L \frac{\alpha}{1+\alpha} + \delta_2^L \frac{\alpha'}{1+\alpha'} \right) \cdot b_0 + \\
&\qquad + \gamma_1^L s_{\mathrm{L}}' + \gamma_2^L u' + 100 \cdot \left((1+\alpha)(\delta_s^L d_s' + r + \beta_0^L) + (1+\alpha')\delta_0^L \right)
\end{aligned}
\tag{39}
$$

$$
\begin{aligned}
b_{\mathrm{R}} &= \left(\beta_1^R \frac{1}{1-\alpha} + \delta_1^R \frac{1}{1-\alpha'} \right) \cdot s_{\mathrm{R}} + \left(\beta_2^R \frac{-\alpha^2}{1-\alpha^2} + \delta_2^R \frac{-\alpha'^2}{1-\alpha'^2} \right) b_0 + \\
&\qquad + \gamma_1^R s_{\mathrm{L}}' + \gamma_2^R u' + 100 \cdot \left((1+\alpha)(\delta_s^R d_s' + r + \beta_0^R) + (1+\alpha')\delta_0^R \right)
\end{aligned}
\tag{40}
$$

where s_{L}' denotes the signal the bidder got in the previous auction, α' denotes the weight from the previous round of auctions, u' is the profit the bidder obtained in the

34

previous auction (if at all), d'_s is one if the previous round was under the second-price sealed-bid auction format and zero otherwise, and r is the index of the round.

We use a censored regression as described above and adjust standard errors for correlations within experiments.

[Table 3 about here.]

Results are shown in table 3. Maintaining the assumption of possibly correlated observations within experiments implies that we can not calculate robust standard deviations for all coefficients. In the table these entries are marked with dots. However, the only point of this exercise is to show that the influence from past auctions or past rounds is sufficiently small.

As in the estimation of equations 18 and 19, only β_1 and β_2 should be one, all other coefficients should be zero. Indeed, most of them are. β_0 takes a similar value as in the estimation of equations 18 and 19. A positive δ_2 shows that subject only slowly adapt to the current value of α when reacting to the first bid. The α' from the previous round plays still a role. All other factors seem to play an ambiguous or small role.

D   Instructions

Welcome to a strategy experiment

This strategy experiment is financed by the Deutsche Forschungsgemeinschaft. The instructions are simple. If you take them into account carefully, decide sensibly, and also take into account the reasoning of the other players, you will gain a serious amount of money payed to you in cash at the end of the game.

Your profit depends on your success. For each "Taler" that you obtain in the experiment you receive 0.05 Euro. We have already carried out similar experiments. From the experience that we have gained there, we expect that, depending on your strategy, you will today obtain between 15 Euro and 35 Euro.

Please note that we do not want to pay you less money than what you deserve. All the money that we do not give to participants, must be returned to the Deutsche Forschungsgemeinschaft.

Rules of the game

Please note that we do not cheat during this experiment. Everything that you read in these instructions is true. This may sound trivial, but, sometimes psychologists do experiments where participants are deceived about parts of the experiment. This is not the case with economic experiments, like this one. We will explain the rules of the game and we will stick to them!

The game will be played in groups of 3 persons each. Allocation to groups will be determined by a random process. During the experiment groups will be reallocated repeatedly, again using a random mechanism.

Each group will play several auctions. Each member of a group has two neighbours, neighbour one and neighbour two. Imagine that the members of a group are sitting around a table.
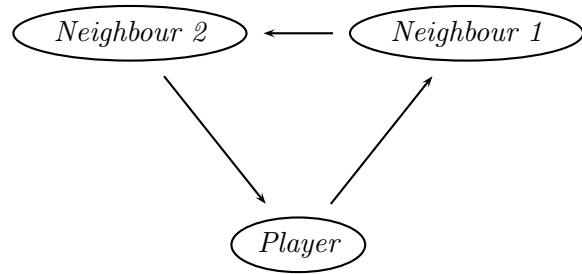
Neighbour one is always the right neighbour. Neighbour two is, in turn, to the right of neighbour one. Also neighbour one has two neighbours. His or her right neighbour is the person that is, for you, neighbour two. His or her left neighbour is you. Finally also neighbour two has two neighbours. His neighbour one is you. His neighbour two is the person that is your right neighbour.

If this sounds complicated to you, please recall that in a sense all members of a group are sitting around a round table, and neighbour one for each player is always the person sitting to the right.

Auction 10

Your Signal is **37** . The Value of the object for you is **37** plus 0.5 times **the signal of your right neighbour**

Your Value		Signal of your right neighbour	Value for your right neighbour depending on the signal of your left neighbour			Signal of your left neighbour	Value for your left neighbour depending on the signal of yourself		
			0	50	100		0	50	100
87	$= 37 + 0.5 \cdot 100$	100	100	125	150	100	100	125	150
74.5	$= 37 + 0.5 \cdot 75$	75	75	100	125	75	75	100	125
62	$= 37 + 0.5 \cdot 50$	50	50	75	100	50	50	75	100
49.5	$= 37 + 0.5 \cdot 25$	25	25	50	75	25	25	50	75
37	$= 37 + 0.5 \cdot 0$	0	0	25	50	0	0	25	50

In each auction each person receives a signal. This signal is a number drawn randomly between 0 and 100. All numbers between 0 and 100 are equally likely. The signal of a person in the current auction is only known to the person itself. Signals are shown at the top border of each individual screen.

When all members of a group of 3 persons have received their signal, an object will be auctioned. The person that manages to obtain the object receives a certain amount of "Taler" on his or her account. This amount is determined as the person's own signal plus 0.5 times the signal of the person's right neighbour. The signal of the person's left neighbour is of no influence on the value.

To make this relationship more clear, the screen (left part of the table) shows a table that represents the value of the object depending on the signal of your right neighbour. Since the same relation also holds for your right neighbour we also show (middle part of the table) how the value of the object for your right neighbour depends on the signal of his right neighbour (your left neighbour). The right part of the table also shows this relation for the value of the object for your left neighbour. Notice that the middle part of the table and the right part of the table are identical and do not change during the course of the game. The left column, however, is different in each auction.

It changes always with your signal.

You always know your own signal. You can deduce the signal of your right neighbour from the behaviour of the other players in the auction.

Middle part of the screen: Bids

Your Bid	Right Neighbour	Left Neighbour
54	54	22

In the middle of the screen, on the left you see a button that shows your bid slowly counting upwards like a clock. When you push this button, you leave the auction. When only one person remains in the auction, this person leaves automatically and obtains the object at the price that is currently indicated, i.e. the price where the previous bidder left the auction.

The bids of your left and right neighbour will be visible on the screen in some rounds. As long as they have not yet left the auction their bid is also counting upwards and shown on a red background. As soon as they leave the auction their clock stops and is shown on a blue background.

We will play some auctions, where you will not receive this information. In this case you see question marks in place of your neighbours' bids. Please note that in this case also your neighbours do not receive any information about your bids.

Lower part of the screen: Past

Round 3 Auction...	Your data (profit=8.16 Euro)			Right neighbour			Left neighbour		
	Signal	Bid	Profit	Signal	Bid	Profit	Signal	Bid	Profit
9	60	80	(-10)	20	20	(-10)	100	80	**50**
8	...	...	...	...	...	...	...	...	...

In the lower part of the screen we give you an overview about the past auctions.

The first line in the table shows your total profit in Euro.

The following lines show for you as well as for your neighbours the signal, bid and the profit. The profit of the person that has obtained the object is shown on red background. We show a (hypothetical) profit also for the other persons. This is the (hypothetical) amount that the person had obtained if the person had not left the auction until its end.

Let us consider auction 9 from the example. Your signal is 60, the signal of your right neighbour is 20. The value of the object for you is, hence, $60 + 0.5 \cdot 20 = 70$. The highest bid is 80. Were you to obtain the object at this price your profit would be $70 - 80 = -10$.
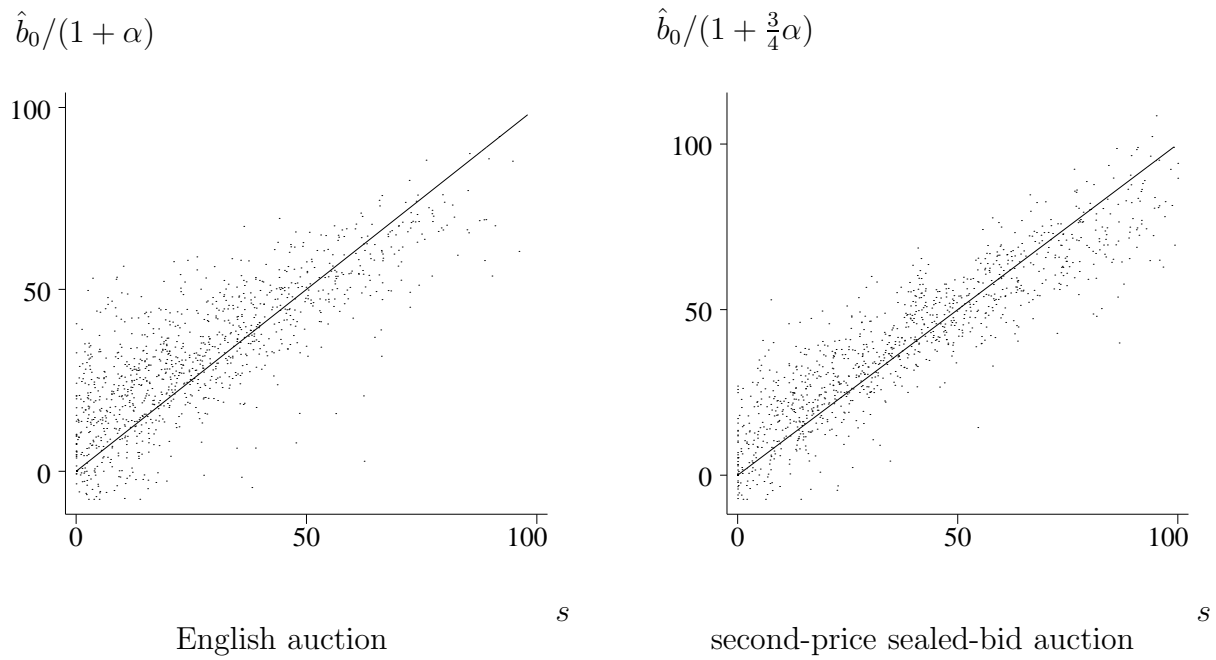
Let us now consider your left neighbour. This person has signal 100. Your signal was 60. The value of the object for your left neighbour is, hence, $100 + 0.5 \cdot 60 = 130$. The price payed by your left neighbour was 80. His profit is $130 - 80 = 50$. Since the object is indeed obtained by your left neighbour, the profit of 50 is shown on red background.

If you have questions, you have now the opportunity to ask them. You can always ask questions during the experiment.

We will first play some auctions to get used to the game. Then we will make a little break to give you the opportunity to ask questions.
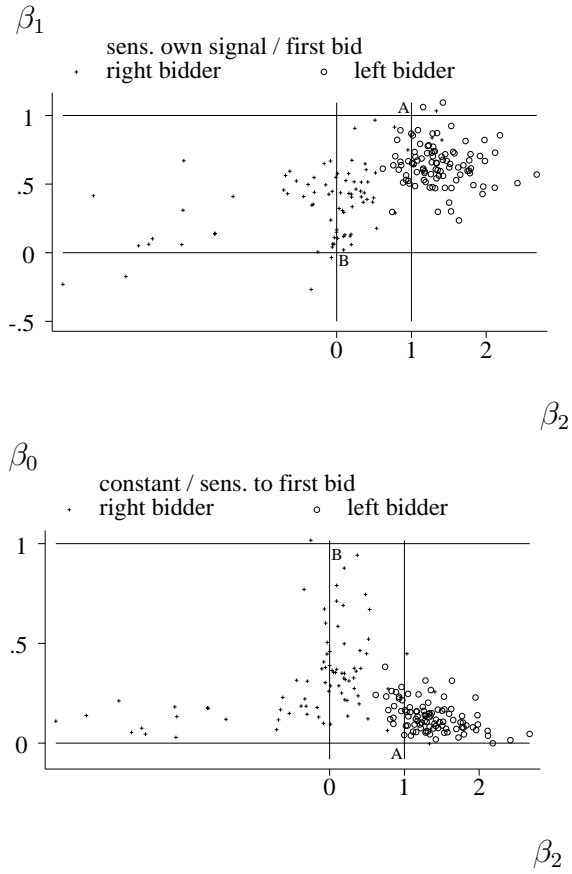
List of Figures

Dots show $\hat{b}_0/(1+\alpha)$ for the English auction, and $\hat{b}_0/(1+\frac{3}{4}\alpha)$ for the second-price sealed-bid auction. Only auctions with $\alpha < 1$ are shown. The diagonal line shows the equilibrium values.

Figure 1: Monotonicity of first bid

β_1

sens. own signal / first bid
· right bidder   ○   left bidder

1 ──────────────────────────── A
.5
0 ──────────────────── B
-.5

   0   1   2

β_2

β_0

constant / sens. to first bid
· right bidder   ○   left bidder

1 ──────────── B
.5
0 ──────────── A

   0   1   2

β_2

The figure shows normalised estimations of the individual censored bidding functions from equation 18 for all auctions with $\alpha < 1$, and for equation 19 for all auctions with $\alpha < \min\{1, (1 - s_R)/(1 - b_0/(1 + \alpha))\}$. Outliers have been eliminated from the graph using Hadi's method (Hadi 1992, Hadi 1994).

In equilibrium we have both for the left (○) and for the right (+) bidder that $\beta_1 = 1$, $\beta_2 = 1$, and $\beta_0 = 0$ (point 'A' in both graphs). The case of a naive right bidder (see section 4.3) is located at point 'B'.

Figure 2: Individual estimates for normalised bidding functions of the second stage
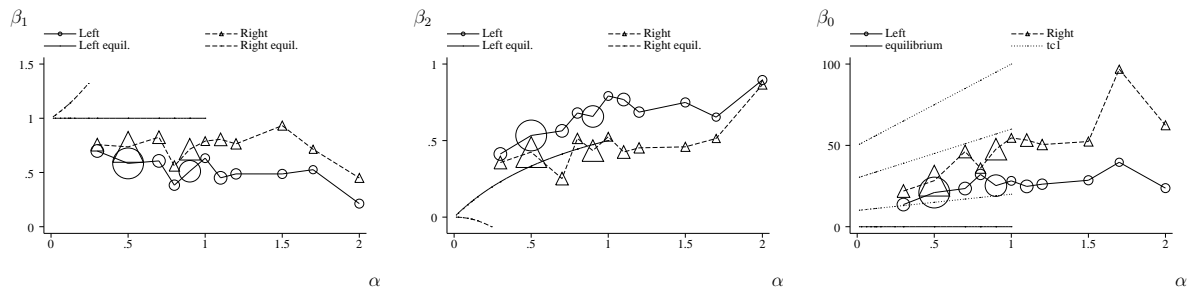
The figure shows estimates of coefficients for the bidding functions 20 and 21. The estimation was done using a censored normal regression approach for each α separately. The area of the symbols is proportional to the number of observations. Smooth curves and lines show values in equilibrium. Dotted lines in the right figure indicate the normalisation chosen for β_0 in equations 18 and 19 (the lines correspond to $\beta_0 = 0.1, \beta_0 = 0.3, \beta_0 = 0.5$).

Figure 3: Estimates of absolute bidding functions in the second stage

relative frequency of
efficient allocations

$(v^* - v_\mathrm{rand})/(v_\mathrm{max} - v_\mathrm{rand})$

all cases

only 'simple' cases
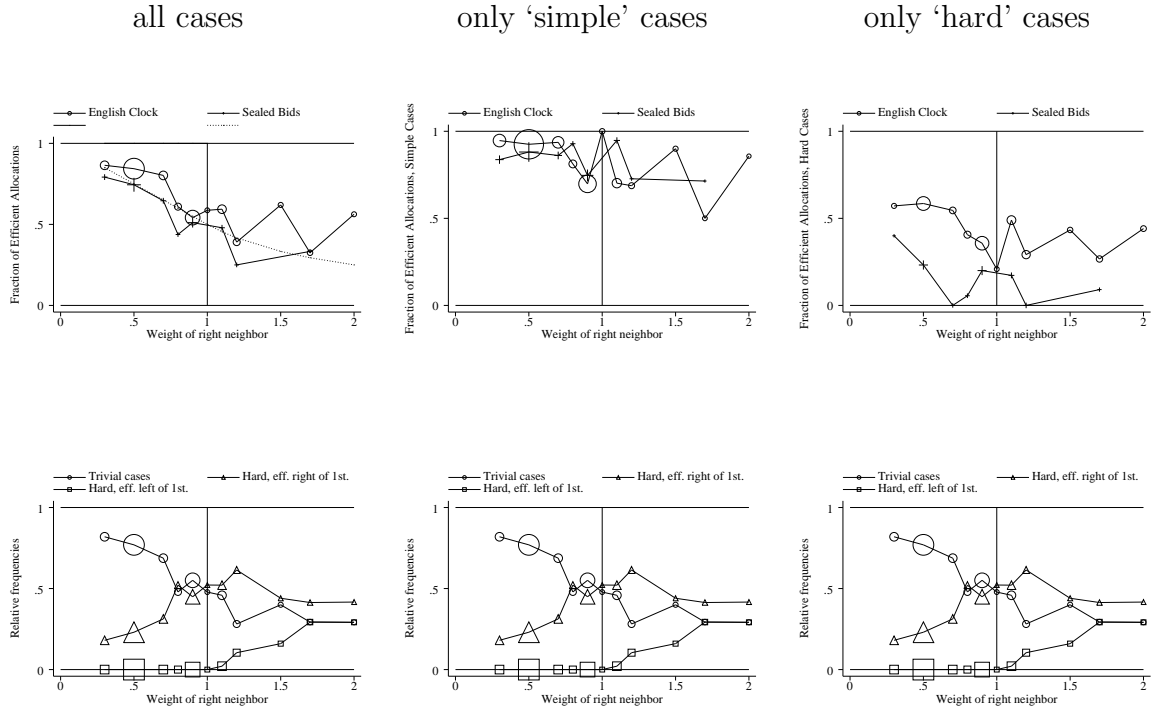
only 'hard' cases

The area of the symbols is proportional to the number of observations. The figure shows that the higher efficiency of the English auction is obtained primarily in 'hard' cases.

Figure 4: Empirical efficiency

4
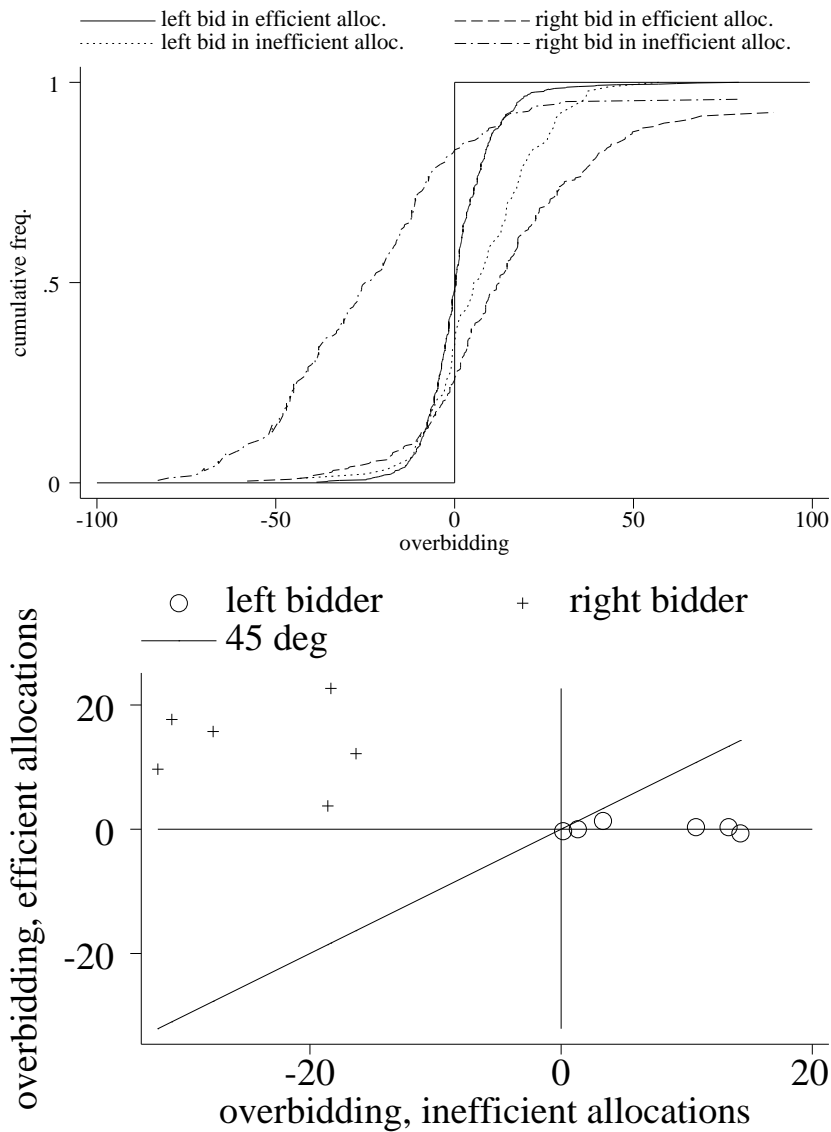
The figure shows average overbidding for left and right bidders and for auctions yielding efficient and inefficient allocations. The upper diagram shows cumulative frequencies, the lower diagram shows average values for each experiment. In equilibrium overbidding should be zero (straight line in the upper diagram).

Figure 5: Overbidding in the second stage of the English auction

seller's revenue / bidder's profit
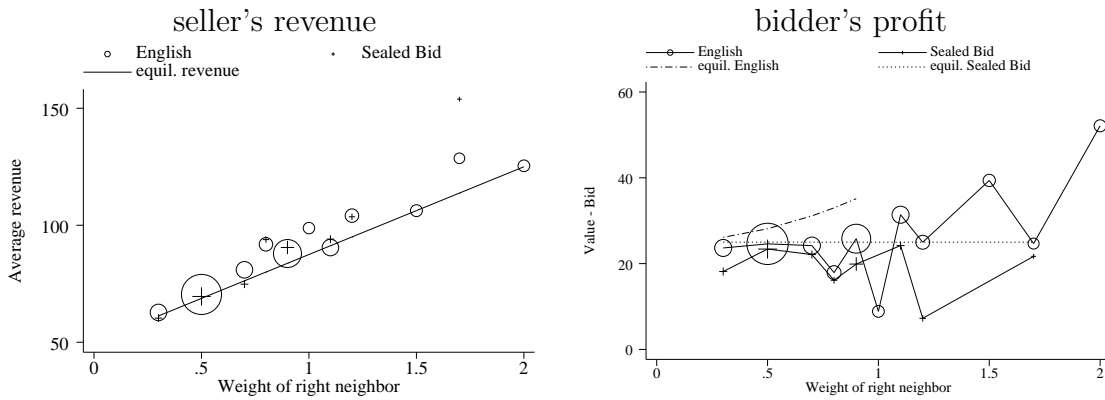
The left part of the figure shows average revenue for English auction and second-price sealed-bid auction. The straight line shows revenue in equilibrium (for $\alpha \geq 1$ only for the second-price sealed-bid auction).

The right part of the figure shows total bidders' profit (value minus price). The curved line shows the equilibrium profit for the English auction, the straight line shows the equilibrium profit for the second-price sealed-bid auction.

Figure 6: Empirical revenue and profit

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List of Tables

	β	robust σ_β	t	$P > t$	95% conf. interval		$F_{1,5}(\beta = 1)$	$P > F$
β_e	1.045328	.0156778	66.675	0.000	1.005027	1.085629	8.36	0.0341
β_s	1.063437	.0317638	33.480	0.000	.9817854	1.145088	3.99	0.1023

Robust regression estimate of equation 23. Following equations 13 and 16, we should expect $\beta_e = 1$ and $\beta_s = 1$. Tests of these equalities are shown in the two rightmost columns. Tests allow for correlations of observations within experiments.

Table 1: Estimation of seller's revenue (equation 23)

	β	robust σ_β	t	$P > t$	95% conf. interval		β^*	$F_{1,5}(\beta = \beta^*)$	$P > F$
β_E	.4198302	.1266414	3.315	0.021	.0942882	.7453722	1	20.99	0.0059
β_S	-.1480403	.1507582	-0.982	0.371	-.5355767	.239496	0	0.96	0.3712
c	.8728101	.0270843	32.226	0.000	.8031877	.9424325	1	22.05	0.0054

Robust regression estimate of equation 24. By equations 14 and 17 we should expect $\beta_E = 1$, $\beta_S = 0$, and $c = 1$. Tests of equality of estimated coefficients β with equilibrium values β^* are shown in the three rightmost columns. Tests allow for correlations of observations within experiments.

Table 2: Estimation of bidders' expected profit (equation 24)

$n = 2877$	β	robust σ	z	$P > |z|$	95% conf. interval	
β_1^L	.6059076	.0379019	15.99	0.000	.5316211	.680194
β_2^L	1.029934	.0670605	15.36	0.000	.8984974	1.16137
δ_2^L	.2978833	.0749272	3.98	0.000	.1510287	.444738
γ_1^L	.0379351	.	.	.	.	.
γ_2^L	$-.0429036$	.0203235	-2.11	0.035	$-.0827369$	$-.0030702$
δ_s^L	$-.0040583$	.	.	.	.	.
r	$-.00159$	.	.	.	.	.
β_0^L	.1524918	.	.	.	.	.
δ_0^L	$-.0021043$	.	.	.	.	.

$n = 2751$	β	robust σ	z	$P > |z|$	95% conf. interval	
β_1^R	.1262103	.	.	.	.	.
δ_1^R	.0226251	.	.	.	.	.
β_2^R	.0058564	.035857	0.16	0.870	$-.064422$	.0761348
δ_2^R	.0912468	.	.	.	.	.
γ_1^R	$-.0361427$	.	.	.	.	.
γ_2^R	.126368	.0522627	2.42	0.016	.0239351	.228801
δ_s^R	.0401919	.022191	1.81	0.070	$-.0033018$	.0836855
r	.0050345	.	.	.	.	.
β_0^R	.5292542	.0572241	9.25	0.000	.417097	.6414114
δ_0^R	.0265782	.	.	.	.	.

Estimation of equation 39 and 40

Table 3: Impact from past auctions or past rounds