How (Not) to Sell Nuclear Weapons

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We consider situations where a sale affects the ensuing interaction between potential buyers. These situations are modeled by assuming that an agent who does not acquire the object for sale incurs an identity-dependent externality. We construct a revenue-maximizing auction for the seller. We observe that: 1) outside options and participation constraints are endogenous. 2) The seller extracts surplus also from agents who do not obtain the auctioned object. 3) The seller is better-off by not selling at all (while obtaining some payments) if externalities are much larger than valuations. (*JEL* D44, D62, L14)

After the breakup of the Soviet Union, the independent Ukraine has inherited a huge nuclear arsenal. In particular, 176 intercontinental missiles were stationed on its soil. It is estimated that the maintenance of a functioning nuclear arsenal would impose a substantial economic burden on Ukraine. Steven Miller (1993) calculates a sum in excess of $5,000 million per year. Moreover, there is no doubt that several countries (in particular some Middle Eastern ones) are interested to acquire fissionable material, or even complete weapon systems from Ukraine. Russia and the United States have no direct interest in Ukraine's weapons (which are old-fashioned, and basically superfluous for the two big nuclear powers), but the danger of proliferation (exposed or covert) is considered to be extremely high. This danger was a subject of major concern for the presidents of the two superpowers, and they repeatedly emphasized it at their meetings.

It is interesting to recall what recently happened: Ukraine has agreed to sign the Nuclear Nonproliferation Treaty as a *nonnuclear* state. It dismantled all its tactical nuclear weapons and begun to dismantle the strategic ones as well. As an "encouragement" Ukraine received high payments from *both* United States and Russia. The first U.S. payment, pledged under the Nunn-Lugar legislation, was $175 million. Ukraine has become, almost overnight, the fourth largest recipient of American aid. In 1994 it got $350 million in economic aid and $350 million towards dismantling nuclear weapons, besides an extra present of $200 million bestowed by President Clinton (see *The Economist*, November 26, 1994 for more American promises). It is assumed that Russia canceled Ukrainian debts worth around $900 million. The Ukraine demands a total of $3,000 million to complete the job (see U.S. Policy, Information and Text, 1993).

In another recent case, China signed an agreement *not to sell* its M-9 and M-11 missiles to Arab countries (see *The Economist*, November 5, 1994). The Economist calls the agreement "binding." In reply, the United States agreed to lift its one-year-old embargo on satellite exports to China.

Trying to model the main features of the above examples, we construct a model where a sale creates negative externalities on nonacquirers. The magnitude of those external effects depends both on the identities of the acquirer and the sufferer. Therefore, potential

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buyers have preferences over which other agent may get the good. In this framework, our goal is to find an optimal selling procedure from the point of view of the seller. Quite surprisingly, we show that an optimal procedure will have some of the qualitative features displayed by Ukraine's strategy.

Generally speaking, our model encompasses instances in which the result of the sale affects the nature of the ensuing interaction between potential buyers. We mainly consider the case of negative externalities (which is typical in competitive situations), and briefly describe the changes induced by the presence of positive external effects. The externalities can represent some physical effect (for example, pollution), or, more generally, can stand for expected profits in future interaction.

There are many situations where our basic model applies: changes of ownership (for example, mergers, acquisitions, privatizations) in oligopolistic markets; the sale of an important input (for example, a patent) to downstream competitors; the award of major projects that change the nature of the industry (for example, projects that lead to the creation of a new industry standard); the decision over the location of environmentally hazardous enterprises; the decision over the location of important international institutions (for example, the location of the European Central Bank); the provision of a public good by a single agent.

The main contribution of this paper is that it applies the *mechanism design* methodology to a generalization of a problem studied by the multilateral vertical-contracting literature. A good motivation is provided by the following excerpt from Michael L. Katz's (1989 p. 656) survey of vertical relationships:

> ... the buyers of an intermediate good typically are involved in a game in the downstream product market, and the sales contract for the upstream product may affect the equilibrium of the downstream game. ... A simple uniform posted price often is held up as the typical contract form in final good markets. Given the sophistication of buyers and the large scale of individual transactions, more complex schemes may be practicable in intermediate good markets. Sellers may utilize sophisticated price mechanisms ...

A vast literature on mechanisms that maximize seller's profits exists,[1] but these papers do not consider situations where buyers have preferences over which other buyer gets the good (that is, agents' utilities depend only on whether they obtain the object or not, and on payments made to the seller as required by the rules of the "game").[2] Hence, situations where buyers' behavior must take into account the effects of the sale on the equilibrium of a subsequent game are not analyzed in the standard framework. As mentioned above, such situations are common in vertical contracting between a monopolist selling inputs to several downstream competing firms. A major theme in the vertical-relationships literature (surveyed in Katz, 1989) has been the study of a relatively narrow set of contractual arrangements (the so-called "vertical restraints"). Practices such as two-part tariffs, royalties, exclusive dealings, resale-price maintenance, are often observed, and their study has great empirical relevance. This pattern may suggest that more sophisticated procedures are unnecessary, or infeasible (say because high implementation costs). Nevertheless, as Katz points out, the somewhat neglected analysis of the relationship between a principal and several game-playing agents may yield new insights. There are several studies trying to bridge this gap in the context of patent licensing.[3] The papers by Katz and Carl Shapiro (1986), and by Morton I. Kamien and Yair Tauman (1986) compare "classical" licensing mechanisms (that is, by means of first-price auctions, fixed fees, royalties) in a framework where all potential buyers are identical. License

[1] William Vickrey (1961), Roger B. Myerson (1981), and Paul R. Milgrom and Robert J. Weber (1982) study the problem in the context of auctions; Robert E. Wilson (1993) offers an extensive treatment of nonlinear pricing and its various applications; D. Fudenberg and J. Tirole (1991) survey in Chapter 7 the literature on mechanism design.

[2] Jeremy Greenwood and R. Preston McAfee (1991) do consider a model with externalities in consumption, but look at welfare-maximizing allocation schemes.

[3] This literature is discussed in the survey of Jennifer R. Reinganum (1989) which is devoted to the sale of a particular type of input (knowledge).

auctions are shown to be superior from the point of view of the patent holder. Kamien et al. (1992) describe a mechanism in which the patent holder can extract surplus from nonacquirers. This mechanism is shown to be optimal if the buyers are engaged in Cournot competition and the cost reduction is modest. All papers mentioned above, and most of the literature, assume that externalities depend on the number of licenses and not on their identity (buyers are assumed to be symmetric).[4] Our emphasis on heterogeneous buyers and identity-dependent externalities allows us, among other things, to identify a delicate interplay between revenue maximization and the endogenous participation constraints (this feature remains unexploited in the above mentioned literature). Moreover, our treatment enables the study of the incentive effects connected to the revelation of privately held information about externalities. On the other hand, our present analysis is limited by the assumption that there is only one unit which may be sold (or licensed) to a single agent.

Another recurrent theme in the vertical-contracting literature is the study of opportunistic behavior: each downstream firm fears that the supplier might renegotiate another's contract to increase bilateral profit at the firm's expense.[5] McAfee and Schwartz (1994) argue that "... committing efficiently to customers about one's dealing with third parties can be especially difficult." The fear of future opportunistic behavior may impede the monopolist's attempt to maximize revenue. This difficulty is implicit in our model since we assume that the magnitude of the externalities is fixed, and cannot be contracted upon. For example, consider the case of a patent holder selling a cost-reducing innovation to one of several oligopolists. A negative externality is due to the fact that a firm that does not obtain the patent will produce with higher relative costs, and its profit will decrease. The assumption of fixed externalities implies that there is no efficient contractual commitment concerning a buyer's output and pricing policy once he holds the patent. Such contracting (which affects third parties) is prone to renegotiation. Apart from this constraint on commitment possibilities, we assume here, as in most of the mechanism-design literature, that the seller can fully commit to the announced form of the selling procedure.

Finally, an intriguing area where some of the basic features of our model apply is the so-called "Economics of Superstars," as discussed by Sherwin Rosen (1981), and by R. Frank and P. Cook (1995). These works draw attention to the fact that modern communication technology ensures that the costs of distributing talent do not increase nearly in proportion to market size. By serving very large market shares at low cost, the first place firms, teams, or individuals obtain huge portions of the rents, while all other competitors earn relatively small profits. In these contexts, the price to be paid for acquiring the services of a superstar will also reflect the large negative externality (that is, decrease in profits) caused if the superstar is employed by a competitor.

The paper is organized as follows. Section I presents the market model: the seller owns an indivisible good, and there are several potential buyers. A buyer is characterized by her valuation for the object, and by the incurred externalities (positive or negative) in case that another buyer acquires the object. The external effects are assumed to be directed and identity dependent.

In Section II we describe the seller's revenue-maximization problem. The main feature to be considered is that a buyer's willingness to pay is endogenous, and can be determined only given an equilibrium of a chosen sale procedure.

Trying to capture the obvious features of all the illustrations offered above, we must assume that potential buyers cannot avoid the external effects by simply refusing to participate in the market. Since externalities are identity dependent and, possibly, asymmetric, participation constraints play a major role in determining an optimal selling procedure. This

[4] Some exceptions are as follows: Katherine E. Rockett (1990) studies an interaction between a patent holder and two asymmetric potential licensees; Susan Scotchmer (1994) differentiates between early and late innovators, and points out the external effects created by patenting second-generation products.

[5] For example, see Patrick J. DeGraba and Andrew Postlewaite (1992), and McAfee and Marius Schwartz (1994). Further references can be found in the later paper.

is to be contrasted with the usual mechanism-design set-up where "outside options" are considered to be exogenous.

In Section III we present an optimal mechanism for the complete-information case. The treatment of this simpler case serves two purposes: first, it allows us to separate the effects due to the presence of externalities from the incentive constraints which are inherent to incomplete-information setups. Second, we identify some of the features of optimal selling procedures that remain valid in more complex setups.

We describe an auction form that has an equilibrium in strictly dominant strategies (this is the unique Nash equilibrium). The seller's revenue in that equilibrium is maximal among all revenues possible in equilibria of feasible mechanisms.

The properties of the optimal auction in the complete-information case can be summarized as follows: 1) agents' outside options are endogenously determined by the selling mechanism itself and by the played strategies. Accordingly, participation constraints and the "threats" in case of nonparticipation play an important role in the revenue-maximization problem of the seller. The seller's action must be specifically tailored to different groups of participating buyers. 2) By using suitable threats, an optimizing seller can extract surplus also from agents who do not obtain the object. 3) The seller is better-off by not selling at all (while obtaining some payments) if externalities are large compared to valuations. 4) The equilibrium outcome is efficient.[6] Moreover, we show that the revenue-maximizing equilibrium is coalition proof if buyers cannot arrange side payments among themselves (say, because of high penalties attached to possible detection by antitrust authorities).

In Section IV we analyze whether the above mentioned properties continue to hold in a model where buyers privately know valuations (that is, the value of the object to themselves) and the externality they *impose on others*. In this section we assume that this externality does not depend on the sufferer's identity, and agents' types are two-dimensional vectors.

The informational constraints turn out to be very strong, and ex-post efficiency is not always achieved by the revenue-maximizing mechanism. In particular, it may happen that the seller does not sell in cases where a sale is efficient, but also the opposite effect may occur—a sale occurs although efficiency requires that the good stays with the seller. The latter effect contrasts with the result of standard auction theory. All other insights (see 1–3 above) continue to hold.

In Jehiel et al. (1995) we study the problem of optimal mechanism design for auctions with externalities in a complementary information setup: there we assume that buyers' private information concerns valuations and externalities *imposed by others*. Many real-life situations probably combine elements of both models.

In Section V we gather some concluding remarks. Proofs are given in the Appendix.

I. A Market with Externalities

The market we consider consists of one seller and n different buyers. The seller owns an indivisible good. The buyers will be denoted by i, j, and so on. If no trade takes place, then, for the theoretical discussion, the utilities of all agents are normalized to be zero (in applications the no-trade situation is a given status quo). If buyer i buys the indivisible good at a price p, then his utility is given by the term $\pi_i - p$, where π_i represents i's valuation, that is, the profit made by i when owning the good. The seller's utility is then given by p. We introduce a matrix, $\mathbf{A} = \{\alpha_{ij}\}_{1 \le i,j \le n, i \neq j}$, of external effects. The interpretation is as follows: if buyer i buys the object, then the utility of j is given by $-\alpha_{ij}$, where $\alpha_{ij} \geq 0$ (respectively $\alpha_{ij} \leq 0$) if externalities are negative (respectively positive). Hence, external effects are directed and identity dependent. Denoting the seller as agent 0, we have $\forall j$, $\alpha_{0j} = 0$ (see normalization above). We can easily allow for the case where the seller himself suffers from potential externalities, that is the payoff to the seller if he sells at price p to buyer i is given by $p - \alpha_{i0}$. Since the main strategic

[6] Since we assume that there are no income effects, efficiency is here equivalent to the maximization of the total value for the involved parties.

effects occur because of the externalities among buyers, we do not explicitly consider this case here.

For an illustration, take n firms competing in an oligopoly. The cost functions may differ from firm to firm. Consider a technical innovation, licensed to only one firm, whose effect is a reduction in marginal costs. The magnitude of that effect will, in general, differ from firm to firm. The firm that acquires the innovation will increase its market share and its profit. The market share and profit of the other firms will decrease. Formally, denote by P_i the profit of firm i before the innovation appears (this is the "status-quo" profit). Denote by P_j^i the profit of firm j if firm i acquires the innovation. This situation exactly fits in our framework by setting:

$$(1)\quad \pi_i = P_i^i - P_i, \qquad \text{for all} \quad i,\ 1 \leq i \leq n \qquad \text{and}$$

$$(2)\quad \alpha_{ij} = P_j - P_j^i, \qquad \text{for all} \quad i \neq j,\ 1 \leq i, j \leq n.$$

II. Auction Design and Participation Constraints

We first describe the class of procedures among which the seller chooses an optimal mechanism. The idea is basic to the theory of auction design. In step 1, the seller designs a *mechanism*. A mechanism is a game-form in which agents send costless messages, and, based on the realized messages, the seller implements an allocation—here this consists of an allocation of the indivisible good (which may be random) and a vector of monetary transfers. An important assumption is that the seller can commit to the proposed mechanism. In step 2, the buyers simultaneously accept or reject the mechanism, that is, decide whether to participate or not in the proposed game. The idea is that the seller does not have the power to coerce buyers to participate in the auction. In step 3, the buyers who decided to participate play the game specified by the mechanism.

In the sequel we incorporate the acceptance stage (step 2 above) in the description of mechanisms, that is, we consider only mechanisms that contain a prior acceptance stage as above. A mechanism is said to be *feasible* if:

(A) At each terminal node it specifies a feasible allocation for the economy.

(B) If buyer i decides not to participate, the seller cannot extract a positive payment from that buyer, and cannot "dump" the object on that buyer.

In a complete information setting step 3 is relatively simple: as the seller does not have to discover privately held information, she can implement any allocation subject to conditions (A) and (B) above. Of course, the seller's decision at step 3 affects buyers' decision whether to participate or not at step 2. Step 3 is much more complex in an incomplete-information setting, since the mechanism must then take into account the incentive constraints.

In most auction-design frameworks it is usually assumed that buyers who decide not to participate obtain some exogenously specified outside-option utility. In contrast, step 2 plays here a major role for the following reason: because of the externalities, a nonparticipating buyer will be affected by the outcome induced by the participating buyers in step 3. (We make the realistic assumption that buyers cannot "escape to the moon.") Outside options are endogenously determined by the mechanism itself, and by others' strategies. Therefore, they play an important role in the revenue-maximizing considerations.

A mechanism is called *optimal* if the game it defines among buyers has a Nash equilibrium (or a Bayes-Nash equilibrium in the incomplete-information case), where the seller achieves the highest payoff among all payoffs possible in equilibria of feasible mechanisms. A problem with the above definition is that an optimal mechanism may possess other equilibria where the seller's revenue is not necessarily maximized. Uniqueness of equilibrium can be obtained, in general, only under very restrictive conditions. In the complete information case, the mechanism we present is optimal in a much stronger sense: the seller achieves maximum revenue in an equilibrium where all buyers use strictly dominant strategies (this is, a fortiori, the unique Nash equilibrium).

III. An Optimal Selling Procedure (Complete Information)

We explicitly consider here the case of negative externalities. We describe a mechanism that will be denoted by Γ. For each buyer i we first define: $\alpha^i = \max_{j\neq i} \alpha_{ji}$. Let $v(i)$ denote a selection out of the set $\{h | \alpha_{hi} = \max_{j\neq i} \alpha_{ji}\}$. Generically $v(i)$ is unique, and $\alpha^i = \alpha_{v(i)i}$. To avoid problems related to the presence of indifferences and a tedious case differentiation, we assume here that there exists a smallest money unit $\varepsilon > 0$, and that $\forall i, j$, $\pi_i - \varepsilon \geq 0$, $\alpha_{ij} - \varepsilon \geq 0$.

Definition of Mechanism Γ.—The buyers simultaneously decide whether to participate or not. Let $\mathcal{B}^*$ denote the set of buyers who decided to participate. For each subset $\mathcal{B}^*$ of $\mathcal{B}$ we now define the winner of the object and the payments made to the seller:

1) When $\mathcal{B}^* = \emptyset$, the seller keeps the object and no payments are made.
2) When $\mathcal{B}^*$ has cardinality no bigger than $n - 2$, the winner is the smallest element in $\mathcal{B}^*$ (this is just an arbitrary rule). Let i be the winner, let $k \in \mathcal{B}^*$, and let j be the smallest element in $\mathcal{B}^* \setminus \{k\}$. (To be consistent in notation, if $\mathcal{B}^* \setminus \{k\} = \emptyset$ we simply define $j = 0$.) If $k = i$, then k is required to pay $\pi_k + \alpha_{jk} - \varepsilon$. Otherwise, k is required to pay $\alpha_{jk} - \alpha_{ik} - \varepsilon$. Note that this last payment may be negative, in which case the seller subsidizes buyer k.
3) If $\mathcal{B}^* = \mathcal{B} \setminus \{h\}$, the winner is $v(h)$. Let $k \in \mathcal{B}^*$, and let j be the smallest element in $\mathcal{B}^* \setminus \{k\}$. If $k = v(h)$ then k is required to pay $\pi_k + \alpha_{jk} - \varepsilon$. Otherwise, k is required to pay $\alpha_{jk} - \alpha_{v(h)k} - \varepsilon$ (this last payment may be a subsidy).
4) If $\mathcal{B}^* = \mathcal{B}$, there are two cases:
   4.1) If $\max_i (\pi_i - \Sigma_{j\neq i} \alpha_{ij}) < 0$, the seller keeps the good and each buyer i is required to pay $\alpha^i - \varepsilon$. The revenue to the seller is

$$R_1 = \sum_i (\alpha^i - \varepsilon). \tag{3}$$

   4.2) If $\max_i (\pi_i - \Sigma_{j\neq i} \alpha_{ij}) \geq 0$, the seller sells to a buyer k such that $k \in \operatorname{argmax}_i (\pi_i - \Sigma_{j\neq i} \alpha_{ij})$ for a price $p = \pi_k + \alpha^k - \varepsilon$. (Generically k is unique.) All other buyers $j \neq k$ are required to pay $\alpha^j - \alpha_{kj} - \varepsilon$ (which is nonnegative by the definition of α^j). The revenue to the seller is given by

$$R_2 = \sum_i (\alpha^i - \varepsilon) + \pi_k - \sum_{k\neq i} \alpha_{ki}. \tag{4}$$

PROPOSITION 1: A) *For each buyer, participation is a strictly dominant strategy in mechanism Γ. The strategy profile where all buyers participate is the unique Nash equilibrium of Γ. The seller's revenue in this equilibrium is given by*

$$\bar{R} = \sum_i (\alpha^i - \varepsilon) + \max\left\{0, \max_i \left(\pi_i - \sum_{j\neq i} \alpha_{ij}\right)\right\}. \tag{5}$$

B) *Let G be a feasible mechanism, and let σ be a Nash equilibrium of G.*

Let $R = R(\sigma)$ denote the seller's revenue when σ is played. It holds that $R \leq \bar{R} + n\varepsilon$.

PROOF:

See the Appendix.

When the smallest money unit ε tends towards zero, Proposition 1 shows that the seller's revenue attains its maximal value in the full-participation equilibrium of the mechanism Γ.[7] Moreover, note that the equilibrium outcome of Γ is always efficient.

If, for each buyer i, the total externality imposed by that buyer is larger than her valuation (that is, $\forall i$, $\Sigma_{j\neq i} \alpha_{ij} > \pi_i$), the seller is better-off by not selling at all while extracting from each buyer a payment equal to his worst fear

[7] If we restrict attention to equilibria of a feasible mechanism G where ties are broken in favor of nonparticipation (that is, buyers participate only if they are strictly better-off than nonparticipating) then it is readily verified that the statement in Proposition 1.B holds without the addition of the term $n\varepsilon$.

(see case 4.1 in the definition of Γ). It is this case which seems to have the flavor of our opening story about Ukraine's nuclear arsenal.

Note that the mechanism Γ has the feature that, in some cases, the seller needs to subsidize some buyers, but this never happens on the equilibrium path.

In order to simplify arguments, we have focused on the case of negative externalities (this is also motivated by the leading examples of selling to downstream competing buyers). Our analysis immediately extends to the general case where externalities may be both positive or negative. When externalities are positive, the object has the features of a public good supplied by a single agent. The incentives not to participate are then well known as the *free-rider effect*. In the general case, a revenue-optimizing seller threatens a nonparticipating buyer (see case 3 in the definition of Γ) with the allocation of the good that leaves this buyer with the lowest possible level of utility (this may be strictly positive). Payments to the seller are adjusted accordingly to make all buyers willing to participate.

The dominant-strategy, revenue-maximizing equilibrium of Γ is *collusion* proof if side payments between members of a deviating coalition are not feasible. Although joint deviations that strictly improve the payoff for all members of the deviating coalition exist, none of these deviations is immune against further deviations of subcoalitions.[8] Hence, all rings are unstable when the seller uses mechanism Γ. For the derivation of this result we implicitly assume that buyers' strategy spaces are exactly those implied by mechanism Γ, that is we assume that no transfer payments are possible between buyers. In the terminology of McAfee and John McMillan (1992) we look at *weak rings*.[9] In the context of Ukraine's nuclear disarmement, some kind of collusive agreement between United States and Russia is likely. However, it is harder to imagine a credible agreement in which the United States makes payments to potential buyers of fissionable material (say in the Middle East) if they agree not to buy.

IV. A Model with Incomplete Information

The seller's main considerations have, so far, been driven by the fact that the presence of externalities endogenized the buyers' outside options and, therefore directly influenced buyers' participation decisions. The major theme in the standard mechanism design literature is, however, the interplay between the actions deemed necessary in order to maximize revenue (or achieve efficiency) and the constraints imposed by the fact that, in practice, the seller has to elicit privately held information about willingnesses to pay. We turn therefore to a more realistic setup where buyers possess private information.

We assume here that buyer i only knows her own valuation π_i and the externality α_{ij} *she imposes on buyer* j, $j \neq i$. These two components may, in general, be correlated. In the nuclear weapons example the private valuation depends on the additional deterrence earned and on the cost of maintenance, while the externality imposed depends on the probability that the weapons will be used, on the probability of proliferation, and so on. In the case of a patent holder who sells to competing buyers, consider the following situation that fits in our framework: a status quo is given where all buyers produce with known marginal costs.

[8] As an example, consider the coalition of all buyers whose members could jointly decide not to participate. This deviation strictly improves the payoff of all buyers. Since the good is not sold, and no payments to the seller are made, buyer i receives a payoff of zero. This is better than $-\alpha^i + \varepsilon$, which is the payoff in the unique equilibrium of Γ. Given this deviation, any buyer i will find it advantageous to further deviate and participate! In this case the deviator obtains the object at a price of $\pi_i - \varepsilon$, which leaves him with a strictly positive payoff. Since participation is a strictly dominant strategy, a similar reasoning for all coalitional deviations shows that the unique equilibrium of Γ is coalition proof (see Douglas B. Bernheim et al., 1987).

[9] If transfers payments between buyers are possible, we can show the following: for any allocation that can be implemented by the seller (using any feasible mechanism) there exists a collusive agreement with transfer payments that makes the members of the ring better-off, unless a buyer exists whose valuation π_i is very large compared to all other variables. Conversely, for any (anticipated) collusive agreement there exists a mechanism that gives incentives to some ring members to deviate from the collusive agreement.

The acquirer of the patent may produce with lower marginal costs if he makes a certain initial investment (say, for buying machines needed for the new process). Both initial investment and future reduced marginal cost are private information at the bidding stage, but become common knowledge if a sale occurs. The fixed-investment influences only valuations, while the magnitude of the reduction in marginal-cost influences both valuations and imposed externalities.

To represent buyer i's private information (that is, his *type*) we use a two-dimensional vector $\mathbf{t}_i = (t_i^1, t_i^2) = (\pi_i, \alpha_i)$, where π_i is buyer i's payoff when she gets the object, and α_i is the externality caused by i to all other buyers (hence, we assume here that $\forall j \neq i$, $\alpha_{ij} = \alpha_i$). We again focus on the case of negative externalities, that is, on the situation in which any potential buyer receives a *positive* payoff when he obtains the object, and a *negative* payoff when anybody else gets it (vis-à-vis the case where the seller keeps the object). The seller has an analogous type represented by the vector $\mathbf{t}_0$, which is publicly known. We assume here for simplicity that $\mathbf{t}_0 = (0, 0)$. For each i, buyer i's type is drawn from $T_i = [\underline{\pi}_i, \overline{\pi}_i] \times [\underline{\alpha}_i, \overline{\alpha}_i] \in \mathbb{R}_+^2$ according with the density f_i, and is independent of all other players' types. Buyers' true types are private information.

We use the following notation: $T := T_1 \times T_2 \times \cdots \times T_n$, with a generic element denoted by $\mathbf{t} = (\mathbf{t}_1, \ldots, \mathbf{t}_n)$; $\phi := f_1 \times f_2 \times \cdots \times f_n$; T_{-i} denotes the product $T_1 \times \cdots \times T_{i-1} \times T_{i+1} \times \cdots \times T_n$, and analogously for ϕ_{-i}, $\mathbf{t}_{-i}$, and so on.

Buyer i's utility is additively separable: if he gets the object and pays x_i to the seller, then his utility is given by $t_i^1 - x_i$; if he pays x_i to the seller and buyer j, $j \neq i$, gets the object, then i's utility is given by $-t_j^2 - x_i$. If each buyer i makes a payment x_i to the seller, then the seller's utility is $\Sigma_{i=1}^n x_i$.

By the Revelation Principle (see Myerson, 1981), there is no loss of generality in restricting attention to direct revelation mechanisms where each buyer reports a type. Moreover, it is enough to consider mechanisms that are incentive compatible and that satisfy the participation constraints, that is, mechanisms for which it is a Bayesian equilibrium for each buyer to participate and report his type truthfully.

Let $\Sigma := \{q \in \mathbb{R}_+^n \mid \Sigma\, q_i \leq 1\}$ be the set of *probability vectors*. The seller specifies the rules of the auction in terms of a *revelation mechanism* (x, p, ρ), where $x_i: T \to \mathbb{R}$, $1 \leq i \leq n$; $p: T \to \Sigma$; $\rho = (\rho^1, \rho^2, \ldots, \rho^n)$ with $\rho^i: T_{-i} \to \Sigma$. The interpretation is as follows: the seller asks each of the buyers simultaneously to report a type. If all buyers submit a type and the reported profile is $(\mathbf{t}_1, \ldots, \mathbf{t}_n) \in T$, buyer i must pay the seller $x_i(\mathbf{t}_1, \ldots, \mathbf{t}_n)$, and he gets the object with probability $p_i(\mathbf{t}_1, \ldots, \mathbf{t}_n)$. If buyer i refuses to participate while all other buyers submit the profile of types $\mathbf{t}_{-i}$, the object is given to buyer j with probability $\rho_j^i(\mathbf{t}_{-i})$, $1 \leq j \leq n$, and no buyer makes a payment to the seller.[10] If two or more buyers refuse to submit a report, then, say, the seller keeps the object with probability 1 and nobody makes any payments.[11]

Suppose player i believes everybody else participates and reports truthfully. Then, to assess the expected value of any of his reports, he only needs to know the conditional expected value, given his own type, of his payment and the probability assignment vector. Define the functions $y_i: T_i \to \mathbb{R}$ and $q_i: T_i \to \mathbb{R}$ as follows:

$$(6) \quad y_i(\mathbf{t}_i) = \int_{T_{-i}} x_i(\mathbf{t}_1, \ldots, \mathbf{t}_n)\phi_{-i}(\mathbf{t}_{-i})\, d\mathbf{t}_{-i},$$

$$(7) \quad q_i(\mathbf{t}_i) = \int_{T_{-i}} p_i(\mathbf{t}_1, \ldots, \mathbf{t}_n)\phi_{-i}(\mathbf{t}_{-i})\, d\mathbf{t}_{-i}.$$

We will refer to these functions as buyer i's *conditional expected payment* and *conditional expected probability assignment* in the mechanism (x, p, ρ). If buyer i believes his opponents

[10] If the domain of the function p were extended to include profiles where some agents report "nonparticipation" then the vectors ρ^i could be included in the definition of p. We have chosen the domain T to simplify notation.

[11] Since we study here Bayes-Nash equilibria in which all buyers choose to participate, joint deviations are irrelevant.

will report truthfully, and reports type s_i when his true type is $\mathbf{t}_i$, his expected utility is

$$(8)\qquad U(\mathbf{s}_i, \mathbf{t}_i) := q_i(\mathbf{s}_i)t_i^1 - \sum_{j\neq i}\int_{T_{-i}} p_j(\mathbf{s}_i, \mathbf{t}_{-i})t_j^2 \times \phi_{-i}(\mathbf{t}_{-i})\, d\mathbf{t}_{-i} - y_i(\mathbf{s}_i).$$

The auction mechanism (x, p, ρ) is said to be *incentive compatible* for buyer i if

$$(9)\qquad U_i(\mathbf{t}_i, \mathbf{t}_i) \geq U(\mathbf{s}_i, \mathbf{t}_i) \quad \text{for all} \quad \mathbf{s}_i, \mathbf{t}_i \in T_i.$$

We now look at the participation constraints. Buyers' outside options are not exogenous: they depend on the seller's action in case that one buyer does not participate, that is on ρ. Since the seller's goal is revenue maximization, and since the revenue collected is constrained by the outside options, it is clear that the optimal threat is to leave a nonparticipating buyer with the lowest possible level of utility. Hence, in case that buyer i does not participate, the seller should threaten to sell the good to the agent that creates the highest possible externality for i (see also the construction of mechanism Γ in the complete information case). For each profile $\mathbf{t}_{-i}$, let $v(i, \mathbf{t}_{-i}) \in \text{argmax}_{j\neq i}\{t_j^2\}$. We obtain that, optimally,

$$(10)\qquad \rho^i_{v(i,t-i)}(\mathbf{t}_{-i}) = 1, \quad \text{and} \quad \rho^i_j(\mathbf{t}_{-i}) = 0, \quad \text{for} \quad j \neq v(i, \mathbf{t}_{-i}).$$

The expected utility of buyer i if i does not participate, while all other buyers participate and report their true types, is given then by

$$(11)\qquad \mathcal{A}_i = -\int_{T_{-i}} \max_{j\neq i}\{t_j^2\}\phi_{-i}(\mathbf{t}_{-i})\, d\mathbf{t}_{-i}.$$

We say that a mechanism satisfies the *participation constraints* for buyer i if

$$(12)\qquad U_i(\mathbf{t}_i, \mathbf{t}_i) \geq \mathcal{A}_i \quad \text{for all} \quad \mathbf{t}_i \in T_i.$$

The participation constraints together with the incentive constraints ensure that the profile of strategies where each buyer participates and truthfully reveals the type constitutes a Bayes-Nash equilibrium in the revelation game.[12]

Since we have already determined the optimal form of the threats ρ, in the sequel we denote mechanisms simply by (x, p). The auction mechanism is *feasible* if it is incentive compatible and if it satisfies the participation constraints for every buyer. Clearly, if (x, p) is feasible, so is $(\bar{x}, p)$, where $\bar{x}_i(\mathbf{t}) := y_i(\mathbf{t}_i)$ for all $\mathbf{t} \in T$. Moreover, with $\bar{x}$ the buyers expect to make the same payment to the seller as with x. Thus, there is no loss of generality in restricting attention to mechanisms for which the payment of each player depends only on his own report. Consequently, we specify below auction mechanisms in terms of (y, p).

The seller wants to maximize total expected revenue. Therefore her problem is

$$(13)\qquad \max_{(y,p)} \sum_{i=1}^{n} \int_{T_i} y_i(\mathbf{t}_i)f_i(\mathbf{t}_i)\, d\mathbf{t}_i$$

subject to incentive compatibility and participation constraints.

PROPOSITION 2: *Consider the mechanism (y, p), and let q_i, $1 \leq i \leq n$, be the corresponding conditional probability assignment func-*

[12] Asher Wolinsky has pointed to us that this equilibrium may employ weakly dominated strategies: in the case that one buyer does not participate (this is off the equilibrium path), the remaining buyers have an incentive to exaggerate the reported externality, since the buyer reporting the largest externality receives the good for free. While a more elaborate description of our mechanism is not necessary to ensure that participation and truthful revelation indeed constitute a Bayesian-Nash equilibrium, we can easily remedy the problem of weak dominance by requiring that the winner j must pay $\pi_j + \max_{k\neq i,j} \alpha_k$ in cases where buyer i does not participate. This procedure, which mimics a second-price auction in the announced externalities, ensures that truthful revelation of externalities is also optimal off the equilibrium path.

tions. The mechanism is incentive compatible if and only if the following conditions hold:

A) $\forall i$, $\forall t_i^2 \in [\underline{\alpha}_i, \overline{\alpha}_i]$, *the function* $t_i^1 \to q_i(t_i^1, t_i^2)$ *is nondecreasing on* $[\underline{\pi}_i, \overline{\pi}_i]$.
B) $\forall i$, $\forall t_i^1 \in [\underline{\pi}_i, \overline{\pi}_i]$, *the function* $t_i^2 \to q_i(t_i^1, t_i^2)$ *is constant on* $[\underline{\alpha}_i, \overline{\alpha}_i]$.
C) $\forall i$, $\forall \mathbf{t}_i = (t_i^1, t_i^2)$, $y_i(\mathbf{t}_i) = -U_i((\underline{\pi}_i, \underline{\alpha}_i), (\underline{\pi}_i, \underline{\alpha}_i)) + t_i^1 q_i(\mathbf{t}_i) - \int_{\underline{\pi}_i}^{t_i^1} q_i(v, t_i^2)\, dv - \sum_{j \neq i} \int_{T_{-i}} p_j(\mathbf{t}_i, \mathbf{t}_{-i}) t_j^2 \phi_{-i}(\mathbf{t}_{-i})\, d\mathbf{t}_{-i}$.

PROOF:
See the Appendix.

By the form of the transfers in Proposition 2.C, and by equation (8) we obtain that for all i, and for all $\mathbf{t}_i \in T_i$,

$$(14) \quad U_i(\mathbf{t}_i, \mathbf{t}_i) = U_i((\underline{\pi}_i, \underline{\alpha}_i), (\underline{\pi}_i, \underline{\alpha}_i)) + \int_{\underline{\pi}_i}^{t_i^1} q_i(v, t_i^2)\, dv.$$

Since the functions q_i are nonnegative, we obtain that the participation constraint (see equation (12)) is satisfied for all types of buyer i if it is satisfied for the type $(\underline{\pi}_i, \underline{\alpha}_i)$. Moreover, the participation constraint of type $(\underline{\pi}_i, \underline{\alpha}_i)$ must bind at the optimum. Otherwise, the seller could increase by a fixed amount the payments required from all types (and hence increase revenue) without affecting the incentive constraints. Hence we have $\forall i$, $U_i((\underline{\pi}_i, \underline{\alpha}_i), (\underline{\pi}_i, \underline{\alpha}_i)) = \mathcal{A}_i$. We now define

$$(15) \quad h_i(t_i^1, t_i^2) := \int_{t_i^1}^{\overline{\pi}_i} f_i(\tau, t_i^2)\, d\tau / f_i(t_i^1, t_i^2).$$

Note that

$$(16) \quad \int_{\underline{\pi}_i}^{\overline{\pi}_i} \left(\int_{\underline{\pi}_i}^{t_i^1} q_i(v, t_i^2)\, dv \right) f_i(t_i^1, t_i^2)\, dt_i^1 = \int_{\underline{\pi}_i}^{\overline{\pi}_i} q_i(v, t_i^2) h_i(v, t_i^2) f_i(v, t_i^2)\, dv.$$

Incorporating in program (13) the expressions for buyers' payments (see Proposition 2.C), and using the above observations, the seller's problem becomes:

$$(17) \quad \max_p \sum_{i=1}^n (-\mathcal{A}_i) + \int_T \left(\sum_{i=1}^n [t_i^1 - (n-1)t_i^2 - h_i(t_i^1, t_i^2)] p_i(\mathbf{t}) \right) \phi(\mathbf{t})\, d\mathbf{t}$$

such that:

1) $\forall i$, $\forall t_i^2 \in [\underline{\alpha}_i, \overline{\alpha}_i]$, the function $t_i^1 \to q_i(t_i^1, t_i^2)$ is nondecreasing on $[\underline{\pi}_i, \overline{\pi}_i]$.
2) $\forall i$, $\forall t_i^1 \in [\underline{\pi}_i, \overline{\pi}_i]$, the function $t_i^2 \to q_i(t_i^1, t_i^2)$ is constant on $[\underline{\alpha}_i, \overline{\alpha}_i]$.
3) $\forall i$, $\forall \mathbf{t} \in T$, $p_i(\mathbf{t}) \geq 0$; $\forall \mathbf{t} \in T$, $\sum_{i=1}^n p_i(\mathbf{t}) \leq 1$.

An inspection of program (17) reveals that for each i, at the optimum, the function $p_i(\mathbf{t})$ must have a property similar to that of the function $q_i(\mathbf{t}_i)$ (see Proposition 2.B): if $\mathbf{s}_i$, $\mathbf{t}_i$ are such that $s_i^1 = t_i^1$, then $\forall \mathbf{t}_{-i}$, $p_i(\mathbf{s}_i, \mathbf{t}_{-i}) = p_i(\mathbf{t}_i, \mathbf{t}_{-i})$. Hence the probabilities p_i are only functions of the valuations $t_1^1, t_2^1, \ldots, t_n^1$. Recalling that $\mathbf{t}_i = (t_i^1, t_i^2) = (\pi_i, \alpha_i)$, we denote by $p_i(\pi_1, \pi_2, \ldots, \pi_n)$ the probability that buyer i gets the object if the reports are $(\pi_1, \alpha_1), (\pi_2, \alpha_2), \ldots, (\pi_n, \alpha_n)$. We can now transform the seller's maximization problem into a standard one-dimensional program since we can first integrate over externalities in program (17).

We now discuss several applications. To simplify calculations, we assume below that the situation is symmetric in the sense that $\forall i$, $T_i = [\underline{\pi}, \overline{\pi}] \times [\underline{\alpha}, \overline{\alpha}]$, and $f_i = f$. We denote by Π the set $[\underline{\pi}, \overline{\pi}]^n$.

Application I.—We assume here that valuations and externalities are independent, that is, $f(t_i^1, t_i^2) = f^1(t_i^1) f^2(t_i^2)$. Assume without loss of generality that $\int_{\underline{\pi}}^{\overline{\pi}} f^1(\tau)\, d\tau = \int_{\underline{\alpha}}^{\overline{\alpha}} f^2(\tau)\, d\tau = 1$. This is just a normalization. Let $F^1(t_i^1)$ and $F^2(t_i^2)$ denote the distributions of the densities f^1 and f^2, respectively.

The expected utility of a nonparticipating buyer is given by $\mathcal{A} = -(n-1)\int_{\underline{\alpha}}^{\bar{\alpha}} \tau \cdot (F^2(\tau))^{n-2} f^2(\tau)\, d\tau$.[13] Denote the expected value of the imposed externality by $E = \int_{\underline{\alpha}}^{\bar{\alpha}} \tau f^2(\tau)\, d\tau$. By first integrating in (17) with respect to the second coordinate of buyers' types, we obtain that the seller's problem reduces to

$$\text{(18)}\quad \max_p -(n\mathcal{A}) + \int_\Pi \left(\sum_{i=1}^n \left[\pi_i - \frac{1 - F^1(\pi_i)}{f^1(\pi_i)} - (n-1)E \right] p_i(\pi_1, \dots, \pi_n) \right) \times f^1(\pi_i) \dots f^1(\pi_n)\, d\pi_1 \dots d\pi_n$$

such that:

1) $\forall i, \forall \alpha \in [\bar{\alpha}, \underline{\alpha}]$, the function $\pi \to q_i(\pi, \alpha)$ is nondecreasing on $[\underline{\pi}, \bar{\pi}]$.
2) $\forall i, \forall (\pi_1, \dots, \pi_n) \in \Pi$, $p_i(\pi_1, \dots, \pi_n) \geq 0$; $\forall (\pi_1, \dots, \pi_n) \in \Pi$, $\Sigma_{i=1}^n p_i(\pi_1, \dots, \pi_n) \leq 1$.

The *regular* case occurs when the function $\pi - (1 - F^1(\pi))/f^1(\pi)$ is increasing (see Myerson, 1981). Then it is possible to solve the program (18) by pointwise maximization, while disregarding the monotonicity constraints. The resulting functions q_i will necessarily satisfy those constraints. The solution requires selling to $i^* = \text{Argmax}_i\, \pi_i$ provided that $\pi_{i^*} - (1 - F^1(\pi_{i^*}))/f^1(\pi_{i^*}) - (n-1)E \geq 0$, and keeping the good otherwise (while obtaining payments from all buyers).

Application II.—We now assume a perfect correlation between the valuation and the externality imposed on others, that is, α_i is a function, $g(\pi_i)$, of π_i. Assume that valuations are distributed on $[\underline{\pi}, \bar{\pi}]$ with density f^1, and denote by F^1 the distribution of f^1. In the special case where the function g is monotonically increasing, the expected utility of a nonparticipating buyer is given by $\mathcal{A} = -(n-1) \times \int_{\underline{\pi}}^{\bar{\pi}} g(\tau)(F^1(\tau))^{n-2} f^1(\tau)\, d\tau$. The seller's program reduces to a one-dimensional standard problem:

$$\text{(19)}\quad \max_p -(n\mathcal{A}) + \int_\Pi \left(\sum_{i=1}^n \left[\pi_i - \frac{1 - F^1(\pi_i)}{f^1(\pi_i)} - (n-1)g(\pi_i) \right] p_i(\pi_1, \dots, \pi_n) \right) \times f^1(\pi_1) \dots f^1(\pi_n)\, d\pi_1 \dots d\pi_n$$

such that:

1) $\forall i, \forall \alpha \in [\bar{\alpha}, \underline{\alpha}]$, the function $\pi \to q_i(\pi, \alpha)$ is nondecreasing on $[\underline{\pi}, \bar{\pi}]$.
2) $\forall i, \forall (\pi_1, \dots, \pi_n) \in \Pi$, $p_i(\pi_1, \dots, \pi_n) \geq 0$; $\forall (\pi_1, \dots, \pi_n) \in \Pi$, $\Sigma_{i=1}^n p_i(\pi_1, \dots, \pi_n) \leq 1$.

The regular case occurs when the function $\pi - (1 - F^1(\pi))/f^1(\pi) - (n-1)g(\pi)$ is increasing. The solution to program (19) requires selling to $i^* = \text{Argmax}_i\, \pi_i$ provided that $\pi_{i^*} - (1 - F^1(\pi_{i^*}))/f^1(\pi_{i^*}) - (n-1)g(\pi_{i^*}) \geq 0$, and keeping the good otherwise.

Discussion and Comparison with the Complete Information Case.—The main new phenomenon in the incomplete information case is the effect of the incentive constraints. Since the externality imposed on others does not directly influence buyers' willingness to pay, the seller takes into account only the expected externality caused by a given buyer, and not the true one. The seller's action directly depends only on announced valuations. Moreover, the relevant variables are the *virtual* valuations $\pi - (1 - F^1(\pi))/f^1(\pi)$, which are always smaller than the true valuations. As a consequence, the outcome is not anymore efficient. Depending on the case, the seller may sell both "too much" or "too little." Consider first Application II: assume that the function $\pi - (n-1)g(\pi)$ is increasing, and let $i^* = \text{Argmax}_i\, \pi_i$,

[13] This is the expectation of the first-order statistic for the random variables governing the externalities imposed by all other buyers.

where $\pi_1, \ldots, \pi_n$ are the reported valuations. Efficiency requires a sale if $\pi_{i*} - (n-1)g(\pi_{i*}) \geq 0$ while a revenue-optimizing seller sells only if $\pi_{i*} - (n-1)g(\pi_{i*}) \geq (1 - F^1(\pi_{i*}))/f^1(\pi_{i*})$. For all $\pi_{i*} \neq \bar{\pi}$ the term $(1 - F^1(\pi_{i*}))/f^1(\pi_{i*})$ is strictly positive, implying that the seller may not sell in cases where efficiency requires a sale. Interestingly, the opposite effect may also occur. Consider a regular case in Application I. Assume, for example, that a buyer announces $(\bar{\pi}, \alpha)$, and that $\bar{\pi} > (n-1)E$. The good is then invariably sold, although if $\bar{\pi} < (n-1)\alpha$, efficiency requires that no sale occurs.

Apart from efficiency, all our main insights continue to hold. In particular, the threats in case of nonparticipation play an important role—they involve here expected externalities. Moreover, the seller is able to extract surplus also from nonacquirers. If the expected imposed externality E is very large compared to pure valuations, the seller is better-off by not selling while extracting payoffs from all buyers. We now compute a simple example to illustrate these facts.

Example.—There are two potential buyers. Buyer i's valuation is uniformly drawn from $[0, 1]$, independently from the externality he imposes, which is also uniformly drawn from $[0, 1]$ (we are in the case of Application I). Let $\mathbf{t}_i = (\pi_i, \alpha_i)$. Using the results for Application I, we now derive the optimal auction form and the seller's revenue. The expected value of the imposed externality is given by $E = \frac{1}{2}$. The expected utility of a nonparticipating buyer is given by $\mathcal{A} = -\frac{1}{2}$. The function $\pi - (1 - F^1(\pi))/f^1(\pi) = 2\pi - 1$ is increasing, and we are in a regular case. We know that an optimizing seller sells to $i^* = \text{Argmax}_i\, \pi_i$ provided that $\pi_{i*} - (1 - F^1(\pi_{i*}))/f^1(\pi_{i*}) - E = 2\pi_{i*} - \frac{3}{2} \geq 0$, and keeps the good otherwise. This implies

$$(20)\quad p_1(\mathbf{t}_1, \mathbf{t}_2) = \begin{cases} 0, & \text{if } \pi_1 \leq \pi_2 \\ 0, & \text{if } \pi_1 > \pi_2 \text{ and } \pi_1 \leq \frac{3}{4} \\ 1, & \text{if } \pi_1 > \pi_2 \text{ and } \pi_1 > \frac{3}{4} \end{cases}$$

and analogously for $p_2(\mathbf{t}_1, \mathbf{t}_2)$. Using equation (7) and the expression for buyers' expected payments in Proposition 2.C, we obtain

$$(21)\quad y_i(\mathbf{t}_i) = \begin{cases} \frac{3}{8}, & \text{if } \pi_i \leq \frac{3}{4} \\ (\pi_i)^2/2 + \pi_i/2 + \frac{9}{32}, & \text{if } \pi_i > \frac{3}{4}. \end{cases}$$

By equation (13), the seller's revenue is $\frac{107}{96} = 1.114$. We note the following: 1) all types (also those that have no chance of getting the object) are required to pay at least $\frac{3}{8}$. In particular, a buyer with valuation $\pi_i \leq \frac{3}{4}$ is indifferent between participation (in which case he pays $\frac{3}{8}$, suffers an expected externality of $(\frac{1}{4})\, E = \frac{1}{8}$, and his expected utility is $-\frac{1}{2}$), and nonparticipating (in which case his expected utility is $\mathcal{A} = -\frac{1}{2}$). 2) The seller's reserve price when the two buyers participate $(\frac{3}{4})$ is higher than the optimal one in case there were no externalities $(\frac{1}{2})$. By selling less often, the seller is able to raise the fee he obtains from types that do not get the object.

V. Concluding Comments

We have studied optimal selling procedures for situations in which the outcome of the sale affects the future interaction between agents. In these situations we have identified several phenomena related to the fact that outside options are endogenous. We have shown that a revenue-maximizing seller will be able to extract revenue also from agents that do not obtain the auctioned object. In particular, the seller is better off by not selling at all (while extracting payments from the buyers that fear a sale) if the sum of externalities created by a sale is larger than all valuations. In our opinion, this last observation, coupled with the fact that the maintenance of a functioning nuclear arsenal imposes a huge financial burden, adds an economic element to Ukraine's strategy of dismantling its nuclear weapons. Ukraine has credibly committed to avoid much feared proliferation, and, accordingly, it has been amply rewarded by both Russia and the United States (and, to a lesser extent, by other Western European countries).

In a complete-information framework the seller's main considerations were driven by the participation decisions of potential buyers. In an incomplete-information context the seller must also take into account the incentive effects connected to the revelation of privately known information. We have shown that the incentive constraints are very strong, and that the outcome of the revenue-maximizing procedure is not always efficient. In particular, a sale may occur even if efficiency requires that the good stays with the seller. This phenomenon contrasts with the standard result in the case without externalities where the outcome may be inefficient only because the seller restricts supply.

Externalities are usually caused by actions taken after the close of the sale. Throughout the paper we have implicitly assumed that contracting upon the magnitude of the external effects is not feasible. As in the main body of work on optimal mechanism design, we have assumed that the seller can commit to the announced procedure. In particular, the seller's action can be tailored to the exact group of participating buyers, and there are commitments not to sell at all in some situations, or to sell to a very specific agent if some other agents choose not to participate. In some situations commitments to future actions are possible (recall Ukraine's destruction of nuclear weapons). If commitment is not possible, the analysis becomes more intricate. Agents that anticipate renegotiation will alter their strategy (in particular, revealing "too much" information at a prior stage may have negative effects at later stages). The benefits to the seller will depend upon how the renegotiation process is specified.

The problems raised by renegotiation are not unique to our model. Consider, for example, a standard second-price auction for an indivisible good without externalities. After bids have been revealed, the seller would prefer to receive an amount equal to highest bid.[14] Yet, if the buyers knew that the seller would try to renegotiate to obtain this value, they would systematically underreport their true valuations.

In general, if the seller cannot commit to future actions (like not selling, and so on), we expect that the outcome of the constrained revenue-maximizing procedure will be even farther away from the social optimum. Even in the complete-information case efficiency will not always be attained since the seller's ability to extract surplus crucially depends on threats that may not be optimal ex post.

[14] Similarly, a seller that announces a minimum price under which she is not willing to sell would nevertheless prefer to sell if the second-highest bid is less than the minimum price, but higher than the seller's true reservation value. Note that an optimizing seller announces a minimum price which is higher than the true reservation value—this is the old story of a monopolist restricting supply.

APPENDIX

PROOF OF PROPOSITION 1:

A) Consider buyer i, and assume that the buyers in $\mathcal{B}\setminus\{i\}$ play a pure strategy profile such that the set of participating buyers is an arbitrary set $\mathcal{B}' \subseteq \mathcal{B}\setminus\{i\}$. There are three possible cases:

1) $\mathcal{B}' = \mathcal{B}\setminus\{i\}$. If i does not participate her payoff is $-\alpha^i$ (see case 3 in the definition of mechanism Γ). If i participates then her payoff is $-\alpha^i + \varepsilon$ (see case 4 in the definition of Γ), and participation is optimal.

2) $\mathcal{B}' = \mathcal{B}\setminus\{i, m\}$. If i does not participate then her payoff is $-\alpha_{ji}$, where the j is the smallest element in $\mathcal{B}'$ (see case 2 in the definition of Γ). If i participates, the good is allocated to $v(m)$ (see case 3 in the definition of Γ). If $i = v(m)$, i is required to pay $\pi_i - \alpha_{ji} - \varepsilon$ and her payoff is $-\alpha_{ji} + \varepsilon$. If $i \neq v(m)$, then i is required to pay $\alpha_{ji} - \alpha_{v(m)i} - \varepsilon$, and her payoff is again $-\alpha_{ji} + \varepsilon$. Participation is therefore optimal for i.

3) $\mathcal{B}'$ has cardinality strictly less than $n - 2$. If i does not participate the good is allocated to j where j is the smallest element in $\mathcal{B}'$, and i's payoff is $-\alpha_{ji}$. If i participates, the good is allocated to k, where k is the smallest element in $\mathcal{B}' \cup \{i\}$. If $i = k$, i is required to pay $\pi_i - \alpha_{ji} - \varepsilon$, and her payoff is $-\alpha_{ji} + \varepsilon$. If $i \neq k$, i is required to pay $\alpha_{ji} - \alpha_{ki} - \varepsilon$, and her payoff is again $-\alpha_{ji} + \varepsilon$. Participation is again optimal for i.

We have shown that for all possible pure strategy profiles of buyers in $\mathcal{B} \setminus \{i\}$, buyer i is strictly better-off by participating. Thus, participation is a best response for player i against any strategy profile (mixed or otherwise) of the opponents. The formula for $\bar{R}$ is obvious by the definition of Γ.

B) Assume that σ is a pure strategy profile. Let $\mathcal{B}^*$ denote the set of buyers that participate when σ is played in mechanism G. Assume first that the seller keeps the object (with probability 1) when σ is played. If buyer i participates then it must be the case that his payment to the seller is at most α^i. This follows by the assumption that σ is an equilibrium and by the fact that by not participating i can secure at least $-\alpha^i$. Hence $R \leq \sum_{i \in B^*} \alpha^i \leq \bar{R} + n\varepsilon$. Assume now that the seller sells to buyer i (with probability 1) when σ is played. (This implies, by the no-dumping assumption, that i participates.) As above, we obtain that 1) buyer i will not pay more than $\pi_i + \alpha^i$. 2) Any other buyer j that accepted to participate will not pay more than $(\alpha^j - \alpha_{ij})$. In this case j's payoff is given by $-(\alpha^j - \alpha_{ij}) - \alpha_{ij} = -\alpha^j$. Hence, for the seller, we obtain

$$R \leq \pi_i + \alpha^i + \sum_{j \in B^* \setminus \{i\}} (\alpha^j - \alpha_{ij}) \leq \pi_i + \alpha^i + \sum_{j \neq i} (\alpha^j - \alpha_{ij}) \leq \bar{R} + n\varepsilon.$$

The second inequality holds because, for each buyer k, we have $(\alpha^k - \alpha_{ik}) \geq 0$. The last inequality follows by the definition of $\bar{R}$ in the statement of the Proposition. In cases where the equilibrium σ uses mixed strategies, or where the mechanism uses randomized allocations, the proof easily follows from the discussion above.

PROOF OF PROPOSITION 2:

By a standard argument in the theory of multidimensional mechanism design with linear utility functions (see Jean-Charles Rochet, 1985; or McAfee and McMillan, 1988), a revelation mechanism is incentive compatible *if and only if* $\forall i$, the equilibrium interim utility of the agents, $S_i(\mathbf{t}_i) = \max_{\mathbf{s}_i \in T_i} U_i(\mathbf{s}_i, \mathbf{t}_i) = U_i(\mathbf{t}_i, \mathbf{t}_i)$, is a convex potential and the transfer $y_i(\mathbf{t}_i)$ is the convex conjugate of $S_i(\mathbf{t}_i)$. Details can be found in Jehiel et al. (1995). Note that

$$\text{(A1)} \quad S_i(\mathbf{t}_i) = q^i(\mathbf{t}_i)t_i^1 - \sum_{j \neq i} \int_{T_{-i}} p_j(\mathbf{t}_i, \mathbf{t}_{-i}) \times t_j^2 \phi_{-i}(\mathbf{t}_{-i})\, d\mathbf{t}_{-i} - y_i(\mathbf{t}_i).$$

Note also that convexity implies differentiability almost everywhere, and that convexity of $S_i(\mathbf{t}_i)$ is equivalent to monotonicity of gradient $\nabla S_i(\mathbf{t}_i)$. Moreover, by the Envelope Theorem we have

$$\text{(A2)} \quad \frac{dS_i}{dt_i^1}(\mathbf{t}_i) = \frac{dU_i}{dt_i^1}(\mathbf{t}_i, \mathbf{t}_i) = \frac{dU_i}{dt_i^1}(\mathbf{s}_i, \mathbf{t}_i)\bigg|_{\mathbf{s}_i = \mathbf{t}_i} = q_i(\mathbf{t}_i);$$

$$\text{(A3)} \quad \frac{dS_i}{dt_i^2}(\mathbf{t}_i) = \frac{dU_i}{dt_i^2}(\mathbf{t}_i, \mathbf{t}_i) = \frac{dU_i}{dt_i^2}(\mathbf{s}_i, \mathbf{t}_i)\bigg|_{\mathbf{s}_i = \mathbf{t}_i} = 0.$$

These equations, together with the fact that $S_i(\mathbf{t}_i)$ is a potential function, imply

$$\text{(A4)} \quad S_i(\mathbf{t}_i) = S_i(\underline{\pi}_i, \underline{\alpha}_i) + \int_{(\underline{\pi}_i, \underline{\alpha}_i)}^{\mathbf{t}_i} [q_i(\tau), 0] \cdot d\tau,$$

where the integral can be taken on *any* continuous curve from $(\underline{\pi}_i, \underline{\alpha}_i)$ to $\mathbf{t}_i$. The monotonicity of $q_i(\mathbf{t}_i)$ with respect to the first coordinate, t_i^1, directly follows from the monotonicity of $\nabla S_i(\mathbf{t}_i)$.

We next show that $q_i(\mathbf{t}_i)$ is constant with respect to the second coordinate, t_i^2. Integrating in (A4) on the curve formed by the line segments $[(\underline{\pi}_i, \underline{\alpha}_i), (\underline{\pi}_i, t_i^2)]$ and $[(\underline{\pi}_i, t_i^2), (t_i^1, t_i^2)]$, yields

$$(A5) \quad S_i(\mathbf{t}_i) = S_i(\underline{\pi}, \underline{\alpha}_i) + \int_{\underline{\pi}}^{t_i^1} q_i(v, t_i^2)\, dv.$$

Integrating in (A4) on the curve formed by the line segments $[(\underline{\pi}_i, \underline{\alpha}_i), (t_i^1, \underline{\alpha}_i)]$ and $[(t_i^1, \underline{\alpha}_i), (t_i^1, t_i^2)]$, yields

$$(A6) \quad S_i(\mathbf{t}_i) = S_i(\underline{\pi}_i, \underline{\alpha}_i) + \int_{\underline{\pi}}^{t_i^1} q_i(v, \underline{\alpha}_i)\, dv.$$

Subtracting (A6) from (A5), we obtain

$$(A7) \quad \int_{\underline{\pi}}^{t_i^1} [q_i(v, t_i^2) - q_i(v, \underline{\alpha}_i)]\, dv = 0.$$

Since q_i is monotonic in the first coordinate (and hence continuous almost everywhere), and since equation (A7) holds for all $t_i^1 \in [\underline{\pi}, \overline{\pi}]$ and for all $t_i^2 \in [\underline{\alpha}, \overline{\alpha}]$, we can conclude that the function $t_i^2 \to q_i(t_i^1, t_i^2)$ is constant on $[\underline{\alpha}, \overline{\alpha}]$.

The formula for the transfer $y_i(\mathbf{t}_i)$ immediately follows from (A1) and (A5).

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