License Auctions and Market Structure

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Abstract

We analyze the interplay between license auctions and market structure in a model with several incumbents and several potential entrants. The focus is on the competitiveness induced by the number of auctioned licenses. Under plausible conditions, we show that auctioning more licenses need not result in a more competitive final outcome, contrary to what common sense suggests. This is due to the nature of competition among incumbents, which sometimes exhibits free-riding. We illustrate some results with examples drawn from the recent European license auctions for 3G mobile telephony.

1. Introduction

License auctions shape the market structure of the respective industry. As a consequence, firms competing to acquire a license are not indifferent about the final form of the market structure (in particular about how many and which other firms are going to be licensed)[1]. From a theoretical viewpoint, this means that license auctions should be analyzed in the setup of auctions with externalities[2] in which bidders' valuations depend not only on whether they get licensed or not (as traditional auction theory assumes), but also on the whole architecture of who gets which license.

The main goal in many license auctions is economic efficiency, which implies the maximization of some weighted sum of consumers' and producers' surpluses.[3] A difficulty appears since consumers do not directly participate at the auctions.[4] Therefore, a flexible design where bidding firms have the freedom to determine the number of licenses is unlikely to induce a market structure favorable to both, firms and consumers. In other words, the doctrine of 'letting the market decide' on the number of licenses may be suboptimal.

Unfortunately, ex-ante estimates of expected consumers' surplus in future market scenarios are difficult to make. Since standard oligopoly models predict that, in reasonable ranges, both consumers' surplus and overall efficiency increase with increased competition among firms, the creation of sufficient market competition - as measured by the number of active firms - becomes a proxy goal that can be more or less successfully implemented by the regulatory agency[5]. This goal has been named in a variety of licensing exercises and we focus on it in this paper. But we also point out possible cases where entry does not necessarily increase welfare.

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[1] This aspect seems to be well-understood by managers and analysts. For example, a major investment bank estimated per-licenses values of Euro 14.75, 15.88, and 17.6 billion for a German UMTS market with 6, 5, and 4 firms, respectively.

[2] See for example Jehiel-Moldovanu 2000, 2001 and Jehiel-Moldovanu-Stacchetti 1996.

[3] Although it is invariably less advertised, revenue is also an important part of the agency's objective function because governments tend to prefer solutions that require less subsidies (or even provide budgetary surpluses).

[4] Another, more technical hurdle is presented by the fact that, in complex situations fitting well some spectrum auction environments, multi-unit efficient auctions simply do not exist and second-best mechanisms are not yet known (see Jehiel and Moldovanu, 2001).

[5] Entry cannot be extended without limit since infrastructure costs are very high. We regard here only those situations where (at least marginally) more firms are associated with more efficiency.

A feature present in many license auctions is that earlier allocations of licenses have already established incumbents, possibly operating according to some previous technological standard. Potential new entrants (i.e., firms that do not already operate a network) face two handicaps: 1) The fixed cost of setting up a network (say of antennae and relays) is very large. In contrast, a substantial part of the incumbents' fixed costs are already sunk, since they can use parts of their already existing facilities. 2) Incumbents are also driven by entry pre-emption motives which translate into increased willingness to pay for new licenses.

We consider here a situation where several incumbents are already present in the market, and we ask how the number of auctioned licenses affects competitiveness in the industry. We measure competitiveness by the post-auction number of active firms, e.g. the number of incumbents augmented by the number of new entries. We will also mention how revenue is affected. Potential acquirers of new licenses include the incumbents and entrants who are not yet present in the market. The downstream competition among licensed firms is modeled via a reduced-form industry profit function. This modeling approach implies that values for licenses are endogenous, and depend on the final number of active firms. For simplicity, but also in order to isolate the effect of market structure considerations, we assume that there are no informational asymmetries among the potential acquirers[6].

Our analysis is related to the literature on patent licensing[7], pioneered in Arrow (1962). Gilbert and Newbery (1982) use an auction model to study the interaction between a monopolist incumbent and a potential entrant competing for an innovation. Their main result is the persistence of the monopolist who takes into account the potential negative externality and therefore uses preemptive patenting. Krishna (1993, 1999) and Gale and Stegeman (2000) study sequential auctions of inputs[8] and show that monopoly may not persist in that context. Rodriguez (1997) studies sequential license auctions in a model with incumbents and entrants. He imposes conditions on the reduced-form downstream profit functions which directly induce sure entry at each auction[9]. Kamien and

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[6]Papers that focus on informational asymmetries in market design are Auriol and Laffont (1992), Dana and Spier (1994), Jehiel and Moldovanu (2004), McGuire and Riordan (1995), and Milgrom (1996).

[7]See the survey of Kamien, 1992.

[8]McAfee (1998) studies capacity auctions in oligopolies where some firms are capacity constrained, and he shows that unconstrained firms may win the auction in some cases.

[9]This holds unless the initial market structure is monopolistic, in which case the Gilbert-Newbery result applies.

Tauman (1986), Kamien, Tauman and Oren (1992) and Katz and Shapiro (1985, 1986) study patent licensing in oligopolistic downstream industries. These authors assume that all firms are ex-ante symmetric[10] - this is the key difference between theirs and our work.

Our main insight is that auctioning the maximum possible number of licenses need not induce a higher degree of competitiveness. This can easily be illustrated in the case where two incumbents currently earn large profits, old and new licenses are close substitutes, and the addition of licensed entrants would cause a significant drop in per-firm profit in the industry[11]: Suppose first that only one license is being auctioned. Then each incumbent is willing to avoid entry, but would rather prefer that the other incumbent pays the price of preemption (this follows from the substitutability assumption). As a result, sometimes an entrant acquires the license because each incumbent is relying on the other to prevent entry. Suppose now that two licenses are auctioned: now there is an equilibrium in which each incumbent buys a license (thus sharing the cost of preemption), and there is no entry. From a game-theoretic viewpoint, the form of the interaction has moved from a war of attrition to a coordination type of problem, thus inducing a drastic change of outcome[12]. The free-riding phenomenon among incumbents is connected to the positive externality identified in the literature on mergers[13], and in the literature on entry deterrence[14]. But, those literatures did not discuss the resulting war of attrition[15].

The above example is pretty extreme, and we provide an analysis under more general assumptions. When the number of auctioned licenses is greater than the number of incumbents, we provide plausible conditions under which all incumbents get licenses. When it is smaller, the possibility of a war of attrition phenomenon arises whenever old and new licenses are sufficiently substitutable. We also note

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[10]This assumption is common in practically the entire literature on vertical relations - see Segal, 1999 for a theoretically unifying approach. An exception is Rockett, 1990 who studies the externalities caused by asymmetric licensees on the licensor (but not on each other).

[11]Possibly, because prior to entry the incumbent firms managed to achieve some form of collusion.

[12]We also analyze below the incentives to collude in such a situation.

[13]See Perry and Porter (1985), McAfee and Williams (1988) and Farrell and Shapiro (1990).

[14]See Bernehim (1984), Gilbert and Vives (1986), and Waldman (1987).

[15]For the literature on entry deterrence, this is because partial contributions to entry deterrence are allowed. Thus, in a complete information setting no inefficiency arises (see Bernheim, 1984 and Gilbert-Vives, 1986) except if uncertainty is added (see Waldman, 1987). For an analysis of a war of attrition in a bargaining context, see Jehiel and Moldovanu (1995)

that competitiveness and revenue may be both positively or negatively correlated, depending on the parameters of the model.

The 3G mobile telephony industry   To illustrate the scope of our analysis, it is instructive to very briefly consider the British, Dutch and German license auction for "third generation (3G)" mobile telephony[16]. Both Germany and the UK had 4 incumbents offering 2G services according to the GSM standard[17], and various economic viability estimates, together with physical spectrum limitations implied that no more than 6 firms could be licensed. Holland had 5 GSM incumbents.

The UK designers first considered an auction for 4 licenses[18], but then settled on 5, one more than the number of incumbents. The frequency capacities attached to each license were fixed ex-ante, but different licenses came with different capacities, and the largest license has been reserved for a new entrant. Hence, the UK design actively tried to level the playing field among incumbents and new entrants. In contrast, the Dutch designers did not recognize that reserving licenses for entrants was necessary, and sold exactly 5 licenses. The German design was more flexible, since it allowed outcomes with 4, 5 or 6 licenses. Besides an endogenous number of licenses, the design also allowed for endogenous capacity endowments[19]. This was, in our opinion, its main weakness, since, in principle, it allowed incumbents to completely preempt entry by bidding for additional capacity.

In all auctions mentioned above, all respective incumbents got licenses. While there was no entry in Holland, an entrant (unavoidably) bought the reserved license in the UK. There were two new entries in Germany. The process by which the outcome was reached is amusing: after the stage where 6 firms were left in the auction (which equaled the maximal possible number of licenses and meant that the auction could end immediately) and an aggregate bid of DM 63

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[16]See Klemperer, 2000 and Jehiel and Moldovanu, 2003 for more detailed accounts. See McMillan, 1994, McAfee and McMillan, 1996, Cramton, 1997 and Milgrom, 2000 and the entire issue 3 of *JEMS* **6**, 1997 for accounts of the FCC auctions conducted in the US. Our model applies to examples from other industries, such as power generation (see Cameron, Cramton and Wilson, 1997).

[17]Some other firms buy services from incumbents and resell them, but do not have networks.

[18]That design called for an ascending-price auction followed by a sealed bid stage among the 5 last bidders on the 4 licenses. See Klemperer (2000) for a discussion.

[19]Bids were made on 12 spectrum packages. A firm had to acquire at least two packages in order to be licensed, but could acquire up to three packages.

billion, the incumbents continued to try to acquire additional capacity and hence, simultaneously, to reduce the number of available licenses. Faced with determined entrants and nervous investors, they ultimately gave up without any change in the physical outcome. But all firms were another DM 35 billion poorer! It is now almost certain that, after spending so much money on licenses, the two new entrants do not have resources to build the networks and operate the licenses.

2. The Model

We consider an industry with n incumbents. New firms can enter the market by acquiring licenses from a regulatory agency. We assume that there are m potential entrants.

The regulatory agency organizes an auction for new licenses. New licenses may differ in specification from the licenses owned by incumbents. Our model allows for various forms of substitutability/complementarity between old and new licenses.

We assume that incumbents are all alike, and similarly entrants are all alike. This is to highlight the effect of the asymmetry between incumbents and entrants (rather than the asymmetry within a given group). Under such assumptions, every profit function can be expressed as a function of the number k of licenses and the number z of active firms after the auction. From these two numbers, one can infer how many incumbents get a license and how many entrants get a license. Together, these numbers characterize the state of the oligopolistic market. Specifically, suppose that $k > 0$ licenses are auctioned and suppose that $s \leq k$ entrants acquire a license (and thus $k - s$ incumbents acquire a new license). Then the number of active firms after the auction is $z = n + s$, and all profits depend on k and z as follows:

1. An unsuccessful entrant receives a profit of zero.

2. A successful entrant receives a profit of $w_e^k(z)$.

3. An unsuccessful incumbent receives a profit of $\pi^k(z)$.

4. A successful incumbent receives a profit of $w_i^k(z)$.

We assume that the profit functions π^k, w_e^k, w_i^k are common knowledge among bidders, and that they are decreasing functions of z. This last assumption captures

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the natural feature that all active firms prefer that fewer other firms are active in the market.

The status quo corresponds to the case $k = 0$ where no new licenses are auctioned. In this case, an incumbent receives $\pi^0(n)$.

We assume that $\forall z, k, \ w_i^k(z) \geq \pi^0(z) \geq \pi^k(z)$, and $w_e^k(z) \geq 0$, and we denote $v^k(z) \equiv w_i^k(z) - \pi^k(z)$. Note that we could add to the model explicit assumptions such as $\pi^{k-1}(n+s) > \pi^k(n+s)$, i.e., *holding the total number of firms constant,* an unsuccessful incumbent has a higher profit when less rivals obtain a new license. But, while such assumptions seem plausible in many contexts, we do not need them in our framework since, in an auction for k licenses, the value of $\pi^{k-1}(n+s)$ is not relevant for the analysis (assuming there are sufficient entrants): if, for example, a given rival incumbent "gives up" a license, then the license will go to an entrant for sure.

When the number of licenses k is kept fixed, we will omit the dependence in k of the profit functions in order to simplify notation.

The above setup is sufficiently general to cover many applications of interest. For example, if new and old licenses are perfect substitutes, and if the marginal value of a second license is zero, then $\pi^0 \equiv \pi^k \equiv w_e^k \equiv w_i^k$ and $v^k \equiv 0$. If they are imperfect substitutes and a new license is more valuable to incumbents than an old one, then $v^k > 0$. If entrants have to incur a fixed cost c to catch up the incumbents' advantage, but old and new licenses are otherwise perfect substitutes, then $\pi^k \equiv w_i^k$ and $w_e^k \equiv \pi^k - c$, and so on[20]...

The analysis in this paper focuses on how the number of entries varies as we vary the number k of licenses. We we will consider different scenarios regarding the dependence of the profit functions with respect to k[21] but our main insight (e.g., more licenses may lead to fewer entries) can be obtained even when these profit functions are independent of k.

Throughout the paper, the analysis is restricted to equilibria where symmetric bidders use symmetric strategies, and where bidders do not use (weakly) dominated strategies[22]. To ensure the existence of equilibria in our complete informa-

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[20]The model also covers the case (which is less interesting from the viewpoint of this paper) in which the activities of the two licenses are completely independent. This corresponds to π being constant.

[21]A plausible assumption might be that these profit functions are non-increasing with k (to reflect the idea that for a given number of active firms it cannot hurt that fewer incumbents get a (new) license).

[22]Equilibrium considerations would automatically yield the restriction to (weakly) undominated strategies if some private information, say on valuations, were introduced.

tion models we need tie-breaking assumptions: these are tailored to the specific auctions (an equivalent alternative is to introduce a smallest money unit).

3. Auctions for one license

In this section, we assume that there is one license for sale, i.e. $k = 1$, and we omit the index k from the profit functions. The license is sold through a Vickrey or sealed-bid second price auction[23]. All bidders simultaneously submit bids, which are non-negative real numbers. The bidder with the highest bid gets the license and pays the second highest bid for it.

The main thing to note is the fundamental difference between incumbents and potential entrants with respect to the nature of their willingness to pay: If an entrant acquires the license at a price $p \leq w_e(n+1)$, then it expects an increase in payoff from zero to $w_e(n+1) - p$. Hence, an entrant is prepared to pay up to $w_e(n+1)$ for a license. In contrast, an incumbent's willingness to pay depends on whether, otherwise, the license will be acquired by another incumbent (*intrinsic valuation*) or by a potential entrant (*preemptive willingness to pay*). While the incumbent's intrinsic valuation is given by $v(n)$, its preemptive willingness to pay is $\pi(n) + v(n) - \pi(n+1)$. The outcome of the auction will vary, depending on the relation between $w_e(n+1)$, $\pi(n) - \pi(n+1) + v(n)$, and $v(n)$. There are several cases of interest:

Case 1. Assume that $\pi(n) + v(n) - \pi(n+1) < w_e(n+1)$. In this case, an entrant's expected payoff $w_e(n+1)$ is higher than the maximum willingness to pay of an incumbent $\pi(n) + v(n) - \pi(n+1)$. Entry occurs for sure, and, assuming that there are at least two potential entrants, the successful entrant has to pay $w_e(n+1)$, which is the revenue of the auction.

Case 2. Assume that $w_e(n+1) < v(n)$. In this case, an entrant's expected payoff is lower than an incumbent's intrinsic valuation of the license. Entry is not possible, and the preemption motive is irrelevant. At the auction the incumbents compete for the license, and for $n > 2$ the expected payoff of an incumbent is $\pi(n)$ (i.e., the premium of the winning incumbent is dissipated in competition), and the revenue is given by $v(n)$.

Case 3. Assume that $v(n) < w_e(n+1) < \pi(n) + v(n) - \pi(n+1)$. In this case, an entrants' willingness to pay is lower than an incumbents' preemptive willingness to pay, but higher than an incumbents' intrinsic valuation of the license. If there

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[23]The English ascending price auction yields here the same results.

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is only one incumbent, then the incumbent's willingness to pay is unambiguously defined by $\pi(n) + v(n) - \pi(n+1)$, and the incumbent will acquire the license with probability one.[24] An interesting phenomenon occurs when there are $n > 1$ incumbents. A bidding "war of attrition" may take place among the incumbents, since their bids must balance two conflicting interests: on the one side they wish to preempt entry, but on the other side they wish to let some other incumbent pay the price of preemption.

The purpose of this section is to analyze the equilibrium bidding in Case 3 when there are $n > 1$ incumbents. In order to ensure equilibrium existence, we use the following tie-breaking rule: an entrant with a highest bid cannot win the license if there exists at least an incumbent that has made the same highest bid.[25] Moreover, if s incumbents tie at the highest bid, then each wins the license with probability $1/s$.

If $n > 1$, the game has several equilibria in Case 3. Obviously, there are asymmetric equilibria where each entrant bids $w_e(n+1)$, one incumbent bids $w_e(n+1)$, and the remaining $n-1$ incumbents bid some lower amount. In these equilibria, entry will be deterred for sure. However, such asymmetric equilibria require a high degree of coordination among incumbents (i.e., which incumbent will be in charge of deterring entry?). Moreover, such coordination may be particularly hard to achieve in practice since the equilibria favor $n-1$ incumbents to the detriment of one incumbent (i.e., the one who is in charge of entry deterring).[26] The following proposition shows that there also exists a symmetric equilibrium where incumbents use mixed strategies and where entry occurs with positive probability.

Proposition 3.1. *Assume that $k = 1$ and $n > 1$. Assume also that $v(n) < w_e(n+1) < \pi(n) + v(n) - \pi(n+1)$, i.e., we are in Case 3. Let $\delta(n) \equiv \frac{\pi(n) - \pi(n+1)}{w_e(n+1) - v(n)}$. The following bidding strategies constitute a symmetric equilibrium: each entrant bids $w_e(n+1)$; each incumbent bids $w_e(n+1)$ with probability q, and bids 0 (or below $w_e(n+1)$) with probability $1-q$, where q is implicitly defined by*

$$\delta(n) = \frac{1-q}{nq} \cdot [(1-q)^{-n} - 1]. \tag{3.1}$$

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[24]This is the standard case of monopoly persistence (see Gilbert and Newbery, 1982).

[25]The obtained equilibrium corresponds to the limit as $\varepsilon \to 0$ of the equilibria obtained when (1) all bidders with highest bid have the same probability of getting the license and (2) bids can only take values of ε, 2ε, 3ε, ...

[26]Note that side-payments between incumbents outside the auction may be illegal.

In any symmetric equilibrium, a potential entrant gets the license with probability $x = (1-q)^n$, and has a zero expected profit. An incumbent's expected profit is given by:

$$V_i = [1 - (1-q)^{n-1}]\pi(n) + (1-q)^{n-1}\pi(n+1).$$

Proof. See Appendix B. ∎

The proposition contrasts Gilbert and Newbery's (1982) finding that entry will be deterred with probability one in the case of a single incumbent. When $n > 1$, the equilibrium entry probability x is entirely determined by the number n of incumbents and the function $\delta(n)$ that aggregates the incumbents' and entrants' profit functions. We have:

Proposition 3.2. *Under the assumptions of Proposition 3.1, the equilibrium probability of entry x is a decreasing function of δ. If, in addition, δ is non-increasing in n, then the probability of entry x is an increasing function of n.*

Proof. See Appendix B. ∎

It is relatively intuitive that the probability of entry decreases in δ: if the profit loss due to entry, $\pi(n) - \pi(n+1)$, is larger, incumbents are more willing to deter entry, while for larger $v(n)$ and smaller $w_e(n+1)$, the net cost of acquiring a license for an incumbent is smaller. It is less straightforward to see that the entry probability monotonically increases in n: on the one hand, as n increases, the free riding problem among incumbents becomes more severe, inducing a higher probability of entry; on the other hand, for a given strategy of incumbents, the probability that all n incumbents bid zero (and hence that an entrant wins the license) is decreasing in n. Proposition 3.2 shows that the first effect is domi-nant if δ is non-increasing in n. Whether this condition is satisfied can be checked in each specific IO context. For example, in the case of perfect substitutabil-ity where $w_i = w_e$ and $w_e = \pi$, the condition reduces to the requirement that $[\pi(n) - \pi(n+1)]/[\pi(n+1)]$ is decreasing, which is satisfied in many oligopoly models.

Example 3.3. *The solution of equation (3.1) for $n = 2$ is $q = \frac{2\delta - 2}{2\delta - 1}$, and the probability of entry is given by $(1-q)^2 = \frac{1}{(2\delta - 1)^2}$. The solution for $n = 3$ is $q = \frac{1}{2(3\delta - 1)}\left(6\delta - 3 - \sqrt{(12\delta - 3)}\right)$ and the probability of entry is given by $(1 - q)^3 = \frac{1}{8}\frac{\left(1 + \sqrt{(12\delta - 3)}\right)^3}{(3\delta - 1)^3}$. The probabilities of entry as a function of δ are depicted in the following figure.*

Insert Figure 1 here

In Appendix A, we analyze the incumbents' incentives to arrange an *explicit* collusion among them. To this end, we define the ratio $\frac{\Delta^C - \Delta^{NC}}{w_e(n+1)}$ where Δ^{NC} stands for the equilibrium profit of incumbents characterized in Proposition 3.1, Δ^C stands for the per-firm profit of incumbents under perfect collusion (assuming that side-payments can be organized outside the auction). The higher this ratio, the higher the incumbents' incentive to collude. Our main finding is that this ratio is highest when $\delta(n)$ is neither too low nor too large. When $\delta(n)$ is lower than $1/n$, incumbents do not find it profitable to avoid entry even though it would occur in the non-cooperative equilibrium. When $\delta(n)$ is very large, the ratio $\frac{\Delta^C - \Delta^{NC}}{w_e(n+1)}$ is also low, which is more surprising. The point is that, despite the war of attrition among incumbents, an entrant very rarely gets the license in the non-cooperative outcome (because each incumbent is so afraid that an entrant gets in).

4. Multi-License Auctions

We now analyze the effect of auctioning several licenses. We consider the Vickrey auction (that extends here the sealed-bid second-price auction used for $k = 1$): each bidder i submits a bid b_i; the bidders with the k highest bids get a license each and pay the $(k + 1)$ highest bid. That is, rearranging the bids in increasing order, $b_{i(1)} \geq \cdots \geq b_{i(k)} \geq b_{i(k+1)} \geq \cdots$, every bidder $i(1), \cdots, i(k)$ gets a license and pays $b_{i(k+1)}$[27]. The simultaneous ascending price version (where the price gradually increases until there are k remaining bidders who each obtains a license and pays the current price) yields here the same results.

4.1. When all incumbents get licensed

In the British UMTS auction, there were five licenses, one more than the number of GSM incumbents. All four incumbents obtained a new license. In light of our model this is not surprising since, besides expecting a higher direct benefit (due to lower infrastructure costs), incumbents are also driven by preemption motives. The following analysis makes this observation precise.

The multi-license auctions may have several equilibria. As above, we restrict attention to symmetric equilibria (in undominated strategies). Moreover, if there

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[27]We describe below the appropriate tie-breaking rule guaranteeing the existence of equilibrium.

are multiple symmetric equilibria, we are interested in the equilibrium where incumbents maximize the degree of entry deterrence.

Proposition 4.1. *Assume that $k > n$ and that $\pi(k) + v(k) - \pi(k+1) > w_e(k)$. A symmetric equilibrium of the $k-$license auction is as follows: entrants bid $w_e(k)$, and incumbents bid above that (say, $\pi(k) + v(k) - \pi(k+1)$). All incumbents get a license, and $k - n$ entrants get a license. All licenses are sold, and the revenue is given by $kw_e(k)$.*

Proof. If the above strategy profile is played, entrants get a payoff of zero, and incumbents get a payoff of $w_i(k) - w_e(k) = v(k) + \pi(k) - w_e(k)$. The above strategies form an equilibrium because: 1) Given that all other firms bid at least $w_e(k)$ and given that all licenses are sold, incumbent i has no incentive to bid below that since this would give him a payoff of $\pi(k+1) < v(k) + \pi(k) - w_e(k)$; 2) Given that an entrant expects that n out of k licenses will be sold to incumbents, the value of a license to an entrant is $w_e(k)$. ∎

Two cases must be qualitatively distinguished in the interpretation of Proposition 4.1. If $v(k) > w_e(k)$, incumbents intrinsically value the license more than entrants, and therefore it is legitimate that they are served first. But they continue to be licensed with probability 1 even if $v(k) < w_e(k)$, as long as $\pi(k) + v(k) - \pi(k+1) > w_e(k)$. In this case incumbents primarily bid i order to maintain a less competitive market structure.

Suppose now that the number of auctioned licenses k coincides with the number of incumbents n. This was the Dutch case (5 licenses, 5 incumbents) where no entry occurred.

Proposition 4.2. *Assume that $\pi(n) + v(n) - \pi(n+1) > w_e(n+1)$. The following strategies define a symmetric equilibrium with $k = n$ licenses: Incumbents bid above $w_e(n+1)$ (say $\pi(n) + v(n) - \pi(n+1)$). Entrants bid $w_e(n+1)$. The incumbents acquire one license each at price $w_e(n+1)$. There is no entry and total revenue is $nw_e(n+1)$.*

Proof. The above strategies form an equilibrium because: 1) Given that incumbent i bids above $w_e(n+1)$, incumbent i' has no incentive to bid below $w_e(n+1)$. Incumbent i' would then leave one license to an entrant, and his resulting payoff would be $\pi(n+1) < \pi(n) + v(n) - w_e(n+1)$; 2) Given that an entrant expects that all other licenses go to incumbents, the value of a license to an entrant is $w_e(n+1)$. ∎

The intuition for Proposition 4.2 is very much the same as for the case $k > n$. Accordingly, if $v(n) > w_e(n+1)$, the direct benefit for the new license is superior for incumbents than for entrants, and therefore it is legitimate that incumbents acquire a license. But, if $v(n) < w_e(n+1)$ and $\pi(n)+v(n)-w_e(n+1) > w_e(n+1)$, it is the preemptive motive that explains the outcome. It should be noted (as mentioned in Introduction) that in the $k = n$ license case, incumbents have an easy way to share the price of preemption by buying one license each.[28]

Comparing the one-license auction with the n-license auction, we get:

Corollary 4.3. *Assume that* $v(n) < w_e(n+1) < \pi(n)+v(n)-\pi(n+1)$. *Restricting attention to the equilibria displayed in Propositions 3.1 and 4.2, respectively, the expected number of entries when one license is auctioned is higher than the expected number of entries when $k = n$ licenses are auctioned.*

Remark: Proposition 4.2 has displayed one symmetric equilibrium of the n-license Vickrey auction, but sometimes several equilibria exist. To illustrate the point, assume that there are $n = 2$ incumbents, and that 2 licenses are sold. If $\pi(3) + v(3) - \pi(4) > w_e(4)$, the above equilibrium outcome is the unique outcome of symmetric equilibria in undominated strategies. But, if $\pi(3) + v(3) - \pi(4) < w_e(4)$, there is another symmetric equilibrium that induces a very different outcome: entrants bid $w_e(4)$, and incumbents bid below $w_e(4)$. Hence two entrants get new licenses at price $w_e(4)$. The multiplicity in the case where $\pi(3)+v(3)-\pi(4) < w_e(4)$ and $\pi(2)+v(2)-\pi(3) > w_e(3)$ is caused, essentially, by a coordination problem among incumbents. If incumbent 1 expects incumbent 2 to make a low bid, the question is whether there will eventually be 3 or 4 active firms in the industry. Since $\pi(3) + v(3) - \pi(4) < w_e(4)$, incumbent 1 is also not willing to acquire a license. If incumbent 1 expects incumbent 2 to make a high bid, the question is whether there will eventually be 2 or 3 active firms, and incumbent 1 is then willing to acquire a new license (since $\pi(2)+v(2)-\pi(3) > w_e(3)$). It is clear that, from the point of view of incumbents,[29] the no-entry equilibrium Pareto-dominates the maximum-entry equilibrium.

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[28]The possibility of similarly collusive-like outcomes in auctions for several objects has been studied by Wilson (1979) and Anton and Yao (1992), and more recently by Ausubel and Schwarz (1999), Brusco and Lopomo (1999), and Klemperer (2000).

[29]Entrants get zero utility anyway.

4.2. Supply restriction

Corollary 4.3 shows that an auction for one license may induce *more* entry than an auction for n licenses if $v(n) < w_e(n+1) < \pi(n) + v(n) - \pi(n+1)$ (i.e., if an incumbent's intrinsic valuation is below $w_e(n+1)$, but its preemptive willingness to pay is above $w_e(n+1)$). If the primary concern is to induce more competitiveness, and if at most n licenses can be auctioned,[30] then restricting the number of auctioned licenses may be desirable. Note that revenue is undoubtedly higher in the n-license auction (where it is equal to $nw_e(n+1)$) than in the one-license auction (where it is equal to $w_e(n+1)$). Hence the tension between competitiveness and revenue may be acute. The rest of this subsection considers several forms of supply restriction.

Given the above general observation, it makes sense to ask how entry is affected by the number of licenses k for $k < n$. This turns out to be a difficult question even in the perfect information setting considered here. The main difficulty is that whenever $v(n) < w_e(n+1) < \pi(n) + v(n) - \pi(n+1)$, the k-license auction with $k < n$ has the structure of a war of attrition with k objects, and it is very hard to compare the probabilities of entries for the various k, $k < n$.

We provide a partial answer to the above question in a setting where the benefit functions v^k and w_e^k depend on the number k of auctioned licenses.[31]

Proposition 4.4. *Suppose the benefit functions v^k and w_e^k depend on the number k of auctioned licenses. Fix $\overline{k} < n$, and assume that $w_e^n(n+1) < \pi^n(n) + v^n(n) - \pi^n(n+1)$ and that $v^k(n) < w_e^k(n+1)$ if and only if $k \geq \overline{k}$. Assume also that there are at most $k = n$ licenses. Then the expected number of entries is maximized for $k' \in [\overline{k}, n-1]$.*

Proof. We have $v^k(n) > w_e^k(n+1)$ for $k < \overline{k}$. Thus, if $k < \overline{k}$ licenses are auctioned, incumbents acquire all of them, and there is no entry. By Proposition 4.2, there is no entry either if $k = n$ licenses are auctioned. If $\overline{k}$ licenses are auctioned, there is entry with positive probability depending on whether or not $w_e^{\overline{k}}(n+1) < \pi^{\overline{k}}(n) + v^{\overline{k}}(n) - \pi^{\overline{k}}(n+1)$. Hence the maximum must occur for $k' \in [\overline{k}, n-1]$. ∎

Remark: It is more likely that $v^k(n)$ is a decreasing function of k, since benefits associated with the new licenses are probably larger when fewer new licenses are available. The assumption on $v^k(n)$ is then plausible.

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[30]For example, in some cases there are capacity limitations that physically limit the number of possible licenses.

[31]In the next Proposition, we consider the Pareto-efficient symmetric equilibrium.

Proposition 4.4 shows that a transition from $k = n$ to $k < n$ may be beneficial for competitiveness. Is it possible that the expected number of entrants can also be increased by auctioning $k < n$ instead of $k > n$ licenses? Obviously, if $2n - 1$ or more licenses are auctioned, at least $n - 1$ entrants will acquire a license, and there is no way to induce a higher competitiveness by auctioning $k < n$ licenses. Assume therefore that at most $k < 2n-1$ licenses can be auctioned. The following proposition identifies simple circumstances where indeed more entry is expected if $k < n$ than if $k \in [n, 2n - 2]$.

Proposition 4.5. *Assume that for all k, $w_e(k) = w_e$, $v(k) = v$, $\pi(k) - \pi(k+1) > 0$ and that $\pi(k) + v - \pi(k + 1) = w_e \cdot (1 + \varepsilon) > w_e$. Then, if ε is small enough, for all $k \in [0, 2n - 2]$, the expected number of entries is maximized when $n - 1$ licenses are auctioned.*

Proof. If $k \geq n$ licenses are auctioned, it follows from arguments similar to those in the proofs of Propositions 4.1 and 4.2 that all incumbents get licensed. Hence, the number of entries is 0 if $k = n$ and it is $k - n$ if $k > n$.

If $k < n$ licenses are auctioned, there is a war of attrition phenomenon. Entrants bid w_e, and incumbents use a mixed strategy[32]: bid w_e with probability q, and bid 0 with probability $1-q$. In equilibrium an incumbent has to be indifferent between bidding 0 and bidding w_e. Bidding w_e is effective and hence advantageous relatively to a bid of 0 only if at most $k - 1$ other incumbents bid w_e. The net gain provided by such a bid is $\pi(k) + v - \pi(k + 1) - w_e = \varepsilon w_e$. If k or more incumbents bid w_e, such a bid has a cost of at least $\frac{k}{n}(w_e - v) > 0$[33]. As ε goes to 0, the probability q must also converge to 0 (so that the indifference condition continues to hold). When q is close to 0, there are approximately k entries on expectation, hence the number of entries is maximized by setting $k = n - 1$. ∎

The conditions displayed in Proposition 4.5 are obviously restrictive.[34] However, Proposition 4.5 clearly demonstrates that auctioning less licenses may induce more entry. The main reason is that preemption takes the strategic form of tacit collusion if $k \geq n$ and the form of a war of attrition if $k < n$.

4.3. Endogenous license supply

In all auction formats analyzed above, the number of licenses did not depend on bidders' behavior at the auction. We now consider an auction format in which

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[32]Remember that we restrict attention to incumbent-symmetric equilibria.

[33]Note that this expression does not converge to 0 as ε gets small.

[34]The independence with respect to k is unlikely to be satisfied in most cases.

the number of licenses is endogenously determined by the bid structure[35]. Specifically, consider the following auction format inspired by a proposal submitted by GTE(1997). All bidders simultaneously submit bids. Let $b^{\max}$ be the highest bid. All bidders i who have submitted a bid b_i in the interval $[(1-h)b^{\max}, b^{\max}]$ get a license. The number of winning bidders is thus endogenously determined. The scalar $h \in [0,1]$ is set exogenously, and is part of the description of the auction format. Suppose there are k winners. Then each winning bidder must pay a price equal to the $(k+1)$-highest bid, that is, $b^{(k+1)}$.

The rationale for this proposal is based on a trade-off between the benefits of entry on the one side, and the cost inefficiency associated with not allocating the market to the presumably most efficient firm on the other. {*new:*In the following proposition we assume that w_e is a constant, i.e., independent of the number of active firms. This assumption is made here in order to simplify the analysis by avoiding an uninteresting multiplicity of cases.

Proposition 4.6. *Assume that there are $n > 1$ incumbents and $m \geq 1$ entrants. Assume that w_e is constant. Assume also that $v(n) < w_e$ and $\pi(n) + v(n) - \pi(n+m) > w_e$. Let $\delta_m(n) \equiv \frac{\pi(n) - \pi(n+m)}{w_e - v(n)}$. The following bidding strategies constitute a symmetric equilibrium[36]: each entrant bids w_e; each incumbent bids $\overline{b} > w_e/[1-h]$ with probability q^{GTE}, and bids 0 with probability $1 - q^{GTE}$, where*

$$q^{GTE} = 1 - (\delta(n))^{\frac{1}{1-n}}.$$

Proof. See Appendix B. ∎

Remark: Despite the fact that several licenses may be sold in equilibrium, the incumbents do not achieve a highly collusive agreement. The equilibrium bidding strategies still reflect the war of attrition among incumbents, and this auction format induces more entry than the n-license Vickrey auction analyzed in Proposition 4.2. For illustrative purpose, we compare the entry induced by the GTE auction with the entry induced by the one-license auction.

There are two effects which go in opposite direction. On the one hand, the GTE format exacerbates the market structure impact and therefore induces a lower probability of entry. This comes from the observation that, for $m > 1$, $\pi(n) - \pi(n+m)$ is likely to be larger than $\pi(n) - \pi(n+1)$. On the other hand,

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[35]Recall that this feature was also part of the German design for the UMTS license auctions.
[36]In the special case where $\pi(n+1) = \cdots = \pi(n+m)$, this is the only symmetric equilibrium in undominated strategies.

assuming that $\pi(n) - \pi(n + m)$ and $\pi(n) - \pi(n + 1)$ are close to each other, the nature of the respective wars of attrition is such that the probability of entry is larger in the GTE auction. The point is that the cost of bidding high is greater in the GTE auction than it is in the one-license auction: In the GTE auction, when a firm bids high, it has to buy a license, whereas in the one-license auction, sometimes it does not need to buy it if other incumbents have made high bids. This effect results in a lower probability of high incumbent bids in the GTE auction, and leads therefore to a larger entry probability than in the one-licene auction[37].

5. Extensions

5.1. License-specific bids

In the multi-license auction case, we have considered an auction format where bidders cannot indicate which license is more valuable to them. This is natural if licenses are homogenous (as in our simple theoretical model). But, in practice, licenses may not be all identical. It is then worth looking at the implications of formats allowing license-specific bids. At first glance, license-specific bids seem to encumber coordination among incumbents (e.g., who bids on which license). Absent such coordination, even the case of $k = n$ may have equilibria in which each incumbent randomizes its bid on each license, resulting in some positive probability of entry - in contrast to the no entry outcome of Proposition 4.2.

On the other hand, in spite of license-specific bids, coordination among incumbents may still be considerably easier to achieve in the case of $k = n$ than in the case of $k = 1$. Recall that for $k = 1$ when $v(n) < w_e(n + 1) < \pi(n) + v(n) - \pi(n + 1)$, coordination on asymmetric equilibria with entry deterrence requires that one incumbent is willing to incur the cost of entry deterrence (i.e., by bidding $w_e(n + 1)$ with probability one) to the benefit of all other incumbent firms. Therefore, there is a high incentive for free riding. In contrast, in the case of $k = n$, coordination on equilibria in which each incumbent obtains one license implies that the cost of entry deterrence is shared among all incumbents - similarly as with non-license-specific bids (Proposition 4.2).

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[37]If *all* bidders are symmetric, then this type of auction may restrict the number of licenses precisely to a point where one additional license would cause the profit to drop a lot. Since this may be a signal that this additional license is very valuable to the consumers, the auction format may be quite inefficient in this case.

In practice, most auction formats (especially in complex environments where uncertainty about values is high) allow for several rounds of bidding, enabling bidders to revise their bids and thereby, in principle, to coordinate on who bids on what license. For example, consider a multi-unit ascending auction format such as the one used by the FCC to sell the US spectrum for the 2G license auctions. Assume that $k = n$ licenses are being auctioned. Since most activity rules allow the participants to switch from license to license,[38] after an initial coordination phase, each incumbent may stick to a specific license. Continuing to assume that entrants do not have higher pure valuations than an incumbent's preemptive willingness to pay, the analysis of such auction formats would yield predictions similar to those developed in the Vickrey auction (cf. Proposition 4.2).

Thus, the main insight developed in this paper, i.e., that putting more licenses for sale may facilitate coordination among incumbents in order to deter entry, would be preserved in more complex environments and auction formats.

5.2. Other instruments to induce competitiveness

The analysis in this paper has revealed that auctioning a greater number of licenses need not result in a higher probability of market entry when there are $n > 1$ incumbents. A priori, there are other instruments that can be used in order to influence the induced market structure: some license(s) could be reserved for entrants; entrants could be entitled to bidding credits; or incumbents could be limited in the capacity that they can purchase.

While a complete welfare analysis of these policy measures is beyond the scope of this paper, we highlight some of the main issues in the following discussion. Reserving licenses for entrants has been advocated for as a way of attracting more entrants to participate in the auction.[39] Our analysis provides an alternative (and new) argument for reserving licenses to entrants. For example, recall that the transition from a one-license setting to a two-license setting may be detrimental to entry. By contrast, consider a two-license auction where one license is a-priori

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[38]A key ingredient of most activity rules used in practice is that the closing of auctions is simultaneous, thus allowing for complete re-adjustment of bids until the very end (see Milgrom 2000).

[39]We have implicitly assumed that participation costs are negligible. Observe that an entrant's participation decision is unaffected if some licenses are reserved since the Bertrand competition between entrants drives here their profits to zero.

reserved for new entrants: one obtains then a new entry for sure[40], and, due to the war of attrition on the license where incumbents are allowed to bid, another new entry with some positive probability. Thus, such a set-aside leads to more entry than simple auctions over one or two licenses.

A perfect use of set-asides should, in principle, allow the regulatory agency to induce whatever market structure it considers to be desirable. This, however, requires that the government has perfect information about how much any potential licensee values a license. If the government's information is imperfect, and if there are not enough licenses, set-asides may result in an inefficient allocation of licenses: Suppose for instance that there are $k \leq n$ licenses and that incumbents intrinsically value the license more than entrants, i.e., $v(n) > w_e(n+1)$. Without set-asides, each license will go to incumbent firm.[41] Set-asides will, however, allocate licenses to low-value (e.g., inefficient) firms. If the resale market in licenses/capacity is either inexistent or functions badly,[42] this inefficiency in the allocation of licenses may outweigh any positive effect on social welfare due to market entry. For this reason, the existing legal framework in many countries does not allow any explicit discrimination among firms.[43] The following simple example illustrates this phenomenon and the necessary caution:

Example 5.1. : *There are three incumbents and two new licenses. The competition among firms is a la Cournot with inverse demand function $P = 1 - Q$, where $Q = \sum_i q_i$ is the total output of active firms. In the status quo incumbents have a constant marginal cost $c < 1$. We look at the case where potential entrants are less efficient users of a new license than incumbents. Assume therefore that, if an incumbent obtains a license, its marginal cost will be reduced to $c - \varepsilon$, $0 < \varepsilon < c$. If a potential entrant obtains a license, its marginal cost will be c.*

Social welfare is defined as the sum of firms' profits and consumer surplus. In this example, an incumbent's and an entrant's valuations are, respectively, given by $v(3) = \frac{1}{2}\varepsilon(1-c)$ and $w_e(4) = \frac{1}{25}(1-c-\varepsilon)^2$. For illustration, let $c = \frac{8}{10}$ and assume that $\varepsilon > \frac{29}{20} - \frac{1}{4}\sqrt{33}$. Then we get $v(3) > w_e(4)$.

If one license is reserved for an entrant, only one incumbent gets a license. Consumer surplus and welfare are given by, respectively, $C^{setaside}(4) = \frac{1}{1250}(4+5\varepsilon)^2$

[40]Given that there are entrants willing to bid.

[41]Even though there is no entry in this case, this is good for welfare because incumbents' intrinsic value is higher than entrants' value for the license.

[42]E.g. an unsuccessful incumbent may not be able to smoothly transfer productive assets to the newly licensed firm.

[43]This was the case, for example, in the German and Dutch UMTS auctions.

and $W^{setaside}(4) = \frac{3}{1250}(8 + 20\varepsilon + 325\varepsilon^2)$. *If no licenses are reserved for entrants, both licenses go to incumbents. Consumer surplus and welfare are, respectively:* $C(3) = \frac{1}{800}(3 + 10\varepsilon)^2$ *and* $W(3) = \frac{1}{160}(3 + 20\varepsilon + 140\varepsilon^2) > W^{setaside}(4)$.

Bidding credits can be used to enhance a potential entrant's willingness to pay for a license. It follows from Proposition 3.2 that a small bidding credit (leading to a slightly lower value of δ) may increase the probability of entry in a one-license auction when there are $n > 1$ incumbents and $v(n) < w_e(n+1) < \pi(n) + v(n) - \pi(n+1)$,i.e., an entrant's valuation is higher than an incumbent's intrinsic valuation but lower than an incumbent's preemptive willingness to pay. Hence, bidding credits may result in a higher degree of competitiveness. On the other hand, if there were $k = n$ licenses, a small bidding credit may not be sufficient to promote entry. In order to induce entry it is necessary that the bidding credit raises an entrant's willingness to pay above the incumbent's preemptive willingness to pay. An optimal usage of bidding credits clearly requires to weigh any positive welfare effects of entry against the costs of paying the credits. Moreover, there are again high information requirements. If the government has only limited information about the potential licensees' valuations, bidding credits may result in an inefficient allocation of licenses (similarly to set-asides) and this inefficiency may outweigh any potential social gains.

Finally, recall that, in our analysis, we have restricted bidders to acquire at most one new license. A non-trivial assessment of capacity caps requires a setting where bidders can buy more than one new license. But, again, the government needs rather precise information about future operational plans in order to formulate sensible caps on capacity. Other overtly discriminatory mechanisms are subject to the same caveat.

6. Conclusion

We have analyzed the auction of new licenses in an oligopolistic industry. The focus was on the role of market structure considerations in determining the auction's outcome (in particular the number of licensed firms, and the revenue obtained at the auction). An important observation is that the auction format determines the incumbents' possibilities to preempt new entry in the market. In this context, the relation between the number of new licenses and the number of incumbents plays a major role.

In contrast to the present work, most of the auction-theoretic literature focuses on informational problems. But, in order to conduct a serious discussion

about the merits of various auction designs in the context of recent privatization and licensing processes, it is necessary to augment those "classical" models by incorporating market structure elements.

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8. Appendix A (Explicit collusion among incumbents in the one-license auction)

In this Appendix, we consider the possibility of *explicit* collusion[44] among incumbents. We compare the highest collusive payoff incumbents could achieve (using any kind of mechanism) to the payoff they obtain in the non-collusive bidding analyzed above[45].

Let Δ^C be the per-firm profit of incumbents under perfect collusion and let Δ^{NC} be the profit of incumbent firms in the above symmetric equilibrium outcome.

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[44]By explicit collusion, we mean a situation where incumbents can fully agree on their bidding behavior at the auction, and can make any kind of transfers between themselves, possibly outside the auction.

[45]Caillaud and Jehiel (1998) study collusion in simpler IO setup, but with asymmetric information among bidders. They show how market structure considerations may complicate the information sharing among colluding bidders.

The entrants' willingness to pay is invariably $w_e(n+1)$. Note that when incumbents collude, the price paid for the license is always $w_e(n + 1)$, since the absence of competition between incumbents drives down the price to entrants' willingness to pay of entrants.

We wish to compare the difference $\Delta^C - \Delta^{NC}$ to $w_e(n + 1)$, and we denote

$$\Delta = \frac{\Delta^C - \Delta^{NC}}{w_e(n + 1)}.$$

The higher this ratio, the higher the incumbents' incentive to collude. There are several cases of interest:

1. If $v(n) > w_e(n+1)$, then $\Delta^{NC} = \pi(n)$, and $\Delta^C = \pi(n) + \frac{v(n)-w_e(n+1)}{n}$. Thus, $\Delta = \frac{v(n)-w_e(n-1)}{w_e(n-1)\cdot n}$. In this case, collusion among incumbents takes the standard form of avoiding wasteful competition.

2. If $v(n) < w_e(n + 1)$, the main incumbents' motive for acquiring the license is to preempt entry. The cost of preemption is determined by the entrants' willingness to pay, i.e. $w_e(n + 1)$. Preemption is thus desirable for the incumbents' ring whenever $n\pi(n) + v(n) - w_e(n+1) > n\pi(n+1)$, that is, whenever $\delta(n) > \frac{1}{n}$, where $\delta(n) \equiv \frac{\pi(n)-\pi(n+1)}{w_e(n+1)-v(n)}$, in which case $\Delta^C = \pi(n) + \frac{v(n)-w_e(n+1)}{n}$. If $\delta(n) \leq \frac{1}{n}$, entry occurs for sure and we have $\Delta^C = \pi(n+1)$. This yields for $v(n) < w_e(n+1)$:

$$\Delta = \begin{cases} 0 & \text{if } \delta(n) \leq \frac{1}{n} \\ \frac{\pi(n)-\pi(n+1)}{w_e(n+1)} - \frac{w_e(n+1)-v(n)}{w_e(n+1)\cdot n} & \text{if } \frac{1}{n} < \delta(n) < 1 \\ \left(\frac{\pi(n)-\pi(n+1)}{w_e(n+1)} + \frac{v(n)}{w_e(n+1)\cdot n}\right)(1-q)^{n-1} - \frac{1}{n} & \text{if } \delta(n) \geq 1 \end{cases}$$

For $\delta(n)$ small enough, collusion is not beneficial for the incumbents: in the non-cooperative equilibrium, an entrant gets the license and there is no point to avoid that entry even when taking into account the profit loss incurred by every incumbent. For $\frac{1}{n} < \delta(n) < 1$, there is some benefit of collusion: In the non-cooperative outcome there is sure entry, because the cost to an individual incumbent does not justify preemption; however, taking into account the loss of every incumbent it is worth preempting entry. For $\delta(n) > 1$ there is a clear benefit of collusion, that of avoiding the risk that an entrant gets the license with some positive probability.

Observe that $\Delta = 0$ for $\delta(n) \leq \frac{1}{n}$ and that Δ tends to 0 as $\delta(n)$ tends to infinity[46]. Collusion is not very beneficial when $\delta(n)$ is very large because, despite

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[46]To see this, recall expression 3.1 and note that q tends to 1 when δ tends to infinity. Then, plug the expression of δ to show that $\delta(1 - q)^{n-1}$ tends to $\frac{1}{n}$.

the war of attrition, an entrant very rarely gets the license in the non-cooperative equilibrium. This suggests that we should expect more collusion among incumbents when the market structure parameter is neither too low nor too large.

Example 8.1. *For illustrative purposes, consider the explicit formulae for* $n = 2, 3$ *and* $v(n) = 0$:

$$
\Delta = \begin{cases} 0 & \text{if } \frac{\pi(2)-\pi(3)}{w_e(3)} \leq \frac{1}{2} \\ \frac{\pi(2)-\pi(3)}{w_e(3)} - \frac{1}{2} & \text{if } \frac{1}{2} < \frac{\pi(2)-\pi(3)}{w_e(3)} < 1 \quad \text{for } n = 2 \\ \frac{\pi(2)-\pi(3)}{2\pi(2)-\pi(3)-w_e(3)} - \frac{1}{2} & \text{if } \frac{\pi(2)-\pi(3)}{w_e(3)} \geq 1 \end{cases}
$$

and

$$
\Delta = \begin{cases} 0 & \text{if } \frac{\pi(3)-\pi(4)}{w_e(4)} \leq \frac{1}{3} \\ \frac{\pi(3)-\pi(4)}{w_e(4)} - \frac{1}{3} & \text{if } \frac{1}{3} < \frac{\pi(3)-\pi(4)}{w_e(4)} < 1 \quad \text{for } n = 3 \\ \frac{(\pi(3)-\pi(4))w_e(4)\left(1+\sqrt{\left(3\frac{4\pi(3)-4\pi(4)-w_e(4)}{w_e(4)}\right)}\right)^2}{(3\pi(3)-3\pi(4)-w_e(4))^2} - \frac{1}{3} & \text{if } \frac{\pi(3)-\pi(4)}{w_e(4)} \geq 1 \end{cases}
$$

The following figure plots the relative benefit of collusion as a function of $d(n) \equiv \frac{\pi(n)-\pi(n+1)}{w_e(n+1)}$ and reveals that this benefit is maximal at $d = 1$.

Insert Figure 2 here.

9. Appendix B (Proofs)

Proof of Proposition 3.1: Bidding $w_e(n+1)$ is a dominant strategy for entrants, and we now focus on incumbents. For the suggested strategy to be optimal, it must be the case that each incumbent is indifferent between bidding zero and bidding $w_e(n + 1)$.

Bidding zero yields an expected payoff of

$$
(1-q)^{n-1} \cdot \pi(n+1) + [1-(1-q)^{n-1}] \cdot \pi(n) =
$$

$$
\pi(n) - (1-q)^{n-1} \cdot [\pi(n) - \pi(n+1)].
$$

Bidding $w_e(n + 1)$ yields an expected payoff of

$$\sum_{j=0}^{n-1} \binom{n-1}{j} \cdot (1-q)^{n-1-j} \cdot q^j [\pi(n) - \frac{w_e(n+1) - v(n)}{j+1}] =$$

$$\pi(n) - [w_e(n+1) - v(n)] \cdot [\sum_{j=0}^{n-1} \binom{n-1}{j} \cdot (1-q)^{n-1-j} \cdot q^j \cdot \frac{1}{j+1}].$$

Equating the expected payoffs from the two bids yields the following:

$$\frac{\pi(n) - \pi(n+1)}{w_e(n+1) - v(n)} = \sum_{j=0}^{n-1} \binom{n-1}{j} \cdot (\frac{q}{1-q})^j \cdot \frac{1}{j+1} \tag{9.1}$$

Noting that $\sum_{j=0}^{n-1} \binom{n-1}{j} \cdot (\frac{q}{1-q})^j \cdot \frac{1}{j+1} = \frac{(1+\frac{q}{1-q})^n - 1}{n(\frac{q}{1-q})}$ we[47] finally obtain:

$$\delta(n) = \frac{1-q}{nq}[(1-q)^{-n} - 1]. \tag{9.2}$$

Let $G(q) = \frac{1-q}{nq}((1-q)^{-n} - 1)$. Observe that $\lim_{q \to 0} G(q) = 1$ and $\lim_{q \to 1} G(q) = \infty$. Moreover, $G''(q) > 0$ for $q \in [0, 1]$. Since $\delta(n) \geq 1$ by assumption, we obtain that equation (9.2) has always a unique solution $q^* \in [0, 1]$. An entrant gets the license only when all incumbents bid 0, hence the probability of entry is $(1-q^*)^n$. It is easy to see that the only other symmetric equilibria involve each incumbent bidding $w_e(n+1)$ with probability q^* and bidding a positive amount below $w_e(n+1)$ with probability $1 - q^*$. In such an equilibrium the resulting payoffs and entry probability remain as defined in Proposition 3.1. ∎

Proof of Proposition 3.2: The equilibrium probability of entry $x(\delta, n)$ is implicitly defined by

$$\delta = \frac{x^{\frac{1}{n}}}{n(1 - x^{\frac{1}{n}})} \frac{1-x}{x}.$$

Let $w(x, n) = \frac{x^{\frac{1}{n}}}{n(1-x^{\frac{1}{n}})} \frac{1-x}{x}$ and $H(x, n) = w(x, n) - \delta(n)$. We will show that (i) $\frac{\partial w}{\partial n}(x, n) > 0$ and (ii) $\frac{\partial w}{\partial x}(x, n) < 0$. If δ is non-increasing in n, we obtain by the implicit function theorem that $\frac{\partial x}{\partial \delta} = \frac{1}{\partial w/\partial x} < 0$ and that $\frac{\partial x}{\partial n} = -\frac{\partial w/\partial n - \partial \delta/\partial n}{\partial w/\partial x} > 0$, as desired.

---

[47]To see this integrate (w.r.t. z) the following identity: $\sum_{j=0}^{n-1} \binom{n-1}{j} \cdot z^j = (1+z)^{n-1}$.

(i) $\frac{\partial w}{\partial n} > 0$ is equivalent to

$$\frac{\partial}{\partial m}[\frac{mx^m}{1-x^m}] < 0,$$

which is equivalent to

$$[1 - x^m + \ln(x^m)]\frac{x^m}{(1-x^m)^2} < 0.$$

Since $1 - z + \ln z < 0$ for $z \in (0,1)$, we obtain $\frac{\partial w}{\partial n}(x,n) > 0$.
  (ii) $\frac{\partial w}{\partial x}(x,n) < 0$ is equivalent (for $y \in (0,1)$) to

$$\frac{\partial}{\partial y}\ln(\frac{y}{1-y} \cdot \frac{1-y^n}{y^n}) < 0,$$

which is equivalent to

$$-n(1-y) + 1 - y^n < 0.$$

This condition is easily checked[48] for $y \in (0,1)$. Hence, $\frac{\partial w}{\partial x}(x,n) < 0$. ∎

Proof of Proposition 4.6: Entrants have a dominant strategy, to bid w_e. Consider now an incumbent. Given the strategies of other bidders, a bid of 0 (or any other bid strictly lower than $(1-h)\bar{b}$) yields:

$$\left[1 - (1 - q^{GTE})^{n-1}\right]\pi(n) + (1 - q^{GTE})^{n-1}\pi(n+m) \qquad (9.3)$$

(When one other incumbent bids $\bar{b}$, no entrant acquires a license; when they all bid 0, m entrants acquire a license.)

Any bid in the range $((1-h)\bar{b}, \frac{w_e}{1-h})$ is dominated by a bid of $\bar{b}$. Finally, a bid of $\bar{b}$ or higher yields:

$$\pi(n) + v(n) - w_e. \qquad (9.4)$$

The last expression follows because the incumbent bidder wins then a license, no entrant is licensed, and every winner pays the entrants' bid.

The probability q^{GTE} is obtained by equating expressions 9.3 and 9.4. ∎

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[48]The function $-n(1-y) + 1 - y^n$ is equal to zero at $y = 1$. Its derivative is positive for $y < 1$.

Figure 1
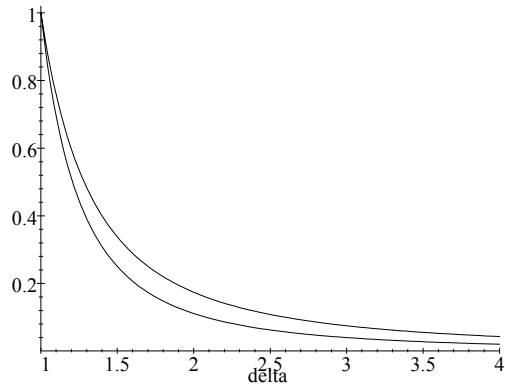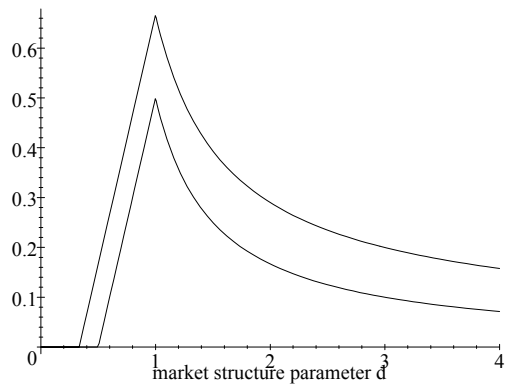
($n = 2$, full line; $n = 3$, interrupted line)

Figure 2

($n = 2$, full line; $n = 3$, interrupted line)