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Source: Econometrica, Vol. 63, No. 6 (Nov., 1995), pp. 1321-1335

Published by: The Econometric Society Stable URL: http://www.jstor.org/stable/2171772

Accessed: 08/01/2010 04:20

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NEGATIVE EXTERNALITIES MAY CAUSE DELAY IN NEGOTIATION

BY PHILIPPE JEHIEL AND BENNY MOLDOVANU¹

We study the strategic equilibria of a negotiation game where potential buyers are affected by identity-dependent, negative externalities. The unique equilibrium of long, finitely repeated generic games can either display delay—where a transaction can take place only in several stages before the deadline—or, in spite of the random element in the game, a well-defined buyer exists that obtains the object with probability close to one.

KEYWORDS: Bargaining, externalities, delay.

1. INTRODUCTION

THE MAIN GOAL OF THIS PAPER is to show that the presence of identity-dependent negative externalities may have unexpected effects on the strategic behavior of agents in a negotiation situation. By "identity-dependent negative externality" we mean that, if buyer j obtains the good to be sold through a negotiation game, then potential buyer $i, i \neq j$, suffers a loss that is dependent on the identity of both i and j. We observe a variety of situations that involve negotiations and externalities. For example, the International Olympic Committee bargains with the big U.S. television networks on the price to be paid for exclusive rights to broadcast the Olympic Games. We quote from the *Economist*, March 19th, 1994:

The networks usually expect to lose money but win prestige from such blockbusters. But Variety, a show-business newspaper, reckons that CBS made a profit on the Lillehamer games and cut Fox's advertising revenues by \$3m, ABC's by \$8m-10m and NBC's by \$12m-14m (because their diminished audiences did not reach the levels promised to advertisers).

Other good examples are provided by the negotiations leading to the privatization of a large public firm, between the government and several private firms that compete in one industry, or by sales of intangible property (say, a patent). Katz and Shapiro (1986) analyze licensing of intangible property, and explicitly point out the external effects. In that paper, as in most of the related literature, the externalities are dependent on the number of the licensees, and not on their identity.

¹ The comments of Martin Hellwig and two anonymous referees greatly improved the quality of the exposition. In particular, one referee provided a more transparent method for presenting the proof of Theorem 3.2. We are grateful to Dieter Balkenborg, Abraham Neyman, Georg Noldeke, Avner Shaked, and Jean Tirole for many helpful remarks. We also wish to thank seminar participants in Bonn, Bielefeld, Dortmund, Jerusalem, Paris, Tel-Aviv, Tilburg, and Toulouse for their comments. We thank Ms. Alexandra Hennemann for her dedicated help at typing this manuscript. Moldovanu gratefully acknowledges financial support from the Deutsche Forschungsgemeinschaft, SFB 303 at the University of Bonn, and from the Landau Foundation, Hebrew University of Jerusalem.

We study the following situation: A seller owns an indivisible object. The game-model consists of finite repetitions of a basic stage game. The basic stage game has the following structure: The seller randomly meets one of the buyers. Then the seller can make an offer—a price to be paid for the indivisible object—or the seller can wait and do nothing ("wait" means here that the seller makes an outrageous offer that is certainly rejected). A buyer that has met the seller and obtained an offer can either accept or reject. If the buyer accepts then he pays the agreed sum to the seller, obtains the object, and enjoys a given profit. All other buyers suffer a nonpositive externality that is dependent on the identity of the actual buyer and on the identity of the sufferer. If the buyer that met the seller rejects the offer or if the seller has chosen to wait, then the game proceeds to the next stage that has exactly the same structure unless it is the last stage where the game ends even if the good does not change hands.

We find that the unique subgame perfect Nash equilibrium (SPNE) of long, finitely repeated generic games can either display delay—where a transaction can take place only in several stages before the deadline—or, in spite of the random element in the game, a well-defined buyer exists that obtains the object with probability close to one. This buyer is the only one to get reasonable offers in almost all periods.

We note that, when externalities are absent, delay is impossible in the unique SPNE of the corresponding game. The final allocation is then always Walrasian, that is, the buyer with highest valuation gets the good with probability close to one.

Studies that explain delay in a complete information framework are rare. Most of the literature on delay phenomena uses some kind of asymmetry of information (see Kennan and Wilson (1993) for a recent detailed survey of this literature). In the complete information case delay has been first observed by Rubinstein (1982) in a model with fixed bargaining costs. Binmore (1987) showed that a finitely repeated version of the Nash demand game has an "embarrassment of equilibria." Some of those equilibria will display delayed agreements. Shaked (as quoted in Sutton (1986)) and Herrero (1985) have observed that versions of Rubinstein's discounted model involving more than two bargainers display multiple equilibria. Haller and Holden (1990) and Fernandez and Glazer (1991) explicitly use the multiplicity of equilibria in other variations of Rubinstein's model to generate inefficient, delayed outcomes. Delays are also obtained in a model where the played game changes along the play path (Fershtman and Seidmann (1991)), and in a model where the transmission time for offers is random (Ma and Manove (1990)). The last two mentioned papers also emphasize the importance of deadlines in negotiation. For an excellent survey of the recent bargaining literature see Osborne and Rubinstein (1990).

The intuition of our delay result is fully in keeping with Rubinstein's (1982) message that it is the cost of rejecting the proposer's offer that determines the proposer's payoff. A main feature of our model is the fact that agents' willingnesses to pay endogenously change along the play path. Although nonbuyers

always suffer a loss, small losses are better than large ones. Hence, the comparison of losses in different potential situations turns some externalities into "positive" ones, leading to a "war of attrition" type of behavior. Delay in our model cannot be equated with inefficiency. Whereas most of the bargaining literature emphasizes situations where part of the "cake" is lost if delay occurs, in our model the size of the shared cake is endogenously determined and may well have a negative value to society. Total welfare will not necessarily be maximized even in situations that do not display delay. If explicit costs of waiting are introduced delay does not disappear and becomes associated with inefficiency also in the "lost cake" sense.

This paper is organized as follows: In Section 2 we describe the model and derive some simple concepts for the equilibrium dynamics. In Section 3 we illustrate the delay phenomenon, and we characterize the general structure of equilibria. Proofs are gathered in Appendixes A–D.

2. EXTERNALITIES AND NEGOTIATION

The Economic Situation

We consider a market consisting of one seller and N buyers, where $N \ge 2$. The seller S owns one unit of an indivisible good. We normalize the utility functions of the agents in such a way that their utility when no trade takes place is equal to zero. The buyers are denoted by i, j, etc..., $1 \le i \le N$. If buyer i owns the indivisible good then his utility is given by π_i , where $\pi_i \ge 0$. If one buyer acquires the good, then all other buyers are subject to an external effect. The utility of j if i owns the good is given by $-\alpha_{ij}$, where $\alpha_{ij} \ge 0$.

The Negotiation Game

We study a bargaining procedure composed of T stages. The first stage will be called stage T, the second stage will be called stage T-1, and so on until the last stage, stage 1. At the beginning of any stage the seller randomly meets one of the buyers. All buyers have the same probability (i.e., 1/N) to meet the seller.

If S and i meet then S proposes a transaction at price p. The price p belongs to an interval [0, P]. The upper bound P satisfies the condition $P > \max_i \{\pi_i + \max_j \{\alpha_{ji}\}\}$. Hence, P is strictly larger than any price that will ever be acceptable to any buyer. By proposing an unacceptable price, say P, the seller basically "waits" (we could equivalently endow the seller with such an explicit action). We assume without loss of generality that, whenever the seller wishes to wait, he proposes the price P.

If S proposes p then i can either accept or reject the proposal. If i accepts then he obtains the good, pays price p to the seller, and the game ends. The utility of the seller is given by p, the utility of buyer i is given by $\pi_i - p$, and the utility of buyer j, $j \neq i$, is given by $-\alpha_{ij}$. If i refuses the proposal, or if S has chosen to wait then there are two possibilities. If the game has already reached

stage 1 (the last stage) then the game ends, otherwise the game continues to the next stage. This stage has the same structure as described above. If the game ends without the good changing hands then the utility of all agents is equal to zero. The T-stage game is denoted by Γ_T .

For the equilibrium analysis we restrict attention to generic situations in the following sense:

DEFINITION 2.1: Let $\varphi = (\{\pi_i\}_{1 \le i \le N}, \{\alpha_{ij}\}_{1 \le i,j \le N,\ i \ne j})$ describe the economic situation. The situation is called *generic* if for all sets of rational coefficients $(\{y_{ij}\}_{1 \le i,j \le N,\ i \ne j}, \{x_i\}_{1 \le i \le N}) \in \mathbb{Q}^{N^2}$ such that at least one of the coefficients is not equal to zero, it holds that

(2.1)
$$\sum_{i=1}^{N} (x_i \cdot \pi_i) + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} (y_{ij} \cdot \alpha_{ij}) \neq 0.$$

The game Γ_T , $T \in \mathbb{N}$, is called *generic* if it results from a generic situation.

For a nontrivial specification of rational coefficients $(\{y_{ij}\}_{1 \le i,j \le N, i \ne j}, \{x_i\}_{1 \le i \le N})$, the set of economic situations φ for which $\sum_{i=1}^{N} (x_i \cdot \pi_i) + \sum_{i=1}^{N} \sum_{j=1,j \ne i}^{N} (y_{ij} \cdot \alpha_{ij}) = 0$ forms a hyperplane of dimension $N^2 - 1$. (Note that the set of economic situations itself has dimension N^2). Since there are countable many possibilities for specifications of a finite set of rational coefficients, the whole set of nongeneric situations has Lebesgue measure zero (and is of Baire Category I).

Equilibrium Dynamics

We now derive formulae that describe the dynamic equilibrium behavior, and we show that a generic game Γ_T has a unique SPNE. Moreover, this SPNE uses pure strategies.

We first fix σ_T , a SPNE in pure strategies for a game Γ_T . For k such that $1 \le k \le T$ we denote by I^k the set of potential buyers at stage k (given σ_T). Buyer i belongs to I^k if, given σ_T and given that S and i meet at stage k, S makes a proposal p such that i accepts and buys the good. Denote by C^k the cardinality of the set I^k . Denote by p_i^k the maximum price that buyer i would accept to pay for the good at stage k (given σ_T). Denote by V_i^k and V_S^k the expected payoff at stage k (given σ_T) of buyer i and of the seller, respectively. These values are calculated before nature has selected whom S meets at stage k. Since in the case where the good does not change hands, the utility of all agents is normalized to be zero, we set $V_S^0 = 0$, $\forall i$, $V_i^0 = 0$. We obtain the following

recursive formulae: $\forall k \geq 1$,

(2.2)
$$V_S^k = \frac{1}{N} \left[\sum_{i \in I^k} p_i^k + (N - C^k) V_S^{k-1} \right];$$

(2.3)
$$V_i^k = \frac{1}{N} \left[\pi_i - p_i^k + \sum_{j \in I^k, j \neq i} (-\alpha_{ji}) + (N - C^k) V_i^{k-1} \right], \quad \text{if} \quad i \in I^k;$$

(2.4)
$$V_i^k = \frac{1}{N} \left[\sum_{j \in I^k} (-\alpha_{ji}) + (N - C^k) V_i^{k-1} \right], \quad \text{if} \quad i \notin I^k.$$

Because σ_T is a SPNE for Γ_T , we also obtain:

(2.5)
$$\pi_i - p_i^k = V_i^{k-1};$$

$$(2.6) p_i^k > V_S^{k-1} \Rightarrow i \in I^k \text{ and } p_i^k < V_S^{k-1} \Rightarrow i \notin I^k.$$

Condition 2.5 sets the limit for what a buyer is prepared to pay at stage k. This maximal price makes the buyer indifferent between buying now and refusing the offer. Condition 2.6 describes the seller's behavior when making an acceptable offer to i is strictly more advantageous than to wait, and vice-versa.

PROPOSITION 2.2: A generic game Γ_T has a unique SPNE, denoted by σ_T^* . Moreover, σ_T^* employs pure strategies.

PROOF: See Appendix A.

It should be clear that, for all pairs T and T' with T > T', the prescriptions of σ_T^* coincide with those of $\sigma_{T'}^*$, for all stages k, $1 \le k \le T'$. Hence, we can unambiguously speak of I^k without reference to a specific stage game and equilibrium, i.e. I^k denotes the set of potential buyers at stage k for every equilibrium σ_T^* with $T \ge k$. An analogous remark holds for V_i^k, V_S^k, p_i^k .

The key observation leading to the result of Proposition 2.2 is that genericity rules out equalities of the type $p_i^k = V_S^{k-1}$. Generically, the seller is never indifferent between making an offer that will be accepted and waiting. The conditions in (2.6) completely determine the structure of the sets I^k , i.e.,

(2.7)
$$i \in I^k$$
 if and only if $p_i^k > V_S^{k-1}$.

To conclude, we summarize the behavior at any stage k, $1 \le k \le T$, in σ_T^* : If the seller meets a buyer $i \in I^k$, then he proposes p_i^k . Otherwise, the seller waits. A buyer i rejects all offers p such that $p > p_i^k$, and accepts all offers p such that $p \le p_i^k$. (An offer $p = p_i^k$ must be accepted because otherwise the seller could deviate by infinitesimally lowering the price offer.)

PROPOSITION 2.3: The following hold for generic situations:

a. If $I^{k} = \emptyset$ then $I^{k+1} = \emptyset$. b. If $I^{k} = \{i\}$ then $i \in I^{k+1}$ and $p_{i}^{k} = p_{i}^{k+1}$.

c. V_s^k is nondecreasing in k.

PROOF: See Appendix A.

3. DELAY AND DETERMINATION

An interesting phenomenon in our perfect and complete information framework is the appearance of delays. Delay is said to occur at stage k if the set of potential buyers at that stage is empty, i.e. $I^k = \emptyset$. We will show that the equilibrium strategy of the seller may be such that in almost all stages he does not offer acceptable prices to any buyer. In this case transactions will take place only a few stages before the end of the game. The length of this brief activity period is independent of the number of stages played (provided the game is sufficiently long).

EXAMPLE 3.1 (Delay): Let N=3, let $\pi_1=\pi_2=\pi+\epsilon$, and let $\pi_3=\pi$, with $\epsilon>0$. Let $\alpha_{31}=\alpha_{32}=\alpha>0$, $\alpha_{21}=\alpha_{12}=1$, and $\alpha_{13}=\alpha_{23}=0$. We construct α,π , such that $I^1=\{1,2,3\}$, $I^2=I^3=\{1,2\}$, and $I^k=\varnothing$ for all $k\geq 4$. Thus, even for very long games activity will take place only in the last three stages. Because $\alpha_{13} = \alpha_{23} = 0$ it is easy to check that $3 \notin I^k$, for all $k \ge 2$. This happens because $p_3^k = \pi$ which is less than π plus some positive amount. Using the recursive formulae (2.2)–(2.4) and conditions (2.5) and (2.7), it easily follows that $I^2 = I^3 = \{1, 2\}$. Moreover, we obtain that

(3.1)
$$V_S^3 = \pi + \frac{26}{27}\epsilon + \frac{6}{27}\alpha + \frac{12}{27}$$
,

(3.2)
$$p_1^4 = p_2^4 = \pi + \epsilon + \frac{4}{27}\alpha + \frac{19}{27}$$
.

For any $\alpha > (7/2) + (\epsilon/2)$ it holds that $p_1^4 = p_2^4 < V_S^3$. For such values we obtain $I^k = \emptyset$ for all $k \ge 4$ (see also Proposition 2.3a). Note that for small perturbations of the parameter values $\{\pi_i\}_{1 \le i \le N}$, $\{\alpha_{ij}\}_{1 \le i,j \le N,\ i \ne j}$ the structure of the sets $\{I^k\}$, $k \in \mathbb{N}$, remains the same. Hence the delay phenomenon occurs in an open neighborhood of the parameters' values. The nongeneric example (where buyers 1 and 2 are symmetric) was chosen only for the sake of simpler calculations.

The delay phenomenon may, at first glance, seem paradoxical. We now verbally explain its intuition: From the viewpoint of the seller, the attractive buyers are 1 and 2 because they suffer high externalities if buyer 3 obtains the object. Buyer 3 is not very attractive because she suffers no externality at all. As the deadline approaches, the threat to sell to buyer 3 becomes more real

because the seller has to get rid of the object. Therefore, the seller may have to wait for a while if he wants to extract higher prices from buyers 1 and 2. The question is then why buyers 1 and 2, who can anticipate all this process, are not prepared to pay at a stage k, $k \ge 4$, the same price that they would pay at the first stage where activity takes place (the third stage from the end)? The answer is that at all stages k, $k \ge 4$, buyer 1 attaches a higher probability (i.e. higher than at stage 3) to the event that buyer 2 will meet the seller and obtain the object. Note that buyer 1 is relatively less afraid of buyer 2 than she is of buyer 3. The same kind of argument applies to buyer 2. Hence, a "war of attrition" is taking place between buyers 1 and 2, where each of those buyers waits for the other one to "save" her from buyer 3.

Note that for large values of ϵ or α maximizing total welfare requires the good to be sold to either buyer 1 or to buyer 2. If delay occurs there is a fixed, positive probability (1/27) that the good will be sold to buyer 3. Total welfare is not maximized even in the limit. For small values of π and ϵ total welfare is maximized if the good remains in the possession of the seller. In this case the outcome will be inefficient even if delay does not occur because the good is eventually sold.

We now briefly discuss whether delay persists in Example 3.1 if the agents discount the future. Note first that the statements in Proposition 2.3 do not hold anymore, but observe that delay first occurs for discount factor $\delta = 1$ because $\forall i, p_i^4 < V_S^3$. Since the equilibrium values with discounting $p_i^k(\delta)$ and $V_S^k(\delta)$ are locally continuous functions of δ , we obtain that, for discount factors close to one, $\forall i, p_i^4(\delta) < \delta V_S^3(\delta)$. Delay persists, but, for a given discount factor less than one, if the game is long enough, the seller cannot wait until the last but few stages to make a reasonable offer. If this were the case, then, evaluated at the beginning of the game, the seller's expected payoff would be very small because of the discounting. The seller would rather prefer to sell earlier and obtain at least a price close to a valuation π_i . Hence, potential activity must resume in a sufficiently high-numbered period. It follows that, while the behavior close to the deadline is not affected by discounting, the situation at earlier stages is qualitatively different. A new phenomenon which we call *cyclical delay* may emerge (see Jehiel and Moldovanu (1992)).

Note that, without externalities (i.e. $\alpha_{ij} = 0$, for all $1 \le i, j \le N$), if a SPNE is played in Γ_T then $I^1 = \{1, 2, ..., N\}$ and $i \in I^2$ if and only if $\pi_i \ge (1/N)\sum_{j=1}^n \pi_j$. Continuing in this way it is readily verified that, if T is large enough, there exists a \bar{k} such that for all $k \ge \bar{k}$, I^k must only consist of those buyers with maximal valuations π_i (generically, a unique buyer). Hence, as $T \to \infty$, the probability of a Walrasian allocation converges to one.

Consider now the general case with negative externalities. A priori one might imagine that, in addition to the delay phenomenon, several other types of equilibrium behavior may occur. For instance, the sequence of sets of potential buyers, $\{I^k\}_{k\in\mathbb{N}}$, might display either a "chaotic" or a nontrivial cyclical structure. The next theorem states that, for long generic games, there are only two possibilities for equilibrium play: we either encounter a delay phenomenon as in

the previous example, or, in spite of the randomness, a unique well defined buyer exists that is the only one to get acceptable offers in almost all stages. In other words, as k goes to infinity, the sequence $\{I^k\}_{k\in\mathbb{N}}$ converges either to a singleton or to the empty set.

THEOREM 3.2: Consider a generic situation. There exist $\bar{k} \in \mathbb{N}$ such that either:

- (1) for all $k \ge \overline{k}$, $I^k = \emptyset$ (delay); or:
- (2) for all $k \ge \overline{k}$, $I^k = \{i^*\}$, where $i^* \in \{1, 2, ..., N\}$ (well defined buyer).

PROOF: See Appendixes B and C.

Intuition For The Proof Of Theorem 3.2

Note first that the sequence $\{V_S^k\}_{k \in \mathbb{N}}$ is bounded (by P, say) and nondecreasing, hence it must converge to a limit. Denote this limit by p^{∞} . In the absence of delay, there must exist at least one buyer that appears infinitely often in the sequence $\{I^k\}_{k\in\mathbb{N}}$. Let i be such a buyer. There necessarily exists a subsequence of $\{p_i^k\}_{k\in\mathbb{N}}$ that converges to p^∞ (see Lemma B.1 in Appendix B). Note that any price of type p_i^k is given by the sum of π_i and some weighted average of externalities caused by the other buyers. The strategy of the proof is then to show that, if none of the alternatives mentioned in the statement of the Theorem holds, we obtain a contradiction to the genericity assumption. More precisely, assuming the Theorem is false, we derive two distinct equations: The derivative equations: $\pi_i - p^{\infty} = -\sum_{h=1, h \neq i}^{N} (q_{hi} \cdot \alpha_{hi})$, and $\pi_j - p^{\infty} = -\sum_{h=1, h \neq j}^{N} (q_{hj} \cdot \alpha_{hj})$, where $i \neq j$, and where all coefficients q_{hi}, q_{hj} are rational numbers. By eliminating p^{∞} from the two equations above we obtain $\pi_i - \pi_j + \sum_{h=1, h \neq i}^{N} (q_{hi} \cdot \alpha_{hi}) - \sum_{h=1, h \neq j}^{N} (q_{hj} \cdot \alpha_{hj}) = 0$, producing the desired contradiction to genericity. The derivation of the two equations is fairly intuitive if, for example, the sequence $\{I^k\}_{k\in\mathbb{N}}$ has a nontrivial cyclical structure involving at least two distinct buyers (for a general argument involving cyclical structures see Lemma B.2 in Appendix B). Since such a structure cannot be assumed a priori, the main difficulty in the proof consists in detecting regularities in specially constructed subsequences of $\{I^k\}_{k\in\mathbb{N}}$, and then using those regularities for the construction of two equations as above.

To conclude, for long generic games the absence of delay implies the existence of a unique "preferred" buyer i^* : in most of the periods the seller will take his chances waiting for i^* ; "threats" to sell to any other buyer are not credible.

The Role Of The Genericity Assumption

The main message of Theorem 3.2 remains unchanged even if the economic situation is not generic. Uniqueness of the SPNE is not ensured. However, all equilibria for long enough games will display either delay, or serious offers in almost all periods *only* to buyers who are, in the limit, identical from the point

of view of the seller in the sense that they all must be willing to pay the same price close to p^{∞} (for instance, two completely symmetric buyers).

It should also be clear that for any other exogenous specifications of matching probabilities we can adapt the definition of genericity to derive the same result as in Theorem 3.2. Since the structure of the sets I^k is, for generic games, determined by strict inequalities, both cases mentioned in the Theorem are robust. To get an idea about the sizes of the parameter sets displaying one or the other case consider again Example 3.1: Delay will occur there, possibly accompanied by more stages of potential activity towards the end of the game, if and only if $\alpha > 2$.

The Identity of the Preferred Buyer i*

What can be said about the identity of i^* , the well-defined buyer in the absence of delay? Since a buyer i is never prepared to pay for the good more than $\pi_i + \max_{g \neq i} \{\alpha_{gi}\}$, it is readily verified that i cannot be the well defined buyer if there exists a buyer j, $j \neq i$, such that $\pi_j + \alpha_{ji} > \pi_i + \max_{g \neq i} \{\alpha_{gi}\}$. Together with an argument showing that delay is impossible for situations where N = 2, the last observation yields the following proposition.

PROPOSITION 3.3: Consider a generic situation with N=2, and assume without loss of generality that $\pi_2 + \alpha_{12} > \pi_1 + \alpha_{21}$. There exists $\bar{k} \in \mathbb{N}$ such that for all $k \geq \bar{k}$, $I^k = \{2\}$.

PROOF: See Appendix D.

Given the result for N=2, one might think that either: (i) i^* is characterized by $i^*=\operatorname{argmax}_i\{\pi_i+\operatorname{max}_{g\neq i}\{\alpha_{gi}\}\}$ or (ii) i^* maximizes total welfare subject to the condition that the good does not remain in the possession of the seller, i.e. $i^*=\operatorname{argmax}_i\{\pi_i-\sum_{g=1,\ g\neq 1}^N\alpha_{gi}\}$. (Note that both characterizations coincide for the case where N=2.) Unfortunately, neither alternative is necessarily correct in the general case. In fact, the identity of i^* is generally given by a rather complicated function involving the economic parameters and the matching probabilities. Since an exact specification of matching probabilities does not seem to be fundamental for the economic problem at hand, the important fact is, in our view, that the message of Theorem 3.2 continues to hold for any specification of those probabilities, as discussed above. Moreover, the fact that, with externalities, the identity of the "preferred" buyer (given that such a buyer exits at all) cannot be directly inferred from the economic parameters alone

² Consider first the following example: N=3; $\pi_1=\pi_2=\pi_3=\pi>0$; $\alpha_{32}=7$, $\alpha_{21}=\alpha_{31}=5$, $\alpha_{13}=\alpha_{23}=3$, $\alpha_{12}=0$. Then $\underset{g\neq i}{\operatorname{argmax}}\{\pi_i+\max_{g\neq i}\{\alpha_{gi}\}\}=\{2\}$, but it is readily verified that: $I^1=I^2=I^3=\{1,2,3\}$, and $I^k=\{1\}$ for all k>4.

Consider next the following situation: N=3; $\pi_1=\pi_2=\pi_3=\pi>0$; $\alpha_{21}=4$, $\alpha_{12}=1$, $\alpha_{31}=\alpha_{13}=\alpha_{23}=\alpha_{32}=0$. Then $\underset{i}{\operatorname{argmax}}_i\{\pi_i-\sum_{g=1,\ g\neq i}^N\alpha_{gi}\}=\{3\}$, but it is readily verified that $I^1=I^2=I^3=\{1,2,3\}$, and $I^k=\{1\}$ for all $k\geq 4$.

contrasts with the case without externalities, where, for any specification of matching probabilities, the Walrasian intuition about the buyer with maximum valuation persists.

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Manuscript received April, 1992; final revision received February, 1995.

APPENDIX A: PROOFS OF PROPOSITIONS 2.2 AND 2.3

PROOF OF PROPOSITION 2.2: We use an inductive argument. The unique SPNE of Γ_1 has the following form: If the seller meets buyer i he proposes a price $p=\pi_i$. A buyer j accepts all offers p such that $p \leq \pi_j$, and rejects all offers p such that $p > \pi_j$. (By genericity $\pi_j > 0$. Hence j must, in equilibrium, accept an offer equal to her valuation. Otherwise the seller could deviate by infinitesimally lowering the offered price.)

Assume now that the statement of the proposition is true for Γ_T , and let σ_T^* be the unique SPNE of Γ_T . Consider the game Γ_{T+1} . By the definition of SPNE, and by the definition of our stage games, it is clear that the prescription of any SPNE of Γ_{T+1} must coincide with the prescription σ_T^* at all stages k, $1 \le k \le T$.

We now look at stage T+1 in Γ_{T+1} . It is optimal for buyer j to accept all offers p such that $p < \pi_j - V_j^T(\sigma_T^*)$, and to reject all offers p such that $p > \pi_j - V_j^T(\sigma_T^*)$. By genericity it cannot be the case that $V_S^T(\sigma_T^*) = \pi_j - V_j^T(\sigma_T^*)$. (Note that, by repeated applications of equations (2.2)–(2.4), $V_S^T(\sigma_T^*)$ and $V_j^T(\sigma_T^*)$ are weighted averages of valuations and externalities, where all weights are rational.) If the seller and buyer j are matched, there are two cases: (i) If $V_S^T(\sigma_T^*) > \pi_j - V_j^T(\sigma_T^*)$ then it is optimal for the seller to wait. (ii) If $V_S^T(\sigma_T^*) < \pi_j - V_j^T(\sigma_T^*)$ then it is optimal for the seller to propose a price $p = \pi_j - V_j^T(\sigma_T^*)$. In equilibrium buyer j must answer by yes to this offer. Otherwise the seller could deviate by infinitesimally lowering the offered price. Hence there is a unique SPNE σ_{T+1}^* of Γ_{T+1} , and σ_{T+1}^* only uses pure strategies.

PROOF OF PROPOSITION 2.3: (a) If $I^k = \emptyset$ then, by 2.7, we obtain that

$$(A.1) p_i^k < V_S^{k-1} for all 1 \le i \le N.$$

If $I^k = \emptyset$ we obtain by (2.2) and (2.4) that

(A.2)
$$V_S^k = V_S^{k-1}$$
,

(A.3)
$$V_i^k = V_i^{k-1}$$
 for all $1 \le i \le N$.

By (2.5) we know that, for all $1 \le i \le N$,

(A.4)
$$\pi_i - p_i^{k+1} = V_i^k$$
, and $\pi_i - p_i^k = V_i^{k-1}$.

By (A.3) and (A.4) we obtain that, for all $1 \le i \le N$,

(A.5)
$$p_i^k = p_i^{k+1}$$
.

By (A.1), (A.2), and (A.5) we obtain that

(A.6)
$$p_i^{k+1} < V_S^k$$
, for all $1 \le i \le N$.

By (2.7) and (A.6) we finally obtain $I^{k+1} = \emptyset$.

(b) If $I^k = \{i\}$ then, by (2.7), it holds that

(A.7)
$$p_i^k > V_s^{k-1}$$
.

By (2.2) and (A.7) we obtain that

(A.8)
$$V_S^k = \frac{1}{N} p_i^k + \frac{N-1}{N} V_S^{k-1} < p_i^k.$$

If $I^k = \{i\}$ we obtain by (2.3) and (2.5) that

(A.9)
$$V_i^k = \frac{1}{N}(\pi_i - p_i^k) + \frac{N-1}{N}V_i^{k-1} = \pi_i - p_i^k.$$

By (A.9) and another application of (2.5) we obtain that

(A.10)
$$p_i^{k+1} = \pi_i - V_i^k = p_i^k$$
.

Finally, by (A.8) and (A.10) we obtain that

(A.11)
$$V_S^k < p_i^k = p_i^{k+1}$$

This together with (2.7) proves that $i \in I^{k+1}$.

(c) The fact that V_S^k is nondecreasing in k follows immediately from equations (2.2) and (2.7). Q.E.D.

APPENDIX B: PRELIMINARY RESULTS FOR THE PROOF OF THEOREM 3.2

LEMMA B.1: For buyer i denote by K_i the set $\{k | i \in I^k\}$, and denote by B the set of buyers i such that K_i is infinite. Then there exists a price p^{∞} such that, for all $i \in B$, $\lim_{k \to \infty, k \in K_i} p_i^k = p^{\infty}$.

PROOF: By Proposition 2.3(c) the sequence $\{V_S^k\}_{k \in \mathbb{N}}$ is nondecreasing. It is clear that this sequence is bounded (by P, say), hence it converges to a limit.

Let $\lim_{k\to\infty}V_S^k=p^\infty$. For $i\in B$ denote by AC_i the set of accumulation points of the sequence $\{p_i^k\}_{k\in K_i}$. Assume, by contradiction, that the claim of the Lemma does not hold. Then the set $\bigcup_{i\in B}AC_i$ must contain at least two distinct points. Let $p=\inf\{p|p\in\bigcup_{i\in B}AC_i\}$, and let $\bar{p}=\sup\{p|p\in\bigcup_{i\in B}AC_i\}$. It must hold that $\bar{p}>p\geq p^\infty$ (see definition of B and condition (2.6)). By the definition of \bar{p} there exists a buyer $i\in B$ and a subsequence K_i' of K_i such that $p_i^k\to_{k\to\infty}, k\in K_i$ \bar{p} . Then, for all $\lambda>0$ there exists a $k_\lambda\in K_i'$ such that for all $k\geq k_\lambda$, $k\in K_i'$, it holds that $p_i^k\geq \bar{p}-\lambda$. By conditions (2.2), (2.7) we obtain for all $k>k_\lambda$, $k\in K_i'$, that

(B.1)
$$V_{S}^{k} = \frac{1}{N} \left[\sum_{j \in I^{k}} p_{j}^{k} + (N - C^{k}) V_{S}^{k-1} \right] \ge \frac{1}{N} (\bar{p} - \lambda) + \frac{N-1}{N} V_{S}^{k-1}.$$

As $k \in K_i'$ tends to infinity we obtain that $p^{\infty} \ge \bar{p} - \lambda$. Because this inequality holds for any $\lambda > 0$ we obtain $p^{\infty} \ge \bar{p}$. This is a contradiction to $\bar{p} > p \ge p^{\infty}$.

We introduce a construction that is repeatedly used in the sequel.

Construction of a Sequence $\{I_{-m}\}_{m\geq 0}$: Denote by I_0 an accumulation point of $\{I^k\}_{k\in\mathbb{N}}$, and consider a subsequence $\{k_n^0\}_{n\in\mathbb{N}}$ such that $\forall n,\ I^{k_n^0}=I_0$. Denote by I_{-1} an accumulation point of $\{I^{k_n^0-1}\}_{n\in\mathbb{N}}$, and consider a subsequence of $\{k_n^0\}_{n\in\mathbb{N}}, \{k_n^1\}_{n\in\mathbb{N}}$, such that $\forall n\in\mathbb{N},\ I^{k_n^1}=I_0$ and $I^{k_n^1-1}=I_{-1}$. Continue to construct for each integer $m\geq 0$, a set I_{-m} and a subsequence of $\{k_n^{m-1}\}_{n\in\mathbb{N}}, \{k_n^m\}_{n\in\mathbb{N}}$, such that $\forall n\in\mathbb{N},\ I^{k_n^m}=I_0,\ I^{k_n^m-1}=I_{-1},\dots,I^{k_n^m-m}=I_{-m}$. The cardinality of I_{-m} is denoted by C_{-m} .

DEFINITION: We will say that, given a sequence $\{I_{-m}\}_{m\geq 0}$, Condition (*) is satisfied for buyer i if there exist nonnegative integers $m_1 < m_2$ (which may depend on i) such that:

(B.2)
$$i \in I_{-m_1} \cap I_{-m_2}$$

(B.3)
$$\exists m, m_1 < m \le m_2$$
, such that $I_{-m} \ne \{i\}$.

LEMMA B.2: Assume that Condition (*) is satisfied for buyer i. Then there exist rational coefficients $\{q_{hi}|1 \le h \le N, \ h \ne i\}$ such that $\pi_i - p^{\infty} = -\sum_{h=1, \ h \ne i}^{N} (q_{hi} \cdot \alpha_{hi})$.

PROOF: Consider u, v, a pair of nonnegative integers such that: (i) $i \in I_{-u} \cap I_{-v}$; (ii) $\exists m, u < m \le v$, such that $I_{-m} \ne \{i\}$; (iii) For all pairs m_1, m_2 satisfying conditions (B.2)-(B.3), it holds that $v - u \le m_2 - m_1$. Condition 3 implies that $i \notin I_{-m}$ for all m such that u < m < v. We now look at the subsequence $\{k_n^v\}_{n \in \mathbb{N}}$. By construction, $\forall n, i \in I^{k_n^v - v} \cap I^{k_n^v - u}$. By equations (2.3)-(2.5) we obtain for all integers n the following chain of equations:

$$\begin{split} \pi_i - p_i^{k_n^v - u} &= V_i^{k_n^v - (u+1)}, \\ V_i^{k_n^v - (u+1)} &= \frac{1}{N} \left[\sum_{h \in I_{-(u+1)}} (-\alpha_{hi}) + (N - C_{-(u+1)}) V_i^{k_n^v - (u+2)} \right], \dots \\ V_i^{k_n^v - v} &= \frac{1}{N} \left[(\pi_i - p_i^{k_n^v - v}) + \sum_{h \in I_{-v}} (-\alpha_{hi}) + (N - C_{-v}) V_i^{k_n^v - (v+1)} \right], \\ \pi_i - p_i^{k_n^v - v} &= V_i^{k_n^v - (v+1)}. \end{split}$$

Combining the equations in the chain we obtain

$$(B.4) p_i^{k_n^v - u} = \frac{1}{N} \left[\sum_{h \in I_{-(u+1)}} (-\alpha_{hi}) \right] + \left(\frac{1}{N} \right)^2 (N - C_{-(u+1)}) \left[\sum_{h \in I_{-(u+2)}} (-\alpha_{hi}) \right] + \cdots$$

$$+ \left(\frac{1}{N} \right)^{v-u} (N - C_{-(u+1)}) \dots (N - C_{-(v-1)}) \left[\sum_{h \in I_{-v}} (-\alpha_{hi}) \right]$$

$$+ \left(\frac{1}{N} \right)^{v-u} (N - C_{-(u+1)}) \dots (N - C_{-(v-1)}) (N - C_{-v} + 1) (\pi_i - p_i^{k_n^v - v}).$$

By Lemma B.1, $\lim_{n\to\infty} p_i^{k_n^{\nu}-u} = \lim_{n\to\infty} p_i^{k_n^{\nu}-v} = p^{\infty}$. The result follows by taking the limit as $n\to\infty$ in (B.4) and rearranging terms.

Q.E.D.

LEMMA B.3: Assume that $I_{-m} = \{j\}$ for all $m \ge 1$, and $I_0 \ne \{j\}$. Then, for all $i \in I_0$ such that $i \ne j$ it holds that: $\pi_i - p^{\infty} = -\alpha_{ij}$.

PROOF: Let $i \in I_0 \setminus \{j\}$. We use the diagonal sequence $\{k_m^m\}_{m \in \mathbb{N}}$. For all $r \ge 1$ and for all m > r, $i \in I^{k_m^m} = I_0$ and $i \notin I^{k_m^m - r} = I_{-r} = \{j\}$. Using formulae (2.3)–(2.5) we obtain the following chain: $\pi_i - p_i^{k_m^m} = V_i^{k_m^m - 1}$; $V_i^{k_m^m - 1} = -(1/N)\alpha_{ji} + ((N-1)/N)V_i^{k_m^m - 2}$; ... $V_i^{k_m^m - (m-1)} = -(1/N)\alpha_{ji} + ((N-1)/N)V_i^{k_m^m - m}$. Combining all equations in the chain we obtain

(B.5)
$$\pi_{i} - p_{i}^{k_{m}^{m}} = -\frac{1}{N} \left[1 + \frac{N-1}{N} + \left(\frac{N-1}{N} \right)^{2} \dots + \left(\frac{N-1}{N} \right)^{m-1} \right] \alpha_{ji} + \left(\frac{N-1}{N} \right)^{m-1} V_{i}^{k_{m}^{m}-m}.$$

The result follows by taking the limit as $m \to \infty$ in B.5. By Lemma B.1, $\lim_{m \to \infty} p_i^{k_m^m} = p^{\infty}$. The geometric series converges to N. The expected payoff $V_i^{k_m^m - m}$ is bounded. The term $((N-1)/N)^{m-1}$ converges to zero.

Q.E.D.

APPENDIX C: Proof of Theorem 3.2

Suppose that $I^k \neq \emptyset$ for all $k \geq 1$, and that $\{I^k\}_{k \in \mathbb{N}}$ does not converge to a singleton. We will show that this, together with the hypothesis that the situation is generic, leads to a contradiction. The proof is organized in 6 steps.

- 1. Let I_0 be any accumulation point of $\{I^k\}_{k\in\mathbb{N}}$ and construct I_{-1},I_{-2},\ldots as in Appendix B.
- 2. If there exists u and v ($u \neq v$) such that $I_{-u} = I_{-v}$ and $C_{-u} > 1$, then (using Lemma B.2 for each $i \in I_{-u}$) the situation cannot be generic. Since there are only finitely many buyers, this observation implies that all but finitely many of the I_{-m} ($0 \le m \le \infty$) are singletons.
- 3. DEFINITION: We will say that $\{i\}$ appears in arbitrarily long sequences in $\{I^k\}_{k \in \mathbb{N}}$ if, for every m > 0 there exists n > m such that $I^{n-k} = \{i\}$ for every k = 1, 2, ... m.

We distinguish three cases:

Case 1: There exist buyers i and j ($i \neq j$) such that both $\{i\}$ and $\{j\}$ appear in arbitrarily long sequences in $\{I^k\}_{k \in \mathbb{N}}$.

Case 2: There exists exactly one buyer i such that $\{i\}$ appears in arbitrarily long sequences in $\{I^k\}_{i\in\mathbb{R}^N}$.

Case 3: There does not exist any buyer i such that $\{i\}$ appears in arbitrarily long sequences in $\{I^k\}_{k \in \mathbb{N}}$.

4. In Case 1, assume that $\{i\}$ appears in arbitrarily long sequences in $\{I^k\}_{k\in\mathbb{N}}$. $\forall m$, define $k_m=\min\{n>m$ such that $I^n\neq\{i\}$ but $I^{n-k}=\{i\}$ for every $k=1,2,\ldots m\}$. k_m is well defined since we assume that I^k does not converge to a singleton. Select an accumulation point, J_0 , of $\{I^k m\}_{m\in\mathbb{N}}$ and take a subsequence $\{k'_m\}_{m\in\mathbb{N}}\subset\{k_m\}_{m\in\mathbb{N}}$ such that $I^{k'_m}=J_0$ for all m. Analogously to the construction of $\{I_{-m}\}_{m\geq 0}$, construct a sequence $\{J_{-m}\}_{m\geq 0}$. Observe that, $\forall r\geq 1$, $J_{-r}=\lim_{m\to\infty}I^{k_m-r}=\{i\}$. By construction, $J_0\neq\{i\}$ but, by Proposition 2.3(b), $J_0\supset\{i\}$. Let $g\in J_0$, where $g\neq i$. By the argument of Lemma B.3 (applied to $\{J_{-m}\}_{m\geq 0}$), we obtain $\pi_g-p^\infty=-\alpha_{ig}$. Performing the same argument for $j\neq i$ such that $\{j\}$ also appears in arbitrarily long sequences, we obtain $\pi_h-p^\infty=-\alpha_{jh}$. Combining these two equations (which are not redundant even if g=h) we conclude that the situation is not generic.

- 5. In Case 2, let i be the unique buyer such that $\{i\}$ appears in arbitrarily long sequences in $\{I^k\}_{k\in\mathbb{N}}$. As in step 4 we begin by defining k_m , the subsequence $\{k'_m\}_{m\in\mathbb{N}}$ and the set J_0 such that $J_0 \neq \{i\}$ but $J_0 \supset \{i\}$. Next, define J_{+1} to be an accumulation point of $\{I^{k'_m+1}\}_{m\in\mathbb{N}}$, and take a subsequence $\{k^1_m\}_{m\in\mathbb{N}} \subset \{k'_m\}_{m\in\mathbb{N}}$ such that $I^{k^1_m} = J_{+1}$ for all m. Similarly, define J_{+2}, J_{+3}, \ldots . By the same logic as in Step 2 all but finitely many sets in $\{J_{+m}\}_{m\geq 0}$ are singletons. Hence, there exists j such that $J_{+m} = \{j\}$ for infinitely many m. There are now two possibilities:
- a. Assume first that j=i. In particular, there exists $r\geq 1$ such that $J_{+r}=\{i\}$. Since $I\in J_0\cap J_{+r}$, and since $J_0\neq \{i\}$, the analog of Condition (*), applied to $\{J_{+m}\}_{m\geq 0}$, is satisfied for i. We obtain that $\pi_i-p^\infty=-\sum_h q_{hi}\cdot\alpha_{hi}$, where all coefficients are rational. As in Step 4, we also have $\pi_g-p^\infty=-\alpha_{ig}$, where $g\in J_0$, $g\neq i$. We can conclude that the situation is not generic.
- b. Assume now that $j \neq i$. By Proposition 2.3(b), $J_{+m} = \{j\}$ implies $J_{+m+1} \supset \{j\}$. Since $\{j\}$ does not appear in arbitrarily long sequences, there exists an infinite sequence $\{m_n\}_{n \in \mathbb{N}}$ such that $\forall n$, $J_{+m_n} = \{j\}$ but $J_{+m_n+1} \supsetneq \{j\}$. We have shown that there are infinitely many sets J_{+m_n+1} which contain more than one element, contradicting Step 2, and implying that the situation is not generic.
- 6. In Case 3 we work with the original sequence $\{I_{-m}\}_{m\geq 0}$ and define I_{+1}, I_{+2}, \ldots analogously. By the same logic as in Step 5, there exists j such that $I_{+m} = \{j\}$ for infinitely many m. Since there is no buyer that appears in arbitrarily long sequences in $\{I^k\}_{k\in\mathbb{N}}$ we obtain, exactly as in Part b of Step 5, that there must exist infinitely many sets I_{+m} containing more than one element. This contradicts Step 2, implying that the situation is not generic. Q.E.D.

APPENDIX D: Proof of Proposition 3.3

Our first step is to prove, by induction, that $I^k \neq \emptyset$ for all $k \in \mathbb{N}$. It is clear that $I^1 = \{1, 2\}$. Assume then that $I^k \neq \emptyset$. We have to prove that $I^{k+1} \neq \emptyset$. There are two cases: If I^k contains a single element then $I^{k+1} \neq \emptyset$ by Proposition 2.3(b). If $I^k = \{1, 2\}$ we obtain by equations (2.2), (2.3) that

(D.1)
$$V_i^k = \frac{1}{2} [\pi_i - p_i^k - \alpha_{ji}], \text{ for } i, j \in \{1, 2\}, i \neq j,$$

(D.2)
$$V_S^k = \frac{1}{2} [p_1^k + p_2^k].$$

By condition (2.5) we know that

(D.3)
$$\pi_i - p_i^{k+1} = V_i^k$$
, for $i \in \{1, 2\}$.

From (D.1), (D.3) we obtain that

(D.4)
$$p_i^k = 2p_i^{k+1} - \pi_i - \alpha_{ji}$$
, for $i, j \in \{1, 2\}, i \neq j$.

From (D.2), (D.4) we obtain that

(D.5)
$$V_{\mathcal{S}}^{k} = \frac{1}{2} [2p_{1}^{k+1} - \pi_{1} - \alpha_{21} + 2p_{2}^{k+1} - \pi_{2} - \alpha_{12}].$$

For all $k \in \mathbb{N}$, $p_1^{k+1} \le \pi_1 + \alpha_{21}$ and $p_2^{k+1} \le \pi_2 + \alpha_{12}$. We obtain

(D.6)
$$V_s^k \le \frac{1}{2} [p_1^{k+1} + p_2^{k+1}].$$

Hence it cannot be that $p_1^{k+1} < V_S^k$ and $p_2^{k+1} < V_S^k$. Then $I^{k+1} \neq \emptyset$ follows by condition (2.7). Since delay is impossible, we obtain by Theorem 3.2 that the sequence $\{I^k\}_{k \in \mathbb{N}}$ converges to a singleton. Assume, by contradiction, that the limit set is $\{1\}$. Buyer 1 is never prepared to pay more than $\pi_1 + \alpha_{21}$. However, in this situation we have $\lim_{k \to \infty} p_2^k = \pi_2 + \alpha_{12}$. Since $\pi_2 + \alpha_{12} > \pi_1 + \alpha_{21}$, the seller could profitably deviate by making an acceptable offer to buyer 2, producing a contradiction. *Q.E.D.*

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