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Order Independent Equilibria*

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We study a noncooperative game of coalition formation, based on an underlying game in coalitional form. We introduce *order independent* equilibria (OIE). A strategy profile is an OIE if, for any specification of first movers in the sequential game, it remains an equilibrium and leads to the same payoff. Our results are: (1) Payoffs in OIE that use pure, stationary strategies must be in the core of the underlying game in coalitional form. (2) If the underlying game has the property that all its subgames have nonempty cores then, for each payoff vector, there exists an OIE with the same payoff. *Journal of Economic Literature* Classification Numbers: C70, C71, C72, C78.

1. INTRODUCTION

We study a noncooperative game of coalition formation and payoff division based on a game in coalitional form with nontransferable utility. We wish to identify, in a formal noncooperative framework, conditions on the equilibrium strategies that ensure that the equilibrium payoff will be in the core of the underlying game in coalitional form. Hence, our goal

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is to contribute to a formal understanding of the similarities and differences between the cooperative and noncooperative approaches to the problem of coalition formation.

Following the pioneering papers by Harsanyi (1974) and Selten (1981) there is a recent, renewed interest in studying strategic games of coalition formation and payoff division. The papers in this literature usually consider specific noncooperative games that represent possible institutional settings for multilateral negotiations where coalitions may form. The results relate outcomes in various types of noncooperative equilibria with outcomes that have certain satisfactory properties from the point of view of cooperative game theory. Binmore (1985), Bloch (1992), Chatterjee *et al.* (1993), Moldovanu (1992), Okada (1991), and Perry and Reny (1994) study variations of a model proposed by Selten (1981) where agents sequentially make proposals consisting of a coalition and a payoff division for that coalition. Although several cooperative solution concepts appear as outcomes, the core gets most of the attention. The papers by Bennett and van Damme (1992), Binmore (1985), Selten (1992), and Winter (1991) analyze models where agents make payoff demands, without explicitly naming a coalition to whom those demand are addressed. These models are usually called "demand commitment models". Coalitions form if the demands of their members can be satisfied. Gul (1989) (in a framework of bilateral bargaining), Hart and Mas-Colell (1992), and Winter (1991) display institutional settings that yield equilibrium payoffs coinciding with the Shapley Value. Lagunoff (1994) identifies a class of sequential voting mechanisms whose status-quo outcomes coincide with core outcomes. Finally, in Moldovanu and Winter (1992), we take a dual approach and we characterize, for a given negotiation procedure, the class of games in coalitional form for which the outcomes in the strategic equilibria coincide with the predictions of cooperative theory.

On the one hand solution concepts such as Walrasian equilibria, the core, the bargaining set, etc. do not take into account procedural aspects such as the exact order of moves, and the time dimension is missing. In a sense, simpler and more basic stability ideas are embodied in the definitions of the above-mentioned solutions. For example, Aumann (1988) writes: "it is the possibilities for coalition forming, promising, and threatening that are decisive, rather than whose turn it is to speak."

On the other hand, it is well known that the results of noncooperative analysis are extremely sensitive to variations in the game form that represents the institutional setting.

Our paper constitutes an attempt to bridge the above-mentioned dichotomy between the cooperative and noncooperative approaches and to formally identify a framework where comparisons are meaningful. Our conceptual innovation is the use of *order independent* equilibria. A strategy profile is an order independent equilibrium if, for any specification of first movers in the sequential game, it remains an equilibrium and leads to the same payoff. Hence, we compare strategies in *different* extensive form games that may be played, and to do this we consider an equivalence relation between possible situations in the games.

We show that payoffs in order independent equilibria that use pure, stationary strategies must be in the core of the underlying game in coalitional form. This result does not hold anymore if order independence or stationarity are removed. A corollary is that, if the core of the underlying game is empty, then the order of the moves can be manipulated to create an advantage for some players.

We also show that the core is contained in the set of payoffs attainable in order independent equilibria if the game in coalitional form has the property that all its subgames have nonempty cores. Note that coalitional games arising from exchange markets have this property. For market games we obtain then that the core coincides with the set of payoffs attainable in order independent equilibria that use pure stationary strategies.

The concept of order independent equilibrium is not meant to be a "refinement" in the usual sense. Apart from the reasons stated above and the established close connection to coalitional stability, we believe that order independent equilibria (or other equilibria close in spirit) are interesting because they may serve as focal points in situations where there is some uncertainty about the exact circumstances and procedures of the negotiation process.

The paper is organized as follows: In Section 2 we introduce games in coalitional form and the core. In Section 3 we describe a noncooperative game of coalition formation and payoff division based on games in coalitional form. In Section 4 we define our main concept—order independent equilibria—and prove the main results. In Section 5 we provide two examples that establish the necessity of the used assumptions. Concluding remarks are gathered in Section 6.

2. Games in Coalitional Form

Let $N = \{1, 2, \ldots n\}$ be a set of players. A coalition S is a nonempty subset of N. A payoff vector for N is a function x: $N \rightarrow \mathbb{R}$.

Notation. We denote by x^S the restriction of x to members of S. The restriction of $\mathbb{R}^S$ to vectors with nonnegative coordinates is denoted by $\mathbb{R}^S_+$. The zero vector in $\mathbb{R}^S$ is denoted by 0^S. For $x, y \in \mathbb{R}^S$ we write $x \geq$

y if $x^i \geq y^i$ for all $i \in S$. Let K be a subset of $\mathbb{R}^S_+$. Then, int K denotes the interior of K relative to $\mathbb{R}^S_+$ and ∂K denotes the set $K \backslash \text{int } K$.

DEFINITION 1. A *nontransferable utility (NTU) game in coalitional form* is a pair (N, V) where V is a function that assigns to each coalition S in N a set $V(S) \subseteq \mathbb{R}^S$ such that:

$$V(S) \text{ is a nonempty, closed, and bounded subset of } \mathbb{R}^S_+; \tag{2.1}$$

$$\forall\, i \in N,\ V(i) = 0^i; \tag{2.2}$$

$$\text{If } y \in \mathbb{R}^S_+,\ x \in V(S) \text{ and } x \geq y \text{ then } y \in V(S); \tag{2.3}$$

$$\text{If } x, y \in \partial V(S) \text{ and } x \geq y \text{ then } x = y. \tag{2.4}$$

Condition 2.2 is a normalization. Condition 2.3 ensures that utility is freely disposable. A set $V(S)$ satisfying 2.3 is said to be *comprehensive*. Condition 2.4 requires that the Pareto-frontier of $V(S)$ coincides with the strong Pareto-frontier, and it implies that utility cannot be transferred at a rate of zero or infinity. A set $V(S)$ satisfying 2.4 is said to be *nonleveled*.

Denote by $A \times B$ the Cartesian product of the sets A and B. We assume that the NTU game (N, V) is *superadditive*, namely:

$$\forall\, S, T \subset N \text{ with } S \cap T = \varnothing, \qquad V(S) \times V(T) \subseteq V(S \cup T) \tag{2.5}$$

We next define a concept that will play a major role in our analysis:

DEFINITION 2. Let (N, V) be an NTU game, and let $x \in V(N)$. x can be *improved upon* if there exists a coalition S and a vector $y^S \in V(S)$ such that $y^i > x^i$ for all $i \in S$. The *core* of (N, V), $C(N, V)$, is the set of all $x \in V(N)$ that cannot be improved upon.

3. A GAME OF COALITION FORMATION AND PAYOFF DIVISION

We describe a simple sequential bargaining procedure based on an NTU game (N, V): First, consider a function φ: $2^N \to N$ where $\varphi(S) \in S$ is the *first* player that has the initiative if the set of players still active in the game is S. An initiator i may shift the initiative to another player, or he may make a proposal. A proposal consists of a coalition S such that $i \in S$, a payoff vector $x^S \in V(S)$, and a responder j, a player of S. The responder can reject or accept the proposal. If the responder rejects, then he becomes the new initiator. If the responder accepts there are two possibilities: If this responder was the last player in S to accept the proposal, then the coalition S forms, it leaves the game, and its members

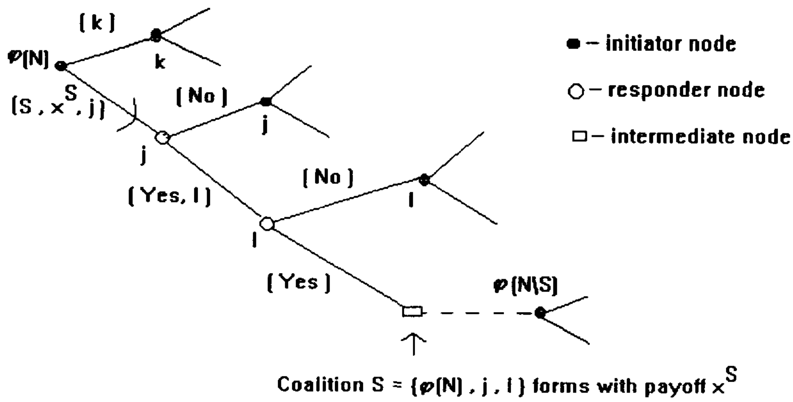

FIGURE 1

are paid according to x^S. Otherwise, the responder must select the next responder to the existing proposal. After the first coalition S has formed, the first player with the initiative is $\varphi(N \backslash S)$, and the game continues in the same fashion. An infinite play results in zero payoffs to all players that remained active in the game. See Fig. 1.

This is an informal description of an extensive form game with perfect information. Selten (1981) studies a version of this game in which the game stops as soon as *one* coalition has formed. Chatterjee *et al.* (1990) study a version with time-discounting. Both papers consider situations described by coalitional games with "side payments" (or with "transferable utility"). This implies rather stringent properties of the utility functions. Moldovanu (1992) studies coalition-proof Nash equilibria in Selten's model with nontransferable utility.

Observe that, in the above described situation, each order mechanism φ generates a *different* extensive form game. We wish to compare strategies in these different games, and therefore we proceed to a formal description of the bargaining procedure that will easily allow such comparisons.

Formally, we consider a game $G_\varphi(N, V) = ((N, V), P, Z, Z', \Pi, C, h, \varphi)$ where (N, V) is a superadditive NTU game, P is a set of *positions*, $Z \subseteq P$ is a set of *final positions*, $Z' \subseteq P$ is a set of *intermediate positions*, $\Pi = (P_1, P_2, \ldots P_n)$ is a *partition* of $P \backslash (Z \cup Z')$ in *player sets*, C is a *choice function* that assigns to each $p \in P$ a subset $C(p) \subseteq P$, h is a *payoff function* that assigns to each $z \in Z$ a payoff vector in $\mathbb{R}^N$, and φ is an *order mechanism* that assigns to each coalition $S \subseteq N$ a player $\varphi(S) \in S$.

There are four kinds of positions:

(1) Initiator positions of the form $(S, x^{N \backslash S}, i) \in P_i$, where $S \subseteq N$ is the set of players still in the game, $x^{N \backslash S} \in \mathbb{R}^{N \backslash S}$ is the agreed payoff for

members that already left the game, and $i \in S$ is the player with the initiative.

When all players are still active (i.e., no coalition has yet formed), initiator positions will be simply denoted by (N, i).

(2) Responder positions of the form $(S, x^{N\setminus S}, j, (T, x^T), Q) \in P_j$, where S and $x^{N\setminus S}$ are like in (1); $j \in T$ is the present responder to the standing proposal (T, x^T) where $T \subseteq S$ and $x^T \in V(T)$; $Q \subseteq T\setminus\{j\}$ is the set of players that have already accepted or proposed the standing proposal.

(3) Intermediate positions of the form $(S, x^{N\setminus S})$ where S and $x^{N\setminus S}$ are like in (1).

(4) Final positions of the form $(\emptyset, x^N)$ where $x^N \in \mathbb{R}^N$.

The choices at an initiator position $(S, x^{N\setminus S}, i)$ are either $(S, x^{N\setminus S}, j)$ with $j \neq i$, which means that i passes the initiative, or a responder position of the form $(S, x^{N\setminus S}, j, (T, x^T), i)$ with $i, j \in T$ and $x^T \in V(T)$, which means that i proposes (T, x^T) and designates j to be the first responder.

At any responder position $(S, x^{N\setminus S}, j, (T, x^T), Q)$ there is the choice:

(a) $(S, x^{N\setminus S}, j)$ which means that j refuses the proposal and becomes initiator.

If $Q \cup \{j\} \neq T$ then j has choice (a) and additionally:

(b) $(S, x^{N\setminus S}, k, (T, x^T), (Q \cup \{j\}))$, where $k \in T\setminus(Q \cup \{j\})$.

This means that j accepts the proposal and designates k to be the next responder to the standing proposal.

If $Q \cup \{j\} = T$ and $S\setminus T \neq \emptyset$ then j has choice (a) and additionally:

(c) $(S\setminus T, x^{N\setminus(S\setminus T)})$, an intermediate position. This means that j accepts the proposal and coalition T leaves the game.

If $Q \cup \{j\} = T$ and $S\setminus T = \emptyset$ then j has choice (a) and additionally:

(d) $(\emptyset, x^N)$, a final position. This means that j accepts, coalition T forms, and the game ends.

The payoff of player i at final position $(\emptyset, x^N)$ is simply x^i. If the play continues indefinitely then there must be a position after which no coalition forms any more, so the first two coordinates of all later reached positions must be constant, say, $(S, x^{N\setminus S})$. Then the payoff for every $i \in N\setminus S$ is x^i, and the payoff for all players in S is zero.

The game begins at initiator position $(N, \varphi(N))$. If an intermediate position $(S, x^{N\setminus S})$ is reached, the game continues automatically at position $(S, x^{N\setminus S}, \varphi(S))$.

We restrict our attention, and formally describe only pure, stationary strategies. Observe that some element of stationarity must be taken into account if one wishes to compare strategies for different games.

DEFINITION 3. A pure, stationary behavioral strategy for player i is a function that assigns to each $p \in P_i$ an element of $C(p)$. This function does not depend on the second coordinate of the position p, i.e., it does not depend on the agreed payoff of players that are already out of the game.

Note that our description of strategies *does not* depend on the mechanism φ, therefore such a strategy applies formally to all games $G_\varphi(N, V)$. Note also the stationarity assumption: all information about previous history of play is gathered in the description of the position reached, and choices can depend only on the position at which they are made.

Subgame perfect Nash equilibria are defined in the usual way—optimal behavior is required at each position. Note that equilibria that employ stationary strategies are also stable against nonstationary deviations. We conclude this section with the following simple observation:

LEMMA. *Let σ be a strategy profile leading to payoff x in a game $G_\varphi(N, V)$. Then $x \in V(N)$.*

Proof. Suppose that, if σ is played, the coalitions $S_1, S_2, \ldots S_m$ form and the members of coalition S_{m+1} (that may be empty) stay in the game indefinitely. We know that $\cup S_k = N$, and that $\forall\, i, j \in \{1, 2, \ldots m + 1\}$ with $i \neq j$ it must hold $S_i \cap S_j = \emptyset$. By the definition of proposals, $x^{S_i} \in V(S_i)$ for all $1 \leq i \leq m$, and $x^{S_{m+1}} = 0^{S_{m+1}}$. By the definition of (N, V) we know that $0^{S_{m+1}} \in V(S_{m+1})$. By super-additivity of (N, V) we obtain $x \in V(N)$. Q.E.D.

4. ORDER INDEPENDENT EQUILIBRIA AND THE CORE

We now define an equilibrium concept that removes, in our framework, the dependence of outcomes on the exact order of the moves.

DEFINITION 4. A strategy profile σ is an *order independent* equilibrium for games of the form $G_\varphi(N, V)$ if it satisfies for any order mechanism φ that:

(1) σ is a subgame perfect Nash equilibrium in $G_\varphi(N, V)$. (4.1)

(2) If σ is played, the payoff in $G_\varphi(N, V)$ is given by a vector $x = x(\sigma, (N, V))$, independent of φ. (4.2)

Proposition A. *Let (N, V) be a game in coalitional form, and let σ be an order independent equilibrium in pure, stationary strategies for games of the form $G_\varphi(N, V)$. Then the payoff vector $x = x(\sigma, (N, V))$ must be in the core of (N, V).*

Proof. Assume on the contrary that $x \notin C(N, V)$. By the Lemma we know that $x \in V(N)$, so it must be the case that $x^S \in \text{int } V(S)$ for a coalition $S \subseteq N$.

Let $i \in S$ and let $\bar{\varphi}$ be a mechanism with $\bar{\varphi}(N) = i$. We choose $\delta > 0$ such that $y^S = (x^j + \delta)_{j \in S}$ belongs to $V(S)$.

Consider the following strategy of i: If i is the initiator, and if the set of players still in the game is N, then propose (S, y^S); otherwise follow strategy σ^i. Denote this strategy of i by τ^i. We will show that this deviation benefits i in $G_{\bar{\varphi}}(N, V)$ (given that all other players follow their strategy given by σ). This will be a contradiction to assumption 4.1.

Take $j \in S$, and assume that all other players in $S \backslash \{i, j\}$ have accepted the proposal (S, y^S) of i. We will show that it is optimal for j to accept as well. A rejection by j leads to the position (N, j), and (τ^i, σ^{-i}) is played.

Consider now the play from position (N, j) on. If no coalition ever forms then the game continues indefinitely, and the payoff to j is $0 < x^j + \delta$. Otherwise, let T be the first coalition that forms and leaves the game, with payoff z^T. Consider then the play between (N, j) and the first intermediate position $(N \backslash T, z^T)$. (If $T = N$, then a final position is reached and the proof is the same). We first claim that the initiator position (N, i) cannot appear on this path, i.e. before $(N \backslash T, z^T)$ is reached. If the position (N, i) is reached once then it will be reached again and again because i proposes and j refuses the proposal. The game continues indefinitely and no coalition can form. This contradicts the assumption that coalition T forms.

Hence, because (N, i) never appears on the path, the play in this instance must coincide with the play under σ given a mechanism φ' with $\varphi'(N) = j$. By assumption 4.2 the payoff of j must be $x^j < y^j = x^j + \delta$, whether j is a member of T or not. Note that the play under (τ^i, σ^{-i}) after the intermediate position $(N \backslash T, z^T)$ coincides in any case with the play under σ. Hence, if j does not belong to T, her payoff cannot be affected anymore by i's deviation.

We have shown that a refusal by j cannot be optimal. By backward induction it is now obvious that i's proposal (S, y^S) must be accepted by all players in S, therefore the coalition S forms, and i benefits from his deviation in $G_{\bar{\varphi}}(N, V)$. Q.E.D.

Corollary 1. *Let (N, V) be a game with empty core, and let σ be a subgame perfect Nash equilibrium in pure stationary strategies for all* *games of the form $G_\varphi(N, V)$. Then there exist order mechanisms φ^1, φ^2 such that, if σ is played, the payoff in $G_{\varphi 1}(N, V)$ is different from the payoff in $G_{\varphi 2}(N, V)$.*

The corollary is an obvious consequence of Proposition A.

Note that our definition of order independence applies formally to a richer class of strategy profiles than the one considered in this paper. Nevertheless, we show in Section 5 that the result of Proposition A does not hold for order independent equilibria that employ more complex strategies.

PROPOSITION B. *Let (N, V) be a game in coalitional form with the property that all its subgames have non-empty cores. Then, for every $x \in C(N, V)$ there exists an order independent equilibrium σ in pure stationary strategies that leads to payoff x in every game of the form $G_\varphi(N, V)$.*

Proof. For every $S \subseteq N$ choose a vector $x_S \in C(S, V_S)$ where V_S is the restriction of V on S. Also, let $x_N = x \in C(N, V)$. Consider the following strategy σ^i for a player $i \in N$:

(1) If i is initiator and if the set of players still in the game is $S \subseteq N$, then i proposes (S, x_S)

(2) If i is a responder when the set of players still in the game is S, the proposal is (T, y^T), and the set of players that already accepted this proposal is R, a subset of T, then i accepts if and only if $y^j \geq x^j_S$ for every $j \in T \backslash R$.

It is clear that no matter what φ is, if σ is played, the grand coalition forms and the payoff is given by $x_N = x \in C(N, V)$. It remains to show that σ is an equilibrium.

Assume that i is the initiator and the set of players still active in the game is S. If i deviates by proposing (T, y^T) where $y^i > x^i_S$, and all other players play according to σ, this proposal will be accepted if and only if $y^j \geq x^j_S$ for all $j \in T$. Because $x_S \in C(S, V_S)$, and because $V(T)$ is nonleveled this implies that $y^T \notin V(T)$. This is a contradiction to the definition of a proposal. Thus i cannot benefit from this deviation. It is also clear that i cannot benefit from a deviation where she proposes (T, y^T) with $y^i < x^i_S$: if the proposal is accepted, then i could have achieved more by playing according to σ; if the proposal is rejected, then another proposal will be made and i's payoff cannot be higher than x^i_S (note that all other players play according to σ).

Assume now that i is a responder, the set of players still active in the game is S, the standing proposal is (T, y^T), and the players in R, a subset of T, have already proposed or accepted. If $y^j \geq x^j_S$ for all $j \in T \backslash R$ (note that $i \in T \backslash R$), then it is optimal for i to accept the proposal. Because all other players follow σ this proposal will be accepted and the payoff of i will be y^i. Be refusing, i becomes initiator, and we have already seen that his payoff can not be greater than $x^i_S \le y^i$. Finally, if $y^j < x^j_S$ for a $j \in T \backslash R$, then the offer (T, y^T) will be rejected by j and i cannot benefit from accepting this offer. This concludes the proof that σ is an equilibrium for every $G_\varphi(N, V)$. Q.E.D.

COROLLARY 2. *Let (N, V) be a game such that all its subgames have nonempty cores. Then the core of (N, V) coincides with the set of payoffs attainable in order independent equilibria in pure, stationary strategies.*

This corollary follows immediately from Propositions A and B. Coalitional games arising from exchange markets have the property required in the corollary, hence we have obtained a rather novel characterization of the core of a market. We note that all our results hold also for the simpler version of the negotiation procedure where the game ends as soon as one coalition forms (see Selten, 1981).

Remark. In Proposition B we require the coalitional game to have the property that all its subgames have nonempty cores. We show then how core outcomes can be sustained by order independent equilibria. The question arises whether it is possible to similarly sustain core outcomes also in games that do not have the above-mentioned property. In fact, the existence of order independent equilibria implies not only that the coalitional game has a nonempty core (see Proposition A), but also that all its subgames have nonempty cores as well. To prove this statement it is enough to observe that the recommendations of an order independent equilibrium for games of the form $G_\varphi(N, V)$ induce an order independent equilibrium also for the reduced games that arise after a coalition has formed and left with its payoff. The result follows then by a repeated application of Proposition A.

Hence, a payoff vector x can be sustained by an order independent equilibrium for games of the form $G_\varphi(N, V)$ if and only if: 1. $C(S, V_S) \neq \emptyset$ for all coalitions S in N; and 2. $x \in C(N, V)$.

5. EXAMPLES

In this section we show that order independence or stationarity alone are not sufficient for the result of Proposition A. Our first example describes an order independent equilibrium that employs nonstationary strategies. We show that the constant payoff vector is not in the core of underlying game.

EXAMPLE 1. Consider a market with two buyers, B_1 and B_2, and a seller A. The seller owns an indivisible object and his reservation price for the object is normalized to zero. Both buyers have reservation prices of 1. This defines the following TU game:

$$v(B_1) = v(B_2) = v(A) = v(B_1, B_2) = 0 \quad (5.3)$$

$$v(B_1, A) = v(B_2, A) = v(B_1, B_2, A) = 1. \quad (5.4)$$

The core of this game consists of the unique vector $(x^{B_1}, x^{B_2}, x^A) = (0, 0, 1)$.

We first construct three auxiliary strategy profiles that are stationary. We only describe the actions for the case where all players are still active in the game. This is sufficient in this simple situation. The reader may easily complete the picture for the other cases.

The profile σ_1 is given as follows:

Buyer B_1 proposes the coalition $\{B_1, A\}$ with division $(1, 0)$ and accepts only proposals where he is offered at least 1.

Buyer B_2 proposes the coalition $\{B_2, A\}$ with division $(1, 0)$ and accepts only proposals where he is offered at least 1.

Seller A proposes the coalition $\{B_1, A\}$ with division $(1, 0)$ and accepts any offer.

The profile σ_2 prescribes for the buyers the same strategy as in σ_1 and:

Seller A proposes the coalition $\{B_2, A\}$ with division $(1, 0)$ and accepts any offer.

The profile σ_3 is an order independent equilibrium sustaining the unique core vector (see the proof of Proposition B).

It is straightforward to check that all these profiles are subgame perfect equilibria for all games of the form $G_\varphi(N, V)$. Note that: If B_i, $1 \le i \le 2$, is the first initiator and σ_i is played then B_i obtains 1 and the other players obtain 0; if A is the first initiator and σ_3 is played than A obtains 1 and the buyers obtain 0.

We next describe a nonstationary strategy profile σ^*:

Each player starts with proposing the grand coalition N, with division $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, and accepting only this proposal. This behavior is followed as long as the standing proposal is $(N, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}))$. If a player has to respond to a different proposal then she rejects it, and till the end of the game the players play according to that equilibrium from the set $\{\sigma_1, \sigma_2, \sigma_3\}$ that yields 1 for the rejecter and 0 to the other players.

We claim that σ^* is an order independent subgame perfect Nash equilibrium for all games of the form $G_\varphi(N, V)$. Because the constant payoff $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ is not in the core we show then that stationarity is necessary for the result of Proposition A. It is clear that σ^* satisfies condition 4.2 (order independence). It remains to show that this profile is a subgame perfect Nash equilibrium (condition 4.1).

Because σ_i, $1 \le i \le 3$, are all equilibria, a deviation cannot be advantageous if the players switched already to play one of these profiles. Hence consider a deviation before switching has occurred. If a player proposes something different from $(N, (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}))$ then the proposal will be rejected, switching will occur, and the deviator will obtain 0. Hence this kind of deviation is not advantageous. Finally, it is optimal to reject an offer that is different from $(N, (\frac{1}{3}, \frac{1}{3}, \frac{1}{3}))$ because then switching occurs, and the rejecter will obtain 1, which is the highest possible payoff in this game.

We next consider a simple game in coalitional form and a profile in stationary strategies that is a subgame perfect Nash equilibrium for any game of the form $G_\varphi(N, V)$, but it is not order independent. The payoff vectors will not in the core of the underlying game.

EXAMPLE 2. Consider the market described in Example 1 where buyers B_1 and B_2 have reservation prices of 1 and 2 respectively. This defines the following TU game:

$$v(\mathrm{B}_1) = v(B_2) = v(A) = v(B_1, B_2) = 0 \tag{5.1}$$

$$v(B_1, A) = 1; v(B_2, A) = v(B_1, B_2, A) = 2. \tag{5.2}$$

The core of this game consists of all payoff vectors that represent a trade between the seller and buyer B_2 at a price of at least 1 and no more than 2.

Consider the following strategies for the bargaining game. Again, it is enough in this simple situation to describe the actions for the case where all players are still in the game.

B_1 proposes the coalition $\{B_1, A\}$ with division (0.5, 0.5) and accepts only proposals where he is offered at least 0.5.

B_2 proposes the coalition $\{B_2, A\}$ with division (1.5, 0.5) and accepts only proposals where she is offered at least 1.5.

A proposes the coalition $\{B_2, A\}$ with division (1.5, 0.5) and accepts only proposals where she is offered at least 0.5.

This profile is a subgame perfect Nash equilibrium for any game of the form $G_\varphi(N, V)$, and trade always takes place at price of 0.5. If the seller A or the buyer B_2 start the game as initiators then the payoff is given by $x = (x^{B_1}, x^{B_2}, x^A) = (0, 1.5, 0.5)$. If the buyer B_1 is the first initiator then the payoff is given by $y = (y^{B_1}, y^{B_2}, y^A) = (0.5, 0, 1.5)$. Both payoff vectors are not in the core. The strategy profile is, of course, not order independent.

6. Concluding Remarks

Our results show that the predictions of the core (which is the most fundamental coalitional stability concept) continue to hold in a noncooperative model only under rather restrictive assumptions. Nevertheless, real life negotiations are quite amorphous, and it is difficult to model exactly just what the procedures are. As a first approximation that abstracts from procedural aspects, the stability idea represented by the core seems quite powerful. On the other hand, the nonexistence of order independence equilibria for a large class of games sheds some light on the importance of procedural aspects in negotiation.

An important issue is the applicability of our, or similar, independence concepts to other models where the institutional framework is flexible in one or more dimensions. We do not think that a universal concept may be easily available. Usually, some aspects of the particular problem at hand must be taken into account. For example, in our framework, the presence of deadlines or time discounting would clearly create asymmetries that should be taken into consideration when defining robustness with respect to order.

The focus on institutional aspects that affect the order of the moves has a more general character than what is immediately apparent. Important aspects of dynamic bargaining situations are closely connected to the sequence of play. For example, consider a bargaining institution where agents negotiate face to face, and the same institution where some parties are represented by delegates. As the delegates may need more time to propose or respond (having to consult their principal) the order of play may differ, although the underlying institution remains the same. The possibility to make "take it or leave it" offers and the ability to commit are other examples of details related to the order of the moves.

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