

Content-based Agendas and Qualified Majorities in Sequential Voting

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We analyze sequential, binary voting schemes in settings where several privately informed agents have single-peaked preferences over a finite set of alternatives, and we focus on robust equilibria that do not depend on assumptions about the players' beliefs about each other. Our main results identify two intuitive conditions on binary voting trees ensuring that sincere voting at each stage forms an ex-post perfect equilibrium. In particular, we uncover a strong rationale for content-based agendas: if the outcome should not be sensitive to beliefs about others, nor to the deployment of strategic skills, the agenda needs to be built "from the extremes to the middle" so that more extreme alternatives are both more difficult to adopt, and are put to vote before other, more moderate options. An important corollary is that, under simple majority, the equilibrium outcome of the incomplete information game is always the Condorcet winner. Finally, we aim to guide the practical design of schemes that are widely used by legislatures and committees and we illustrate our findings with several case studies.

Sequential, binary voting procedures are widely used in democratic legislatures and in committees in order to select one among several alternatives. At each stage, one among two subsets of alternatives is adopted via a simple Yes/No vote by (possibly qualified) majority. This process is repeated until a unique alternative is singled out, and formally elected.

There is significant diversity in the details of observed schemes: For example, *amendment* procedures are used in English-speaking and Scandinavian countries, while most continental European parliaments use the rather different *successive* procedure. Which sequential voting procedures have desirable outcomes? Can we explain the voting patterns observed in reality? Can we offer some guidance for the practical design of sequential voting procedures? These are the issues discussed in this paper.

We analyze sequential, binary voting schemes in settings where several privately informed agents have single-peaked preferences over a finite set of alternatives, and we focus on robust equilibria that do not depend on assumptions about the players' beliefs about each other. Our main results identify two intuitive conditions on binary voting trees ensuring that sincere voting at each stage forms an ex-post perfect equilibrium in the associated extensive form game with incomplete information. In other words, in the identified game trees voters cannot gain by manipulating their vote, regardless of their beliefs about others' preferences, and regardless of the information disclosure policy along the voting sequence. An important corollary is that, as long as our conditions are satisfied and simple majority is required at each stage, the equilibrium outcome of the incomplete information game is always the Condorcet winner. We illustrate that this need not be the case if our conditions are violated, giving a strong rationale for using procedures that satisfy our conditions.

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The two main conditions conducive to desirable outcomes in sequential voting procedures are:

1) *Convexity of divisions (CONV)*. Recall that each vote is taken by (possibly qualified) majority among two, not necessarily disjoint, subsets of alternatives. Convexity says that if two alternatives a and c belong to the left (right) subset at a given node, then any alternative b such that $a < b < c$ (in the order governing single-peakedness) also belongs to the left (right) subset. Intuitively, each of the Yes/No votes in the sequence must be among two options that cover a well-defined, coherent segment of positions in the respective ideological spectrum.

2) *Monotonicity of qualified majorities (MON)*. This condition roughly says that, after a vote that resulted in the adoption of a left (right) subset of alternatives, a subsequent movement left (right) requires a qualified majority that is at least as large as the one that governed the previous move in the same direction. Intuitively, adopting consecutive and more "extreme" positions should become more and more difficult. The standard case of keeping a constant majority requirement at each vote in the sequence- such as a simple majority - satisfies monotonicity.

In order to understand the role played by the above conditions, note that, under incomplete information, the main determinants of optimality are the decisions at pivotality events: only such instances offer the opportunity to directly influence the outcome, and hence the optimal strategy must recommend a "correct" action whenever an agent is pivotal. Due to presence of incomplete information, agents do not a-priori know when and whether they are pivotal, and hence they need to make accurate inferences about future outcomes *conditional* on being pivotal. The combined effect of CONV and MON is to finely tune this inference: a pivotal agent is able to infer that a more preferred alternative will ultimately be elected, either because there are anyway enough other supporters for this alternative, or because the agent will continue to remain pivotal (and hence in control of the decision) at future stages. Thus, sincere voting - according to the preference relation restricted to the remaining set of alternatives - is optimal at each and every stage of the voting process.

Fortunately, many ubiquitous voting procedures do satisfy the above two conditions if simple and intuitive rules of agenda formation are respected. Let us consider two prominent examples:

1) In an *amendment* procedure, alternatives are paired, and the winner competes against the next alternative on the agenda, until all alternatives are exhausted. The amendment procedure will satisfy convexity if the pairing is such that the most "extreme" alternatives compete against each other at each round of voting. If the single-peakedness order is $1 < 2 < \dots < A$, the first vote should be among alternatives 1 and A . If 1 wins this contest then the next vote is among 1 and $A - 1$, whereas if A won at the first round, the second vote should be between A and 2, and so on. Monotonicity holds, for example, if the qualified majority needed to keep alternative 1 in the process is not decreasing along the voting tree.

2) In a *successive* procedure alternatives are considered one after the other, and the process stops as soon as one alternative garners the required majority.¹ It will satisfy convexity if, at each stage, the considered alternative is one of the two most extreme ones. If the single-peakedness order is $1 < 2 < \dots < A$, convex agendas are, for example, to vote on the alternatives in the order $1, 2, 3, \dots, A$,² in the order $A, A - 1, \dots, 1$ or even in a left-right alternating order such as $1, A, 2, A - 1, \dots, A/2$. If the successive procedure uses the order $1, 2, 3, \dots, A$, monotonicity says that the qualified majority required to continue the voting process cannot decrease (i.e., the majority

¹A famous case has been the vote to establish the capital of the united Germany: the successive method was used in order to decide among 4 alternatives, some including split locations of government and parliament. The likely Condorcet winner, Bonn, was ultimately not elected.

²For this particular procedure and agenda, monotonicity of qualified majorities has been shown to be necessary for a robust dynamic implementation by Gershkov, Moldovanu and Shi (forthcoming). These authors were mainly concerned with identification of welfare maximizing mechanisms in settings where monetary transfers are not possible.

	Sequential Voting	Amendment Procedure
Always Vote on Most Extreme Alternative	Austria, Denmark, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Netherlands, Norway, Poland, Slovenia, Spain, European Parliament	Finland
Other procedural rule	Belgium, Czech Republic, Luxembourg, Portugal, Slovakia	Sweden, Switzerland, UK, US

TABLE 1—PARLIAMENTARY FLOOR VOTING PROCEDURES. SOURCE: RASCH (2000)

needed to accept the current alternative does not increase).

As should be clear from the above, for our conditions to be satisfied, the content of proposals (rather than purely procedural considerations) should determine the agenda: extremes should be voted on first. This is the main prescriptive design recommendation coming from our study. Of course, the interpretation of content may be ambiguous - leading to possible manipulations that could be prevented by rigid, procedural rules. Nevertheless, the basic idea of a content-based agenda is anchored, formally or informally, in the rules governing many legislatures. For example, the German parliament uses an informal rule, rooted in custom and practice, to vote on extreme alternatives first. The Standing Orders of its Second Chamber (the Bundesrat) explicitly prescribe

“if several proposals are made to the same subject, then the first vote shall be on the farthest-reaching proposal. Decisive is the degree of deviation from status quo” (Article 30 (2), *Geschäftsordnung des Bundesrates* (1993)).

In contrast, agenda formation in other legislatures follows purely procedural rules rather than being content-based: hence, the resulting sequential binary trees need not satisfy convexity. For example, in a decision of the U.S. Congress involving three alternatives - the status quo, a proposed change and an amendment to that change - the status quo is always put up to vote against the proposal or the amendment (whichever won previously) at the second, final stage. If the status quo is an extreme - more to the “left” or to the “right” relative to the other two alternatives, the procedure is not convex.

The above Table 1 (taken from Rasch 2000) summarizes the existing practices in parliaments.

To illustrate the empirical and theoretical content of our study of qualified majorities and the monotonicity condition, recall first that many legislatures and committees use supermajorities for the passing of various, special laws. Prominent among these are the supermajorities required for constitutional amendments in all democracies, and the Taxes and Expenditure laws (TEs) found in 46 states in the U.S. (U.S. Advisory Commission on Intergovernmental Relations 1995) (see also Knight 2000). Those states have imposed a variety of statutory and constitutional limitations on the fiscal autonomy of their counties, municipalities, and school districts. For example, in Nebraska governing bodies of counties and municipalities can increase property taxes reflecting changes in the Consumer Price Index by a simple majority vote, while larger increases up to 5% require a three-quarters majority. Further increases above 5% require a full referendum in the respective population. Another example, discussed in more detail below, is the U.S. Supreme court where accepting a case for review (*certiorari*) requires 4 out of 9 votes, but decisions on the accepted cases

need to garner a simple majority (5 votes out of 9). We show that the use of any particular voting tree satisfying convexity is not restrictive: varying (in a monotonic way) the qualified majority at each node in the voting tree allows us to replicate, via any partitional binary voting procedure that satisfies convexity, the entire set of anonymous, unanimous and dominant strategy implementable social choice functions for the domain of single-peaked preferences. The proof of this result builds upon the seminal contribution of Moulin (1980).

Besides Moulin's work, our study builds upon a large literature (in Economics, Law and Political Science) that has analyzed both theoretical and institutional/empirical aspects of sequential voting procedures.³ Farquharson (1969) is the first to study voting trees from a game-theoretic perspective, and Miller (1977), McKelvey and Niemi (1978) and Moulin (1979) provide important theoretical contributions. Ordeshook and Schwartz (1987) and Schwartz (2008) describe institutional aspects of sequential procedures used in parliaments, and Bjurulf and Niemi (1978), Enelow (1981), and Ladha (1994) empirically analyze voting behavior in sequential voting procedures.⁴

Almost the entire previous literature assumed that agents are completely informed about the preferences of others. Under complete information, the associated extensive form games are amenable to analysis by backward induction: voters can, at each stage, foresee which alternative will be finally elected, essentially reducing each decision to a vote among two alternatives. If a simple majority is used at each stage, then a Condorcet winner - which always exists with single-peaked preferences - must therefore be selected by sophisticated voters independently of the particular structure of the binary voting tree, and independently of its agenda (Miller 1977). But it is important to note that sincere voting need not constitute an equilibrium!

A pioneering analysis of strategic, sequential voting under incomplete information (and of the subtle effect of pivotality in this case) is Ordeshook and Palfrey (1988), who construct Bayesian equilibria for an *amendment* procedure with three alternatives and with three possible preference profiles that potentially lead to a Condorcet paradox.⁵

While several individual instances of strategic behavior have been identified in the Political Science literature, systematic studies found strategic voting to be rare (see, for example, Groseclose and Milyo 2010, Wilkerson 1999, Poole and Rosenthal 1997, Volden 1998). Two main explanations have been advanced for this relative rarity: (1) Legislators may be bound either by party discipline, or by the need to fulfill constituents' expectations and explain their behavior to them (Fenno 1978), and hence cannot act opportunistically at each voting instance (Austen-Smith 1992, Denzau, Riker and Shepsle 1985); (2) Real life agendas are endogenous, and sincere voting is an equilibrium in the resulting game (of complete information) where the agenda is chosen in a first step (Austen-Smith 1987). Our results offer another, simple explanation for the relative rarity of observed strategic voting: sincere voting constitutes the most compelling equilibrium for all situations that can be described by convex and monotone voting procedures, and hence it cannot be empirically distinguished from sophisticated voting (with which it simply coincides) in those cases.

We conclude the paper with an analysis of case studies from the Legal and Political Science literature, where a lively debate has revolved around the question whether strategic behavior is a common phenomenon in real life voting situations. We first illustrate several environments where strategic manipulations seem rare, and argue that, in these examples, our conditions were satisfied. We next review several case studies that identified strategic manipulations and observe that, in each case, conditions CONV and MON were not satisfied. As a consequence, the Condorcet winner need

³Our general treatment of varying qualified majorities at each stage generalizes the constant, simple majority rule assumed in almost the entire previous literature.

⁴See also Volden (1998), Wilkerson (1999), and Caldeira, Wright and Zorn (1999).

⁵See also Krehbiel and Rivers (1990), who study the voting on the school construction bill of 1956 as an incomplete information game.

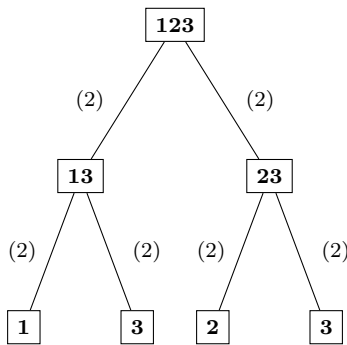


FIGURE 1. ILLUSTRATION OF AN AMENDMENT PROCEDURE

not be elected in equilibrium, in contrast to the complete information case, where, independently of the procedure used, the Condorcet winner always emerges in a sophisticated equilibrium (Miller 1977).

The rest of the paper is organized as follows. In Section I we present several simple examples that illustrate our results. In Section II we present the voting model, the convexity and monotonicity conditions. In Section III we present our main results for voting procedures where sincere voting is a robust equilibrium. In Section IV we characterize the class of social choice functions that can be robustly and dynamically implemented in our setting. In Section V we discuss several case studies in light of our findings. Section VI concludes.

I. Illustrative Examples

The purpose of this section is to illustrate the main ideas and results in the simplest non-trivial setting with three privately informed agents and three alternatives, labeled $\{1, 2, 3\}$. We assume that preferences are strict and single-peaked with respect to the order on natural numbers.⁶ This assumption yields four possible individual preferences: $1 > 2 > 3$, $2 > 1 > 3$, $2 > 3 > 1$, and $3 > 2 > 1$.

A. The Role of Convexity

Assume that voters use an amendment procedure where the first vote is by simple majority between alternative 1 and 2, and the second vote is by simple majority between the winner of the first vote and alternative 3. This can be represented by the voting tree illustrated in Figure 1.

The second (and last vote) is either between 1 and 3, or between 2 and 3. Voting sincerely at the last vote (which is always a simple binary choice between two deterministic outcomes) is clearly optimal for all types of all agents.

Consider now the first vote between alternatives 1 and 2. Sincere voting calls for agents with peaks on 1 to vote for alternative 1, and for agents with peaks on 2 and 3 to vote for alternative 2. But sincere voting does not form of an ex-post perfect equilibrium: consider for example the case where our three agents have the following profile of preferences: $1 > 2 > 3$, $2 > 3 > 1$, $3 > 2 > 1$. Then alternative 2 is ultimately chosen under sincere voting, but a deviation to the left of the agent with peak on alternative 3 at the first vote (vote for 1) would result in a second vote among alternatives

⁶By relabeling alternatives, this is without loss of generality.

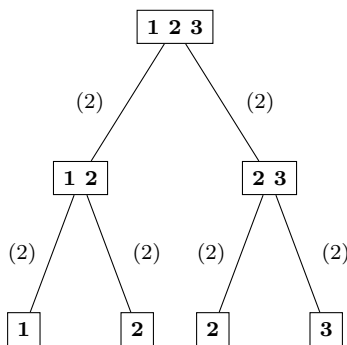


FIGURE 2. ILLUSTRATION OF AN AMENDMENT PROCEDURE SATISFYING CONV

1 and 3, where 3 would win. Hence such a deviation may be profitable, and our agent may regret his first sincere vote.⁷ Thus, sincere voting is not an equilibrium, and the reader may easily work out that only trivial ex-post perfect equilibria exist in this game.

Let us construct a Bayesian equilibrium for this amendment procedure: this illustrates how strategic deviations from sincerity can prevent the Condorcet winner from being selected even in the present setting, where such an alternative exists for any possible profile of preferences.

Example 1. *There are 3 voters that use the voting procedure illustrated in Figure 1. Each of the four possible single-peaked ordinal preferences is associated with a "type": preference $1 > 2 > 3$ with type t_1 , $2 > 1 > 3$ with type t_2 , $2 > 3 > 1$ with type t_3 , and $3 > 2 > 1$ with type t_4 . Types are I.I.D. and, for each agent, type t_i is realized with probability q_i . For an analysis of Bayesian equilibria, we need a cardinal description of utilities: we assume here that each voter values his most preferred alternative by 1, his second-most preferred alternative by v , and his least preferred alternative by 0. We show in the Appendix that, if $v \leq \frac{\min\{q_2, q_3\}}{q_2 + q_3}$, then the following strategies form a Bayes-Nash equilibrium: all types except t_4 vote sincerely, and voters of type t_4 vote left at the first stage and vote sincerely at the second.*

Even though they are aware of the deviation by type t_4 voters, it is optimal for type t_1 voters to vote sincerely because a deviation would significantly reduce their chances of getting their most preferred alternative. Type t_4 voters vote strategically (for their worst alternative!) at the first stage in order to increase the chances of their most preferred alternative at the second stage. This is optimal for type t_4 voters as long as v is small enough.

The strategic manipulations imply that a Condorcet winner, which always exists, will not always be selected in equilibrium: If each alternative is the peak of one of the voters, alternative 2 is the Condorcet winner. However, because the voter with peak at alternative 3 votes left in the first stage, the Condorcet winner is eliminated at the first stage. Note that by voting insincerely, a voter with peak at alternative 3 runs the risk that his least preferred alternative is finally selected in equilibrium.

Consider now the same amendment procedure, but with a different agenda: the first vote is between alternatives 1 and 3, and the second vote between the winner of the first vote and alternative 2. This procedure can be represented by the voting tree illustrated in Figure 2.

The second vote is either between 1 and 2, or between 2 and 3, and voting sincerely at the last vote is again optimal for all types of all agents. Consider now the first vote between alternatives 1

⁷Indeed, in the sophisticated equilibrium under complete information the agents with peaks on alternative 1 or 2 vote right in the first vote and the voter with peak on alternative 3 votes left in the first vote, leading to the Condorcet winner 2 being finally selected.

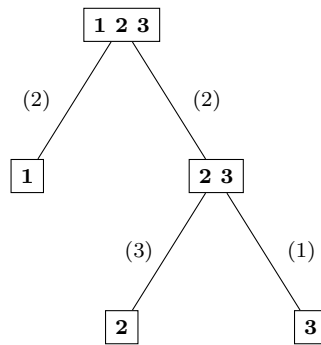


FIGURE 3. ILLUSTRATION OF A SUCCESSIVE VOTING PROCEDURE

and 3. Sincere voting calls for agents with peaks on 1 to vote for alternative 1 and this is indeed optimal since every outcome that can be reached following the left branch at the origin is (weakly) preferred by such a player to any outcome that can be reached following the right branch. An analogous reasoning shows that sincere voting is optimal for agents with peaks on alternative 3.

Consider next an agent i with preference profile $2 > 1 > 3$, for whom sincere voting recommends voting for alternative 1 at the first vote. This vote matters only if such an agent is pivotal, thus only in case there is exactly one other agent that votes for alternative 1, and exactly one agent that votes for alternative 3. But then our agent i can be sure that voting sincerely at the first vote will ultimately lead to a second vote between 1 and 2 where his most preferred alternative is elected. An analogous reasoning for an agent i with preference profile $2 > 3 > 1$ completes the proof that sincere voting is an ex-post perfect equilibrium in this amendment procedure. This equilibrium has the desirable property that the Condorcet winner will always be selected.

As we shall see below, the crucial difference between the two agendas is a (discrete) convexity idea: given the order under which preferences are single-peaked, the votes in the second example are always among sets of alternatives "without holes": the generated divisions are $\{12|23\}$, $\{1|2\}$ and $\{2|3\}$. In contrast, in the first example, the divisions are $\{13|23\}$, $\{1|3\}$ and $\{2|3\}$ and the division $\{13|23\}$ contains the non-convex set $\{13\}$ with a "hole" in place of alternative 2. This creates uncertainty about the actions of others (and hence about the outcome) that cannot be satisfactorily resolved - i.e., without ever experiencing regret - under incomplete information.

B. The Role of Monotonicity

In order to explain the role of our second condition, let us look at the successive voting procedure illustrated in Figure 3. At the first vote the agents decide whether to accept or reject alternative 1. If 1 is accepted voting ends, and otherwise a vote is taken whether to accept or reject alternative 2. If this alternative is accepted voting ends, and otherwise alternative 3 is elected.

Assume first that alternative 1 is adopted by simple majority (so that two votes are sufficient to reject it) while alternative 2 is adopted by unanimity (so that one vote is sufficient to reject it, and hence elect alternative 3). Consider an agent i with preferences $2 > 1 > 3$. Then sincere voting calls for such an agent to vote against alternative 1 at the first vote. However, in case there is one agent with a peak on 1 and one agent with a peak on 3, sincere voting would lead to alternative 3 being elected. Agent i would then be better off by deviating and voting for alternative 1 at the first vote, which would lead to the implementation of alternative 1. Therefore, sincere voting is not an ex-post perfect equilibrium for the above voting procedure.

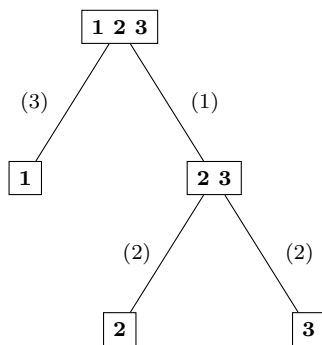


FIGURE 4. SUCCESSIVE VOTING PROCEDURE SATISFYING MON

Note that the partitions generated by this procedure are convex, without "holes": these are $\{1|23\}$ at the first vote and $\{2|3\}$ at the second. So the difficulty lies here elsewhere, namely in the specific thresholds for rejection of consecutive alternatives.

To correct the problem consider the same voting tree, but where alternative 1 is adopted by unanimity (so that one vote is sufficient to reject it) while alternative 2 is adopted by simple majority (so that two votes are required to reject it, and hence to elect alternative 3) (see Figure 4).

Then sincere voting is an ex-post perfect equilibrium: this is obvious for the agents with peaks on alternative 1 and 3 and for an agent with preferences $2 > 3 > 1$, so consider again an agent i with preferences $2 > 1 > 3$. If such an agent is pivotal at the first vote (to accept or reject 1) then the two other agents have peaks on alternative 1, so that at the next vote it must be the case that alternative 2 will be chosen. Hence it is optimal for our agent to also vote sincerely.⁸

This example shows that the thresholds at consecutive votes must be such that pivotal agents can infer (from the instance of them being pivotal) that, roughly speaking, they can still control the outcome at future stages.

II. The Sequential, Binary Voting Model

There is a finite set of alternatives, $A = \{1, 2, \dots, |A|\}$, and a finite set of voters, N . Each voter $i \in N$ has a strict preference ordering $>_i$ over the elements of A . A preference ordering $>_i$ is *single-peaked* (with respect to the natural order \leq on A) if there is an alternative $a \in A$ such that $c < b \leq a$ or $a \leq b < c$ imply $b >_i c$. We denote by \mathbf{P}^{SP} the set of preference profiles that are single-peaked, and assume below that each voter has single-peaked preferences.

A binary tree is a pair $\mathbf{V} = (V, Q)$ where V is a finite set whose elements are called *nodes*, and $Q \subseteq V \times V$ is a partial ordering over V whose elements are called *branches*. It is assumed that V has a unique least element (the *origin*), and that for any $v, v' \in V$ there is at most one Q -chain between v and v' . Any maximal element of V is called a *terminal node*, and we denote by V' the set of non-terminal nodes. If $v Q v'$ for $v \neq v'$, we say v' is a *successor* of v . Every non-terminal node has exactly two successors. If two distinct nodes are successors of the same node, we call them *neighbors*. If there are nodes $v_0 = v, v_1, \dots, v_l = v'$ such that $v_{k-1} Q v_k$ for $k = 1, \dots, l$, we say that v' *follows* v .

⁸Claims in the literature indicate that sincere voting should be a dominant strategy for this procedure. This is not true as the following example illustrates: Suppose agent 1 has preferences $2 > 1 > 3$, agent 2 votes left in the first vote and right in the second, and agent 3 always votes right. Then sincere voting by agent 1 yields alternative 3 being selected. However, if agent 1 deviates and votes left in the first vote instead, then alternative 1, which he prefers to alternative 3, will be selected.

Given a binary tree, a *division correspondence* $L : V \Rightarrow A$ associates with each terminal node a unique alternative, and with each node $v \in V'$ all the alternatives that are associated with terminal nodes following v . We assume that each alternative is associated with at least one terminal node and that for any v and any v' following v , $L(v) \neq L(v')$. The set $L(v)$ denotes the alternatives that are under consideration by the voters at node v .

A *voting tree* (\mathbf{V}, L) is a binary tree \mathbf{V} together with a division correspondence L .

Definition 1. A voting tree (\mathbf{V}, L) satisfies convexity of divisions (CONV) if,

$$\text{for all } v \in V, a \leq b \leq c \text{ and } a, c \in L(v) \text{ imply } b \in L(v).$$

We denote by $\min_{SP} L(v)$ and $\max_{SP} L(v)$ the smallest and largest alternatives of the set $L(v)$ in the order underlying the single-peaked preferences. Given a voting tree (\mathbf{V}, L) and two neighbors $u, v \in V$, we label one of the branches leading to u by ℓ and the other by r . If (\mathbf{V}, L) satisfies CONV, we label the branches leading to u and v as follows: If $\min_{SP} L(u) < \min_{SP} L(v)$, we label the branch leading to node u by ℓ , the *left* branch, and the branch leading to v by r , the *right* branch (and vice versa).⁹

We can identify a node in terms of the branches that lead to this node starting from the origin. Given a path $v \in \{r, \ell\}^k$, we denote by $v \oplus r$ the path of length $k + 1$ whose first k entries coincide with the entries of v and whose last entry is r (and similarly for ℓ).

A *system of thresholds* for a voting tree (\mathbf{V}, L) is a tuple of functions $\tau^\ell : V' \rightarrow \{1, 2, \dots, n\}$ and $\tau^r : V' \rightarrow \{1, 2, \dots, n\}$ such that for any $v \in V'$,

$$\tau^\ell(v) + \tau^r(v) = n + 1.$$

For any non-terminal node, the thresholds determine the number of votes needed in order to continue to the successor node of the left and right branch, respectively. The sum of the two thresholds is $n + 1$, so that no ties can occur. A *voting procedure* (\mathbf{V}, L, τ) is a voting tree (\mathbf{V}, L) together with a system of thresholds τ .

Definition 2. A voting procedure (\mathbf{V}, L, τ) satisfies *monotonicity of thresholds* (MON) if, for all $u, v \in V$ such that $\max_{SP} L(v \oplus \ell) \leq \max_{SP} L(u \oplus \ell)$ and $\min_{SP} L(v \oplus r) \leq \min_{SP} L(u \oplus r)$, it holds that $\tau^\ell(v) \geq \tau^\ell(u)$.

Assuming convexity, monotonicity requires that the adoption of the left branch at v should be governed by a (weakly) larger qualified majority than the adoption of the left branch at u , if the left branch at node v is associated with more extreme alternatives than at node u (and analogously for right branches).

For example, consider a node u such that $L(u \oplus \ell) = \{\dots, k\}$ and $L(u \oplus r) = \{k + 1, \dots\}$ and a node v such that $L(v \oplus \ell) = \{\dots, l\}$ and $L(v \oplus r) = \{l + 1, \dots\}$ where $l < k$. Monotonicity requires that any coalition that can enforce the more extreme left subset at node v should also be able to enforce the more moderate set of alternatives associated with the left branch at u (and any coalition that can enforce the more extreme right branch at u should be able to enforce the more moderate right branch at v). In addition, after adding more moderate alternatives to $L(v \oplus r)$ ($L(v \oplus \ell)$), it should become weakly harder to choose the left (right) branch at v . Monotonicity is always satisfied if, for example, after a vote that resulted in the adoption of the left (right) division, any subsequent movement left (right) requires a qualified majority that is at least as large as the one that governed the previous move in the same direction.

⁹This labeling procedure is well-defined given that (\mathbf{V}, L) satisfies CONV.

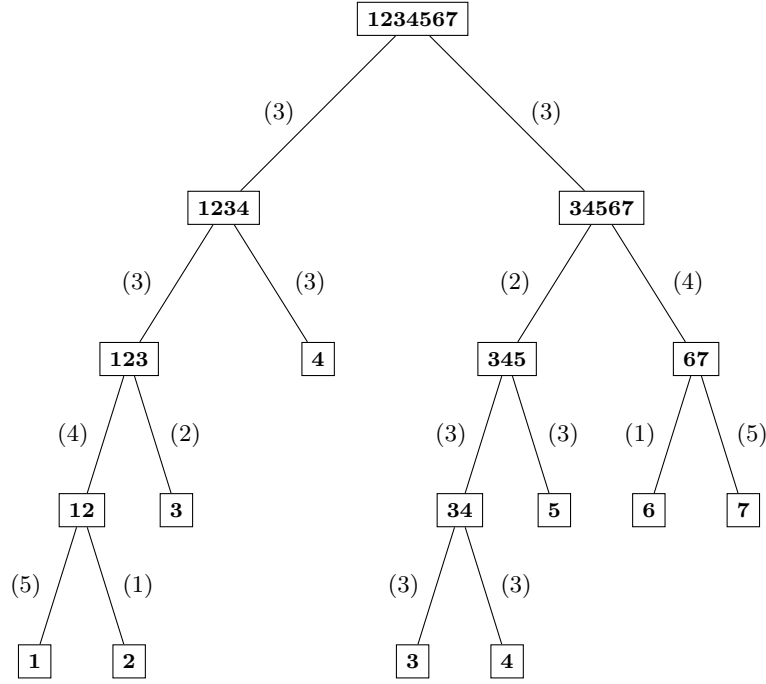


FIGURE 5. EXAMPLE OF A VOTING PROCEDURE SATISFYING CONV AND MON.

Figure 5 shows an example of a general voting procedure satisfying CONV and MON.

Together with a given set of agents, their preferences and beliefs, a voting procedure describes a game of incomplete information: At each non-terminal node, players simultaneously vote either for the left or the right branch. If there are at least $\tau^\ell(v)$ voters voting for the left branch at node v , then the game advances to node $v \oplus \ell$; otherwise the game advances to node $v \oplus r$. If a terminal node v' is reached, then alternative $L(v')$ is implemented.

Ex-post Perfect Equilibrium and Sincere Voting

Even with restrictions on the set of admissible preferences (such as single-peakedness), the analysis of the extensive form voting trees can be daunting if agents are privately informed. Optimal strategies generally depend on the specific, cardinal representation of utilities and on the beliefs about others. In turn, these beliefs are influenced by inferences that can be drawn from the ex-ante probabilities attached to the different possible profiles of preferences, and from new information generated by the employed strategies in the respective institutional setting.¹⁰ We focus instead on the much more robust, *ex-post perfect* equilibrium: in such an equilibrium, the resulting optimal strategies do not depend on ex-ante beliefs, and continue to be optimal irrespective of the information that is revealed during the sequence of votes.¹¹ This is the strongest form of equilibrium possible in our setting since, if there are more than two alternatives, desirable outcomes cannot be dynamically implemented in dominant strategies.

¹⁰In particular, *manipulations* may occur also via voting behavior that attempts to influence the beliefs of other voters, and hence their future behavior (signaling effect), an effect that is absent under complete information.

¹¹Ex-post equilibria in settings with cardinal utility and with monetary transfers have been studied in the literature on *robust* mechanism design, e.g., by Bergemann and Morris (2005) and Jehiel, Meyer-ter Vehn, Moldovanu and Zame (2006).

Let H_i^v denote the part of the history of play that is observable to player i at node v .¹² One sensible specification is that H_i^v consists of the aggregate number of left and right votes at each previous node, and i 's own voting behavior at all previous nodes. Another possible specification is that H_i^v includes the individual voting behavior of every player at all previous nodes. A strategy of player i associates to each node and each history an action.¹³ None of our results below depend on the exact specification of H_i^v .

Definition 3. *A strategy profile constitutes an ex-post perfect equilibrium if for every non-terminal node, and following any history, the agents play best responses for every realization of preferences.*¹⁴

The ex-post equilibrium embodies a notion of no-regret: even if all private information is revealed, no voter regrets her equilibrium strategy, given the equilibrium strategies of the other voters. It is particularly robust because it does not depend on the beliefs voters entertain.

For any preference ordering $>_i$ over A we denote, by the same symbol, its lexicographic extension over sets of alternatives.¹⁵ This allows us to define sincere voting as:

Definition 4. *A voting strategy is sincere given preference $>_i$ if it prescribes at each node $v \in V'$ to vote ℓ if and only if $L(v \oplus \ell) >_i L(v \oplus r)$.*

Under sincere voting, each voter votes for the set that contains his most preferred alternative. If this alternative is contained in both sets, he votes for the set containing his second-most preferred alternative, and so on. This definition of sincere voting goes back to Farquharson (1969) (see also Miller (2010) for a recent discussion of this definition).

III. Sophisticated Sincerity

In order to make our arguments as transparent as possible, we first treat the class of partitional voting procedures, where the sets of alternatives associated with two neighbors (i.e., successors of the same node) are disjoint. A well known example is the successive voting procedure, mentioned in the Introduction: alternatives are considered one at a time, so that, for any non-terminal node, one successor node leads to a single alternative while its neighbor leads to all other remaining alternatives that were not yet eliminated. We shall afterwards analyze the somewhat more complex case where the voting tree need not be partitional.

A. Partitional Voting Procedures

We call a voting procedure (\mathbf{V}, L, τ) *partitional* if $L(u) \cap L(v) = \emptyset$ for all neighbors $u, v \in V$.

Theorem 1. *Consider any partitional voting procedure satisfying CONV and MON. Then the profile of strategies where each player votes sincerely constitutes an ex-post perfect equilibrium. Moreover, if a simple majority is required at each stage, then the Condorcet winner will be selected in this equilibrium.*

¹²And $H^v \subset \times_i H_i^v$ the set of consistent profiles of histories.

¹³Formally, a strategy for voter i is a mapping $\sigma_i : \bigcup_{v \in V'} H_i^v \times P^{SP} \rightarrow \{\ell, r\}$.

¹⁴To formally define the equilibrium, let $g_{v,h^v}(\sigma, >)$ be the alternative that is selected if voters with preference profile $>$ use the strategy profile σ in the subgame starting at node v after observed history $h^v \in H^v$. The strategy profile σ is an ex-post perfect equilibrium if, for all $i, v \in V', h^v \in H^v$, and for all $> \in \mathbf{P}^{SP}$, $g_{v,h^v}(\sigma, >) \geq_i g_{v,h^v}(\sigma'_i, \sigma_{-i}, >)$ holds for all strategies σ'_i .

¹⁵That is, given two subsets B, C of A , we write $B >_i C$ if i 's most preferred alternative in B is preferred by him to his most preferred alternative in C or, if the most preferred alternative is the same in both sets, his second-most preferred alternative in B is preferred to the second-most preferred alternative in C and so on.

Because the equilibrium is ex-post perfect, it is robust to variations in the beliefs agents hold. Moreover, sincerity implies an additional robustness property: even if agents are limited in their use of strategic deviations (as in the home-style model of Fenno (1978)), or even if some agents have limited abilities to reason strategically, sincere voting remains a best response.

PROOF:

Assume, by contradiction, that sincere voting is not an equilibrium. Then there exist a preference profile \succ , a voter i , and a node v such that sincere voting prescribes a left vote for i at v , but this is not a best response to the sincere strategies used by the others (the arguments are analogous if sincere voting prescribes a right vote). Because sincere voting prescribes a left vote for i , his most preferred remaining alternative is contained in the left branch. Because preferences are single-peaked and the voting procedure is partitional and satisfies CONV, i prefers the largest alternative in the left branch, denoted by k , to any alternative in the right branch: $k \succ_i l$ for all $l \in L(v \oplus r)$. We now show that, if i always votes for the branch containing alternative k , then k will ultimately be selected. This contradicts our initial hypothesis that a left vote at v was not a best response.

Since, by assumption, a left vote is not a best response for voter i at v , it must be that i is pivotal at v . Hence, exactly $\tau^\ell(v) - 1$ other voters vote left at v , and therefore exactly $\tau^\ell(v) - 1$ of the other voters have a peak at k or to the left.

Since the voting procedure is partitional and satisfies CONV, for every node v' following $v \oplus \ell$ it holds that, for some alternative $l < k$,

$$\max_{\text{SP}} L(v' \oplus \ell) = l \text{ and } \min_{\text{SP}} L(v' \oplus r) = l + 1.$$

Therefore, only agents with a peak (weakly) to the left of l will vote for the left branch at v' and hence at most $\tau^\ell(v) - 1$ of the other voters will vote left at v' . Since the voting procedure satisfies MON, $\tau^\ell(v) \leq \tau^\ell(v')$. Therefore, if i votes right at v' , the right branch is chosen. Since this holds for all nodes v' following $v \oplus \ell$, and since the right branch always contains alternative k (which is the largest remaining alternative), this alternative is selected at the final node. This contradicts the initial assumption that a left vote is not a best response and shows that sincere voting is an ex-post perfect equilibrium.

If simple majorities are required at each node, then the Condorcet winner will be elected: Because the voting procedure satisfies CONV, and because the agents vote sincerely, at each node at least half of the voters vote for the subset of alternatives containing the Condorcet winner. Therefore, the Condorcet winner will be finally selected.

Remark 1. *As is common in voting games, there are additional trivial equilibria where, for example, voters coordinate to always vote left, no matter what their preferences are. If no decision requires unanimity, then no voter is pivotal, and such strategies do form an ex-post perfect equilibrium. Call a strategy for voter i responsive if, for every node $v \in V'$ and every history leading to this node, there is a preference realization such that i votes left at v , and another preference realization such that i votes right at v . Then, for any partitional voting procedure satisfying CONV and MON, sincere voting is the unique ex-post perfect equilibrium in responsive strategies: given that all other voters use responsive strategies, there is a preference profile such that agent i is pivotal at a given node; by voting insincerely, a strictly less preferred alternative will finally be elected. Hence, sincere voting is the unique best response to responsive equilibrium strategies. Thus, in our opinion, sincere voting constitutes the most compelling equilibrium for any partitional voting procedure satisfying CONV and MON.*

B. Non-Partitional Voting Procedures

There are important voting procedures that are non-partitional, the most common example being the amendment procedure. At each stage of an amendment procedure, a vote is taken among two alternatives, with the winner advancing to the next stage. Thus, for any two neighbors, the intersection of the corresponding sets of alternatives is generally non-empty, and consists of all alternatives that were not yet eliminated and that are not directly considered at the respective stage. An amendment procedure satisfies CONV if, at each stage, the two most extreme alternatives are voted on.

We now extend our results to a much broader class of voting procedures, which includes amendment procedures and partitional procedures as special cases. Specifically, we show that, for any voting procedure satisfying MON and CONV, sincere voting is an ex-post perfect equilibrium.¹⁶

Theorem 2. *Consider a voting procedure satisfying CONV and MON. Then sincere voting is an ex-post perfect equilibrium.¹⁷ Moreover, if a simple majority is required at each stage, then the Condorcet winner will be selected in this equilibrium.*

The proof of Theorem 2 is found in the Appendix.

Remark 2. *By the nature of non-partitional procedures, where the same alternative may be obtained by following both branches of a given node, sincere voting need not be the unique equilibrium in responsive strategies. However, if the voting procedure satisfies MON and CONV, then all equilibria in responsive strategies are outcome-equivalent.*

IV. Dynamic Implementation

In this section we characterize the social choice functions (seen as mappings that associate to each profile of single-peaked preferences a social alternative) that can be implemented by the sincere, ex-post perfect equilibria of sequential binary voting procedures. To do so, let us first focus on the social choice functions that are anonymous, unanimous and dominant-strategy implementable (DIC) on the domain of single-peaked preferences.¹⁸ Unanimity is a weak form of Pareto-Optimality, and requires that an alternative is selected as the social choice when it is preferred by all agents to all other alternatives. Anonymity requires invariance of the social choice with respect to permutations of the voter's names. Note that we do not require neutrality since sequential, binary voting schemes and their agenda are per-se non-neutral.

The set of anonymous, unanimous and DIC social choice functions has been elegantly characterized by Moulin (1980) and further refined by Border and Jordan (1987) and Barbera, Gul and Stacchetti (1993). Moulin showed that any such mechanism can be described as choosing the median peak among n reported real peaks and $n - 1$ phantom peaks (these are constant for each mechanism, and do not depend on the reports of the real voters).¹⁹ In their study about optimal, utilitarian voting schemes Gershkov et al. (forthcoming) showed that the successive voting procedure with a particular convex agenda can be used to replicate, in ex-post perfect equilibrium, every anonymous,

¹⁶Ordeshook and Schwartz (1987) argue that the theoretical concept of amendment procedures as defined above is too narrow, and that many realistic agendas take a different form. Our characterization includes also *asymmetric* and *non-uniform* amendment agendas in Ordeshook and Schwartz's terminology. Note, however, that among amendment procedures, only their *continuous* agendas can satisfy CONV.

¹⁷The proof in the Appendix actually shows that, as long as the strategy sets contain only pure strategies, no coalition has ex-post a profitable deviation. This is particularly important for legislatures where voting is mostly according to party lines, so that coordinated deviations are likely.

¹⁸Given our private value environment, DIC is equivalent to implementability in ex-post equilibrium.

¹⁹Moulin assumed that the social choice functions depend only on the reported peaks, while Border and Jordan, among others, showed that the peak-only assumption is not needed.

unanimous and DIC mechanism. This is done by varying the adoption threshold associated with each alternative in the successive procedure.²⁰ Our result below extends their observation to any convex voting procedure. It offers both a realistic implementation - via sequential, binary voting procedures - of a rich class of non-dictatorial social choice functions, and a much more transparent interpretation of the "phantom voters" that play the main role in Moulin's analysis. It also shows that the use of binary voting trees that satisfy convexity is not restrictive.

Theorem 3. *Consider any unanimous and anonymous social choice function $f : \mathbf{P}^{SP} \rightarrow A$ that is implementable in ex-post equilibrium, and a voting tree (\mathbf{V}, L) satisfying CONV. Then, there exists a system of thresholds τ such that the voting procedure (\mathbf{V}, L, τ) satisfies MON and implements f in ex-post perfect equilibrium.*

This result implies that, as long as we are interested in robust implementation in ex-post equilibrium, we can, without loss, pick any binary voting tree satisfying CONV. In particular, there is no need to consider more complex voting procedures.

PROOF:

It follows from Moulin (1980) and Border and Jordan (1987) that f is a generalized median voting rule, with $\rho_k \geq 0$ phantom voters with peak on alternative k , such that $\sum_{k \in A} \rho_k = n - 1$. For each node $v \in V'$, set

$$\tau^\ell(v) = n - \sum_{m=1}^{\max_{SP} L(v \oplus \ell)} \rho_m \quad \text{and} \quad \tau^r(v) = \sum_{m=1}^{\max_{SP} L(v \oplus \ell)} \rho_m + 1.$$

By construction, $\tau^\ell(v) + \tau^r(v) = n + 1$ for all $v \in V'$. We now show that this system of thresholds satisfies MON. Consider a pair of nodes u and v such that $\max_{SP} L(v \oplus \ell) \leq \max_{SP} L(u \oplus \ell)$. Then, $\tau^\ell(v) \geq \tau^\ell(u)$ and consequently the system of thresholds τ satisfies MON, and Theorem 2 implies that sincere voting is an ex-post perfect equilibrium.

Fix now a preference profile \succ in \mathbf{P}^{SP} and suppose that $f(\succ) = k$. We show that k is selected in the sincere voting equilibrium. Because $f(\succ) = k$, there are at least $n - \sum_{m=1}^k \rho_m$ agents with a peak weakly to the left of k and at most $n - \sum_{m=1}^{k-1} \rho_m - 1$ voters with a peak strictly to the left of k . By construction

$$\tau^\ell(v) = n - \sum_{m=1}^{\max_{SP} L(v \oplus \ell)} \rho_m \leq n - \sum_{m=1}^k \rho_m$$

for any node $v \in V'$ such that $k \in L(v \oplus \ell)$. Since there are at least $n - \sum_{m=1}^k \rho_m$ agents voting for left, the left branch is chosen in this case. If $\max_{SP} L(v \oplus \ell) < k$, then, by construction,

$$\tau^r(v) = \sum_{m=1}^{\max_{SP} L(v \oplus \ell)} \rho_m + 1 \leq \sum_{m=1}^{k-1} \rho_m + 1.$$

By the above argument, there are at least $\sum_{m=1}^{k-1} \rho_m + 1$ voters with a peak weakly to the right of k , and therefore the right branch is chosen. Hence, at each node, a branch that contains alternative k is chosen, and consequently the final choice must be alternative k .

²⁰Note that in that procedure each alternative appears at a unique terminal node.

V. Case studies

We discuss here several real-life cases in light of our findings. Our main goal is two-fold: (1) We first offer examples where convex agendas were used, and we argue that the observed outcome is consistent with sincere voting (and hence lead to the election of a Condorcet winner). This holds even in complex, multi-stage strategic situations with considerable uncertainty about preferences; (2) Conversely, we illustrate how a lack of convexity can lead to apparent strategic manipulations during a sequential voting process.

In Sub-section V.A we look at a decision taken by successive voting in the German parliament. In that legislature the agenda is convex by design, and sincere voting was indeed the most likely outcome.

Sub-section V.B reviews the study of Ladha (1994) on roll-calls in the U.S. Congress. Since agenda formation is procedural rather than ideological, the amendment procedure used by Congress need not be convex. But Ladha identified a large set of roll-calls where the consecutive votes happened to follow an ideologically coherent order. The observed outcomes are again consistent with sincere voting.

In Sub-section V.C we present a case on pension reform from the Swiss second Chamber of Parliament: the amendment procedure used to elect one out of three alternatives was non-convex. Non-convexity implies here the existence of beliefs such that sincere voting is not optimal for some types of voters, and there was indeed evidence of a strategic manipulation.

In Sub-section V.D we look at a decision on gun control taken by the Swedish Parliament. Although in each of the two Chambers of that parliament the procedure happened to be convex, the nature of the opinion aggregation process within the bicameral system lead to a non-convexity, and strategic manipulations for some types became advantageous.

Finally, in Sub-section V.E we briefly sketch the voting procedure used by the U.S. Supreme Court. We argue that the nature of decisions that need to be taken - first to grant a cert or not, and then to decide on merits in case the cert was granted - inherently involves a non-convexity. Moreover, the monotonicity of the qualified majority requirements (4 votes on the cert, 5 votes for the decision on merits) is only apparent. Thus, one should expect strategic voting at the cert stage, and such instances have been often documented in the legal literature.

A. *Stem Cell Research in Germany*

We review here a representative case from the German parliament, a legislature where decisions on several alternatives are taken by successive voting, and where the agenda is convex by design. Sincere voting - predicted by our theory to constitute an equilibrium in this situation - was the likely outcome.

In 2008, the Parliament voted on several proposals to change the restrictive law regulating stem cell research. That law allowed the import of 40 specific stem cell lines developed abroad before 2002, while forbidding research using any newer lines.²¹ According to a common procedure adopted in other cases involving ethical questions (such as laws on abortion, living-wills, assisted suicide, etc.), legislators were formally freed from party discipline, and thus were able to vote purely according to their "conscience". All proposed amendments were formulated by groups of legislators that crossed party lines (including those of the two large centrist ones), leading to considerable uncertainty about the outcome.

There were 4 alternatives which we name and order here by their main characteristic, the number of stem cell lines that are approved for research:

²¹Even researchers working abroad on newer stem cell lines were, theoretically, guilty of a felony.

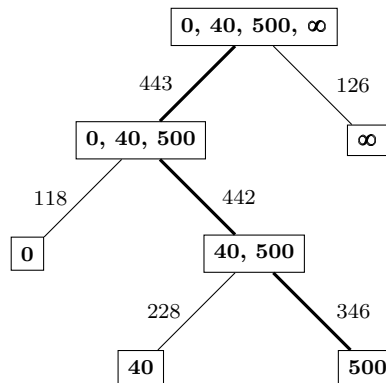


FIGURE 6. VOTING TREE AND RESULTS OF THE VOTES ON THE REGULATION OF STEM CELL RESEARCH

- 1) Proposal **0**, a ban on stem cell research.
- 2) Proposal **40**, retain the status quo.
- 3) Proposal **500**, a relative liberalization, implemented by moving the development deadline to 2007.
- 4) Proposal **∞**, a complete liberalization, freeing stem cell research from further regulation.

Due to the unambiguous, uni-dimensional issue, we assume single-peaked preferences according to the natural order. The German Parliament uses a successive voting procedure, and simple majority is required for each decision. It is important to recall (see Introduction and Table 1) that this Parliament follows the "extremes first" prerogative: in this case the order of votes was $\infty, 0, 500, 40$ (see Figure 6). Note how the agenda oscillates from the extreme on one side to the extreme on the other side! As already mentioned in the Introduction, such an agenda is *convex*, and thus our theory predicts the election of the Condorcet winner via an equilibrium where all legislators vote sincerely.

In the actual vote, there were 3 openly conducted roll-calls. First, alternatives ∞ and **0** were defeated, and then alternative **500** won over alternative **40** with 346 to 228 votes, and was formally elected. The analysis below is based on detailed results, available in disaggregated form at the level of the individual legislator.

The game described above is relatively complex, and it was played by more than 500 incompletely informed voters, each each having one out of 8 possible types and many feasible strategies.²² But, it is important to note that, for any convex voting procedure, we can compute the complete individual expected pattern of vote using only assumptions about the respective location of the individual peak (i.e., without knowing the precise ranking below the peak).²³ Therefore, out of the $2^3 = 8$ possible voting profiles only 4 are consistent with sincere (and hence postulated equilibrium) behavior. Conversely, because in this particular case each alternative was actually put up for vote, we can estimate how many members of parliament had a peak on each alternative. Table 2 below summarizes the results.²⁴

²²Eight is the number of different rankings consistent with single peaked preferences on 4 ordered alternatives. The number of strategies gets very large if detailed information is revealed after each binary vote in the sequence.

²³This need not be true for non-convex procedures.

²⁴We do not show the votes of 7 legislators who did not take part in every vote and of 28 legislators who abstained from a vote at least once. While not consistent with strict preferences, the behavior of all but one of these legislators was consistent with sincere voting according to weak single-peaked preferences.

	Peak on 0	Peak on 40	Peak on 500	Peak on ∞	Inconsistent
Equilibrium profile	ℓ, ℓ, ℓ	ℓ, r, ℓ	ℓ, r, r	r, r, r	r, r, ℓ ; ℓ, r, r ; etc.
Occurrences of profile	115	97	211	116	7

TABLE 2—PREDICTED VOTING PROFILES AND ACTUAL OCCURRENCES

Note how the vast majority of individual voting profiles, 539 out of 546, agrees with one of the 4 expected, sincere voting patterns. Additional evidence for the hypothesis of sincere voting is obtained by juxtaposing: (1) the observed behavior (ℓ, ℓ, ℓ) of 33 out of 48 Green legislators with their party's traditional conservative view on this issue; (2) the observed behavior (r, r, r) of 47 out of 54 Liberal legislators with their party's liberal view. In summary, even when analyzing individual voting behaviour we find no evidence against sincere voting in this example of a convex voting procedure.

B. Roll Calls in the U.S. Congress (Ladha, 1994)

Recall that the U.S. Congress employs an amendment procedure such that agenda formation is procedural rather than content-based (see also Table 1). Consequently, the resulting procedure need not be convex. Nevertheless, Ladha (1994) was able to identify series of votes covering diverse areas of legislation, where the order of votes on amendments in a series closely followed their perceived position on the Liberal-Conservative ideological spectrum. Thus, all conducted votes were among convex sets of alternatives, and sincere voting seems the most likely explanation for the observed outcomes.

Ladha looked at 200 votes selected from more than 8000 roll calls over a period of 8 years. The criterion for selection - the existence of several comparable votes on the same issue - was driven by the author's aim of carefully testing several direct empirical implications of a model proposed by Poole and Rosenthal (1997) in their monumental book on the U.S. Congress. Let us illustrate these general findings with one of Ladha's examples, a series of 1977 Senate votes on the level of tip credit that enable hotels and restaurants to pay less than the minimum wage. The Human Resources Committee has proposed a 20% credit, i.e., allowing pay 20% under the minimum wage. Three amendments were considered, at levels of 50%, 40% and 30%, respectively, in this chronological order. Hence, each observed vote was indeed amongst the two most extreme remaining alternatives. The actual order of votes is depicted, together with the voting results, in Figure 7.

Ladha's empirical hypothesis, *assuming* sincere voting, was that the number of Yes votes (of legislators who prefer higher levels of credit) increases and the number of No votes (of legislators who prefer lower levels) decreases, as voting advances from the high to the low proposal. This is what the actual results suggest for this case, and basically the same picture emerges in all 200 examined cases: smooth increases of the number of Yes votes paralleled by smooth decreases in the number of No votes as the amendment becomes more and more liberal or more and more conservative.²⁵ Thus, there are no instances of large swings characteristic of strategic voting (see, for example, the Swiss example below). Since the roll calls were conducted according to an agenda where the order of vote followed the ideological one, we would indeed expect that incentives to manipulate are significantly reduced. Although we do not have precise information about the hypothetical complete agenda in all these cases, Ladha's many examples suggest that trees with a convex agenda lead to sincere voting in real-life situations.

²⁵One senator voted only for the 40% level, which explains the small increase, from 46 to 47.

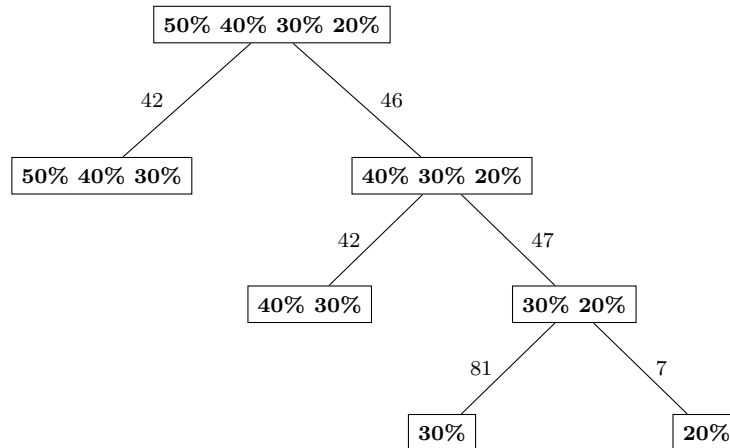


FIGURE 7. OBSERVED VOTING OUTCOMES ON THE LEVEL OF TIP CREDIT

C. Pension and Women's Status in Switzerland (*Senti, 1998*)

As Table 1 documents, Swiss legislatures use an amendment procedure that does not always put the two most extreme alternatives up for vote. Therefore, the procedure will fail sometimes to be convex, leading to possible strategic manipulations. The following example identifies a decision where strategic manipulations led to the rejection of the likely Condorcet winner.

The decision is about a pension reform pertaining to the status of married women, debated in the Swiss Chamber of Cantonal Representatives.²⁶ The alternatives were

- (1) keep the status quo where women's benefits are mainly defined by their husbands' contributions,
- (2) a moderate reform,²⁷ and
- (3) a radical reform, that would make pension contributions and benefits individual rather than family based.

We assume that preferences are single-peaked according to the natural order (for detailed arguments supporting this hypothesis, see Senti 1998). The first vote was among alternatives 2 and 3, and the final vote was among the winner of the first vote and alternative 1. Because the first vote is not among the extreme alternatives, this procedure is not convex, and sincere voting is not an ex-post equilibrium (see also Sub-section I.A). Consequently, there exist combinations of preferences and beliefs such that some types of voters have incentives to strategically deviate from sincere voting.

The observed voting outcomes suggest that voters indeed deviated from sincere voting. At the first stage, alternative 3 won 24 to 19 against alternative 2. At the second stage, however, alternative 3 lost 13 to 30 against alternative 1. This outcome is not consistent with sincere voting based on single-peaked preferences: any voter who votes sincerely for alternative 3 at the first stage has preferences $3 > 2 > 1$, and should consequently also vote for alternative 3 at the second stage.

²⁶This is the second Chamber of the Swiss Parliament, similar to the U.S. Senate.

²⁷In fact there were two moderate proposals, but this does not change the conclusions. We use the simplified version, following Senti.

Therefore, under sincere voting, alternative 3 should obtain weakly more votes on the second stage than on the first stage, contrary to the vote totals we actually observe (recall also the monotonicity of total votes tested by Ladha in the previous case).

The above outcome is typical of so-called "both extremes against the middle" examples of strategic voting discussed in the literature. In a detailed study of this case Senti (1998) concludes that the moderate alternative 2 was the Condorcet winner. His arguments suggest therefore that the strategic deviations from sincere voting led to the rejection of the Condorcet winner at the first stage.

D. Gun Control in Sweden (Bjurulf and Niemi, 1978)

As shown in Table 1, the Swedish parliament also uses an amendment procedure with a procedural agenda setting that need not be convex. In addition, the Swedish parliament consists of two chambers; we show that the rule by which their decisions are aggregated introduces an additional non-convexity that offers possibilities for strategic manipulation.

The Swedish parliament had to decide among three alternatives concerning the government's financial support for the national riflemen's association. These alternatives were

- (1) support the riflemen's association with 500.000 crowns,
- (2) support the riflemen's association with 470.000 crowns, and
- (3) do not support the association.

The main difference between alternatives 1 and 2, and the reason for the political controversy, were the extra 30.000 crowns, targeted at very young riflemen aged 12-15. Bjurulf and Niemi (1978) provide evidence that the Social Democrats had clear preferences $3 > 2 > 1$, while the Conservatives had opposed preferences $1 > 2 > 3$. The Farmers had preference $2 > 1 > 3$, while the preferences of the Liberals were uncertain, but most likely had a peak either on alternative 2 or 3. Given the number of seats of the parties, the arguments presented by Bjurulf and Niemi (1978) clearly suggest that alternative 2 was the Condorcet winner.

In each chamber, voting was according to an amendment procedure with 1 and 3 competing against each other at the first stage, with the winner competing against 2 at the second stage. In this case, the agenda used in each chamber was convex! However, each chamber decides independently, and if the elected alternatives are different, a joint vote is taken among the choices of the two chambers. This special voting architecture may lead to a violation of convexity - see Figure 8 for an illustration.

Because of this non-convexity, some voters may actually prefer to deviate strategically from sincere voting to see their least preferred alternative advance to the last stage: after their most preferred alternative 3 was rejected at the first stage, 36 out of 39 Social Democrats indeed abstained in the second stage, allowing their least preferred alternative 1 to advance (winning by 37 to 32 votes). Because alternative 3 won in the second chamber, there was a final vote between alternatives 1 and 3, with legislators from both chambers taking part. In this final vote, alternative 1 won with 197 to 168 votes. Bjurulf and Niemi (1978) argue that this alternative was not the Condorcet winner, and that some Social Democrats may have experienced ex-post regret; as illustrated in Example 1, such an outcome is nonetheless consistent with equilibrium under incomplete information.

E. Certiorari at the U.S. Supreme Court (Caldeira et al., 1999)

We analyze here the voting procedure used by the U.S. Supreme Court in light of our findings. We argue that the procedure cannot be convex; therefore we should observe strategic manipulations.

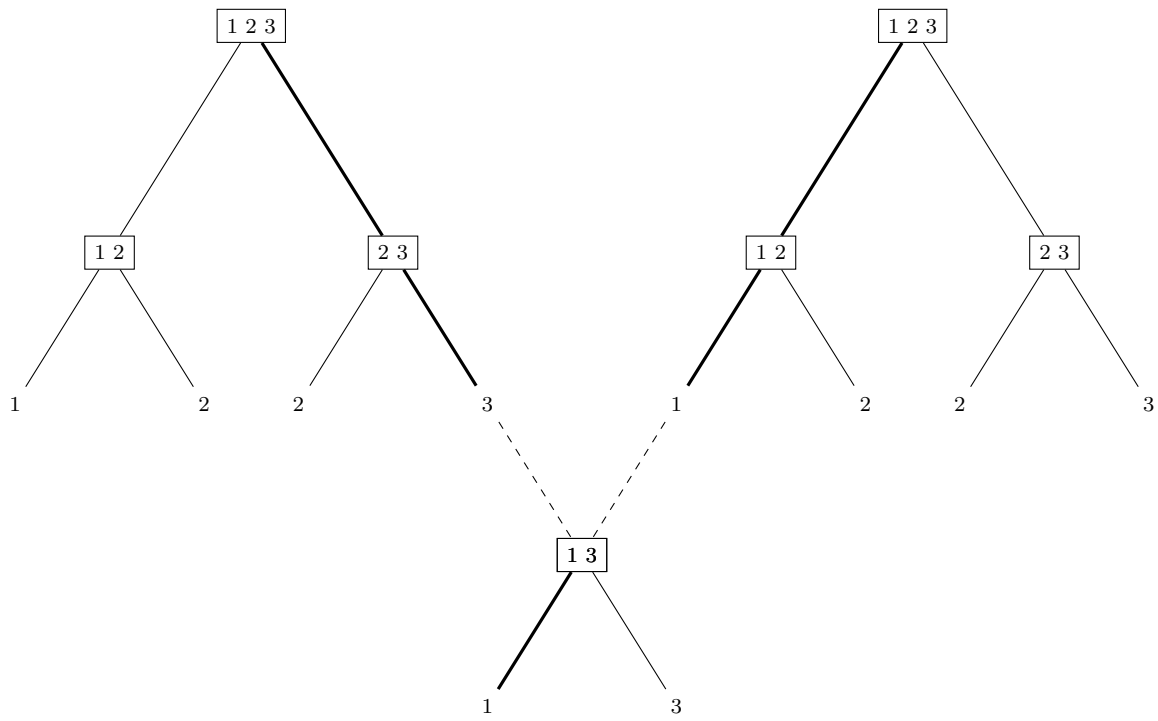


FIGURE 8. GUN CONTROL IN SWEDEN

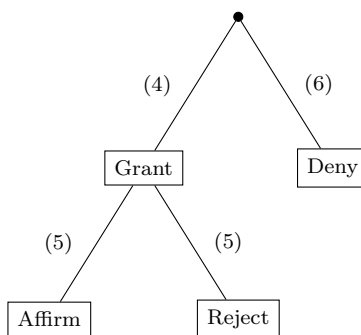


FIGURE 9. STRUCTURE OF THE DECISION PROCESS AT THE U.S. SUPREME COURT

Most cases decided by the U.S. Supreme Court arise from petitions to review decisions of lower courts.²⁸ The decision process is successive: first, a decision to grant or deny the cert is made, where granting requires at least 4 out of the 9 judges to be in favor, thus less than a simple majority.²⁹ This decision is rather secretive and need not be explained to outsiders. If a cert is granted, a decision "on merits", to affirm, or to reverse the opinion of the lower court, follows by simple majority (5 out of 9). A decision on merit is binding and serves as precedent for all consecutive decisions in the lower courts. In contrast, a decision to deny the cert keeps the status quo and allows more latitude to future decisions by lower courts (see Figure 9).

Caldeira et al. (1999) argue that these alternatives often give rise to single-peaked preferences. Consider, for example, a cert to consider a case that has been decided in a perceived liberal way by a lower court (and analogously for a conservative decision). There are three possible outcomes that can be ordered on the liberal-conservative spectrum as follows:

- (1) Affirm the decision;
- (2) Deny the cert and keep the status quo;
- (3) Reverse the decision.

Unless the earlier "status quo" decision is obviously wrong on pure judicial terms, and thus denying the cert is presumably the least preferred alternative of all judges, the assumption of single-peaked preferences on the liberal-conservative spectrum seems reasonable.

Note that the least extreme alternative is alternative 2, to deny the cert. However, the voting process is such that this alternative must be voted on first. Only if this alternative is eliminated and a cert is granted, will the Supreme Court obtain the record of proceedings from the lower court. This leaves the second stage open for a contest among the more "extreme" alternatives 1 and 3. Thus, the procedure used by the Supreme Court cannot be convex: a convex successive procedure must start with the decision on one of the most extreme alternatives, which is simply infeasible here. In particular, the monotonicity of the adoption thresholds is only apparent! In other words, we should expect some strategic manipulation at the decision whether or not to grant the cert (while there is clearly a dominant strategy at the second stage)

Most certs are denied because they are deemed frivolous, but it has been argued, and frequently documented, that some cert decisions are "defensively" denied, for example, by liberal (conservative) judges that are afraid of a reversal (affirmation) of an earlier liberal decision.³⁰ In other words,

²⁸Certiorari (or certs) is the Latin name for requests of more information, used because the Supreme Court requests the relevant documents from the lower court.

²⁹See Godefroy and Perez-Richet (2013) for an analysis of this rule.

³⁰See, for example, the recommendation made by one of judge Marshal's clerks (cited in Caldeira et al. 1999): "In the normal

judges do not vote sincerely at the first stage, but instead vote to deny the cert on defensive strategic considerations. Such a strategic vote is optimal for beliefs that attach a sufficiently high probability to a subsequent unfavorable outcome, and it is more likely to occur for courts where the number of moderates is high enough, so that pivotality at both stages matters. Indeed, the main source of uncertainty about the preferences of others is the presence of "moderate" judges who do not have strong ideological convictions that could be used to predict their opinions with high probability.

Strategic manipulations, as illustrated above, clearly affect the court's final decisions: Alone in 1982, Caldeira et al. (1999) empirically identify 18 cases where a cert was denied on defensive grounds even though a grant was to be expected under sincere voting.

VI. Conclusion

Our study was motivated by observing real voting procedures and their outcomes. Our results offer some guidance for the practical design of sequential voting schemes that are widely used by legislatures and committees. We described a broad class of procedures that yield robust and desirable results even in relatively complex strategic situations that involve incomplete information. We also uncovered a strong rationale for content-based agendas (in contrast to agendas formed by procedural rules): if the outcome should not be sensitive to beliefs about others, nor to the deployment of strategic skills, the agenda should be built "from the extremes to the middle" so that more extreme alternatives are both more difficult to adopt, and are put to vote before other, more moderate options.

Our results also illuminate the empirical discussion about the relative frequency of strategic voting in real life situations. They imply that sincere voting is often a robust, ex-post perfect equilibrium, and hence that sincere and sophisticated voting are often observationally equivalent. On the other hand, assuming incomplete information, we were also able to explain the observed voting behavior and outcomes in situations where the likely Condorcet winner was not elected.

Appendix

Arguments for Example 1

Sincere voting by all types in the second stage is a weakly dominant strategy. Moreover, sincere voting at the first stage is optimal for voters of types t_2 and t_3 : whenever they are pivotal, there is one other voter voting right at the first stage. Since only voters with a peak on alternative 2 vote right at the first stage, they can be sure to obtain their most preferred alternative by voting left at the second stage given that they are pivotal at the first stage.

It is optimal for voters of type t_1 to vote left if, conditional on being pivotal, they prefer the left branch:

$$\begin{aligned} \mathbb{E}U(\ell|t_1, \text{pivotal}) &= \frac{1}{(q_1 + q_4)(q_2 + q_3)} [q_1(q_2 + q_3) + q_4q_2] \\ &= \frac{q_1}{q_1 + q_4} + \frac{q_2q_4}{(q_1 + q_4)(q_2 + q_3)} \\ \mathbb{E}U(r|t_1, \text{pivotal}) &= v \end{aligned}$$

Similarly, it is optimal for type t_4 to vote left if, conditional on being pivotal, they prefer the left

case, this would be a pretty clear grant. Here, though, I would deny. [...] Because every abortion case on which cert is granted creates a new opportunity to overrule *Roe*, I would deny on defensive grounds. Tactical judgments aside, the case is a grant."

branch:

$$\begin{aligned}\mathbb{E}U(\ell|t_4, \text{ pivotal}) &= \frac{1}{(q_1 + q_4)(q_2 + q_3)} [q_4(q_2 + q_3) + q_1q_3] \\ &= \frac{q_4}{q_1 + q_4} + \frac{q_1q_3}{(q_1 + q_4)(q_2 + q_3)} \\ \mathbb{E}U(r|t_4, \text{ pivotal}) &= \frac{1}{(q_1 + q_4)(q_2 + q_3)} [q_4(q_2 + q_3) + q_1(q_2 + q_3)v] \\ &= \frac{q_4}{q_1 + q_4} + \frac{q_1}{q_1 + q_4}v.\end{aligned}$$

It can be easily verified that, given the strategies of the others, if $v \leq \frac{\min\{q_2, q_3\}}{q_2 + q_3}$, then voting left in the first stage is a best response for voters of type t_1 and t_4 . \square

For the proof of Theorem 2 we need the following Lemma.

Lemma 1. *Consider an arbitrary voting procedure satisfying CONV and MON. For any alternative $k \in \{1, \dots, |A| - 1\}$ and for given any $u \in V$ such that $\max_{SP} L(u \oplus \ell) = k$ and $\min_{SP} L(u \oplus r) = k + 1$, let $\tau(k) := \tau^\ell(u)$.³¹ Then the following statements hold:*

- (i) *If $k \in L(v \oplus \ell)$ but $k + 1 \notin L(v \oplus \ell)$, then $\tau^\ell(v) \geq \tau(k)$.*
- (ii) *If $k - 1 \notin L(v \oplus r)$ but $k \in L(v \oplus r)$, then $\tau^\ell(v) \leq \tau(k - 1)$.*
- (iii) *If $k \in L(v \oplus \ell)$ but $k \notin L(v \oplus r)$, then $\tau^\ell(v) \leq \tau(k)$.*
- (iv) *If $k \notin L(v \oplus \ell)$ but $k \in L(v \oplus r)$, then $\tau^\ell(v) \geq \tau(k - 1)$.*

PROOF:

Fix any u such that $\max_{SP} L(u \oplus \ell) = k$ and $\min_{SP} L(u \oplus r) = k + 1$.

(i) CONV implies that $\min_{SP} L(v \oplus r) \leq k + 1 = \min_{SP} L(u \oplus r)$. Since $\max_{SP} L(v \oplus \ell) = \max_{SP} L(u \oplus \ell)$, MON implies $\tau^\ell(v) \geq \tau^\ell(u) = \tau(k)$.

(ii) Analogously.

(iii) Since $\max_{SP} L(v \oplus \ell) \geq k = \max_{SP} L(u \oplus \ell)$ and $\min_{SP} L(v \oplus r) \geq k + 1 = \min_{SP} L(u \oplus r)$, MON implies $\tau^\ell(v) \leq \tau^\ell(u) = \tau(k)$.

(iv) Analogously.

PROOF OF THEOREM 2:

Fix a preference profile \succ , a coalition $C \subseteq N$, and a node v . We prove that there is no profitable pure deviation for C starting at v . This is done by showing that, for any possible deviation, the members of coalition C are weakly better-off by voting sincerely at v , and by following specific strategies afterwards. Since this holds for any v , this observation implies that there is no coalition that has a profitable pure strategy deviation. In particular, sincere voting is an ex-post perfect equilibrium.

To obtain a contradiction, suppose that C has a profitable deviation starting at v . If the decision at v was the same as under sincere voting, then the members of coalition C could, without incurring a loss, vote sincerely at v and then follow the actions prescribed by the deviation from the next

³¹Since the voting procedure satisfies MON, this definition is independent of the exact choice of u .

node on. This holds because all other voters are assumed to vote sincerely, and because sincere voting is a Markovian strategy, i.e., a strategy that does not condition on the past history of play.

Therefore, there exists a subset $\tilde{C} \subseteq C$ such that sincere voting prescribes a left vote at v for $i \in \tilde{C}$, but the deviation prescribes a right vote (or vice-versa, which yields an analogous argument). Moreover, the left branch must be selected if all members of coalition \tilde{C} vote left, but the right branch is selected if all members follow the deviation and vote right. Let k be the alternative chosen if coalition C plays its deviation strategy profile, while all other voters vote sincerely. Hence, k must be contained in the right branch at v . We show first that k is also contained in the left branch.

Claim 1: $k \in L(v \oplus \ell)$

To obtain a contradiction, assume $k \notin L(v \oplus \ell)$. We show that this implies that the outcome after the deviation is strictly worse for some members of C , contradicting the assumption that the deviation was profitable. Let l be chosen under sincere voting. By CONV, $l < k$. Because the deviation is profitable for C , $k \succ_i l$ for all $i \in C$. Since preferences are single-peaked, this implies $l + 1 \succ_i l$ for all $i \in C$.

Since alternative k is selected if the coalition C plays its deviation, a node v' following $v \oplus r$ is reached where $\min_{SP} L(v' \oplus r) = k$ and where the right branch is selected. Consequently, there are at least $\tau^r(v') - |C|$ voters not in the coalition that vote for the right branch at v' under sincere voting. We now show that $\max_{SP} L(v \oplus \ell) = l$. Suppose instead that $l + 1 \in L(v \oplus \ell)$, and consider any node v'' following $v \oplus \ell$ such that $l \in L(v'' \oplus \ell)$ and $l + 1 \notin L(v'' \oplus \ell)$. Lemma 1 implies that $\tau^\ell(v') \leq \tau^\ell(v'')$, or equivalently that $\tau^r(v') \geq \tau^r(v'')$. Hence, at node v'' the right branch is chosen under sincere voting because at least $\tau^r(v') - |C|$ of voters not in the coalition C will vote right, and because the members of coalition C sincerely vote right (as they prefer $l + 1$ to l). This contradicts the assumption that l is selected under sincere voting and we conclude that $l + 1 \notin L(v \oplus \ell)$.

Because l is the largest alternative in the left branch, and because a sincere vote at v for $i \in \tilde{C}$ is to vote left, single-peaked preferences imply that $l \succ_i k$ for all $i \in \tilde{C}$. This implies that the deviation is not profitable for $i \in \tilde{C}$, which contradicts our initial assumption. Thus, we can conclude that $k \in L(v \oplus \ell)$.

Claim 2: If coalition C votes sincerely at v and afterwards always votes for the branch containing alternative k , then alternative k will be selected.

Because alternative k is contained in both branches at v , the right branch must contain a strictly larger alternative. Condition CONV implies then that $k + 1 \in L(v \oplus r)$. Since alternative k is selected if the right branch is chosen at v and if coalition C plays its profitable deviation, a node v' following $v \oplus r$ is reached such that the left branch is chosen at v' and $k + 1 \notin L(v' \oplus \ell)$. By Lemma 1 (i), $\tau^\ell(v') \geq \tau(k)$. Since the left branch is chosen at node v' , there are at least $\tau(k) - |C|$ voters not in coalition C having a peak weakly to the left of alternative k .

Since alternative k is selected if the right branch is chosen at v and if coalition C plays its profitable deviation, a node v'' following v (potentially $v = v''$) is reached such that the right branch is chosen at v'' and $k - 1 \notin L(v'' \oplus r)$. By Lemma 1 (ii), $\tau^\ell(v'') \leq \tau(k - 1)$. Since the right branch is chosen, there are at most $\tau(k - 1) - 1$ voters not in coalition C having a peak weakly to the left of alternative $k - 1$.

We show now that if coalition C always votes for a branch containing alternative k , then k is chosen following the left branch at v as well. Consider any node v' following $v \oplus \ell$ such that k is contained in the left branch, but not in the right branch: that is, $k \in L(v' \oplus \ell)$ and $k \notin L(v' \oplus r)$. By Lemma 1 (iii), $\tau^\ell(v') \leq \tau(k)$. Since at least $\tau(k) - |C|$ of the voters not in coalition C have a peak weakly to the left of alternative k , the left division is chosen if coalition C votes left.

Analogous arguments imply that the right branch is chosen whenever alternative k is only in the right branch and C votes for the right branch. As a consequence, the branch containing alternative k is chosen at each node, and k is finally selected even if the left branch is chosen at v . To conclude, we have shown that, for any deviation of coalition C at v , the same outcome can be obtained by voting sincerely at v and following specific strategies thereafter. Hence, there is no profitable deviation starting at v , contradicting our initial hypothesis.

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