The Art of Compromising: Voting with Interdependent Values and the Flag of the Weimar Republic

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Abstract

We model a situation where ex ante opinions in a legislature are dichotomous but cross traditional left-right party lines, e.g., crucial decisions on ethical issues such as gay marriage and abortion, or joining/exiting an economic/political union. In addition to the two "extreme" positions on the left and on the right, we consider the effect of a compromise alternative whose location may be endogenous. We compare sequential, binary voting schemes conducted by privately informed agents with interdependent preferences: the voting process gradually reveals and aggregates relevant information about the location of preferred alternatives. The Anglo-Saxon amendment procedure (AV) always selects the (complete-information) Condorcet winner. In contrast, the continental successive procedure (SV) does not. This holds because AV allows learning about the preferences of both leftists and rightists, while SV only allows one-directional learning at each step. Moreover, under SV, the agenda that puts the alternative with ex ante higher support last elects the Condorcet winner more often than the agenda that puts that alternative first. The optimal compromise location for various goals is also shown to differ across voting procedures. We illustrate our main findings with a fascinating historical episode, the vote on the flag of the Weimar republic.

1   Introduction

Sequential, binary voting schemes are used by almost all democratic legislatures to decide among more than two alternatives. We analyze the performance of such schemes in settings where several privately informed agents have single-peaked preferences over three alternatives, and where these preferences are interdependent: each agent's peak is determined by his/her own signal and by the mean signal of others. We focus on the effects of a compromise alternative whose location may vary endogenously between two more extreme options. The interdependence of preferences is what makes the compromise salient, whereas the compromise alternative would never be elected under a private values assumption.

We model a situation where ex ante opinions are dichotomous, but cross the traditional left-right party lines. This is the case, for example, in national decisions with far reaching consequences on basic ethical issues (e.g., gay marriage, abortion) or on joining/exiting a trade agreement or an economic/political union. For example, in the German 2017 vote to legalize same-sex marriage, the main Government party, the CDU, was split with 225 MPs against vs. 75 in favor. The CDU and their leader Angela Merkel, who voted against, were defeated since all other parties voted in favor.

While in the above mentioned case the decision itself was dichotomous, we observe many other situations where, in addition to the two "extreme" positions on the "left" and on the "right" a compromise alternative is proposed and sometimes selected. For example, during March 2019 the UK parliament struggled to identify and elect a compromise deal between the "hard" Brexit demanded by a large faction of the Tories, and the "soft" version, closer in spirit to remaining in the EU, supported by Labour and other smaller parties.[1]

In this paper we generalize the standard binary, sequential voting model by introducing interdependent preferences: we vary the dynamic voting procedures, the information disclosure policies, the (possible endogenous) location of the compromise, and the degree of preference interdependence. Since others' signals are private information, each agent is ex ante uncertain about her own preferred alternative. The voting process gradually reveals and aggregates information, and agents respond to new information by adjusting their voting strategy.

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[1] *The Economist* writes: "Both main parties, long divided over Brexit, are seeing their factions splintering into ever-angrier sub-factions." (See "Whatever next?" lead article, March 16, 2019, page 11).

As an application, we discuss the famous Weimar Flag Controversy where a compromise flag—a combination of the pre-WWI German Reich's flag on the one side, and the flag mostly associated with the progressive 1830 and 1848 revolutions on the other—was ultimately selected by the Weimar National Assembly after bitter debates that split the German nation.

Variations on the basic sequential, binary voting procedures studied in this paper are used by democratic legislatures, and by committees, in order to select one among several alternatives (see Rasch [2000]). At each stage, one among two subsets of alternatives is adopted via a simple Yes/No vote by majority. This process is repeated until a unique alternative is singled out, and formally elected. We focus here on two well-known procedures: all English-speaking democracies, several Scandinavian countries and Switzerland regularly use the *amendment* procedure where alternatives are considered two-by-two, and where the majority winner advances to the next stage, as in an elimination tournament. In contrast, most continental European parliaments (including the EU parliament) use the *successive* procedure where alternatives are put to vote, one after another, until one of them gets a majority. Moreover, we consider *convex* agendas where each of the binary Yes/No votes in the sequence must be among two subsets of options such that each of them covers a well-defined, coherent segment of positions in the respective ideological spectrum (see the Literature Review below for a justification of this choice).

The amendment procedure will satisfy convexity if the pairing is such that the most "extreme" alternatives compete against each other at each round of voting. If the single-peakedness order on three alternatives is $1 < 2 < 3$, the first vote should be among alternatives 1 and 3. If 1 wins this contest then the next vote is among 1 and 2, whereas if 3 wins at the first round, the second vote is between 3 and 2.

The successive procedure will satisfy convexity if, at each stage, the considered alternative is one of the two most extreme ones. If the single-peakedness order on three alternatives is $1 < 2 < 3$, convex agendas are to vote on the alternatives in the order $1, 2, 3$ or in the order $3, 2, 1$. For example, the German parliament and its Weimar precursor use an informal rule, rooted in custom and practice, to vote on extreme alternatives first:

> "if several proposals are made to the same subject, then the first vote shall be on the farthest-reaching proposal. Decisive is the degree of deviation from status quo."

In contrast, when the U.S. Congress, say, takes a decision involving three alternatives—the status quo, a proposed change and an amendment to that change—the status quo is usually put up to vote at the second, final stage: either against the original proposal or against the amendment (whichever won previously). If the status quo is an extreme—more to the "left" or to the "right" relative to the other two alternatives—this procedure is not convex.

We compare the equilibrium outcomes of the voting games associated to the successive and amendment procedure, respectively, under several variations concerning the information that is revealed to voters after each binary vote, and concerning the order under which alternatives are put to vote (while respecting convex agenda formation rules). The most interesting equilibria occur when agents get to know not only which alternative won at a previous stage, but also the margin of victory/defeat. This conveys information both about the relatively attractiveness of each alternative (recall that preferences are here interdependent) and about the likely election outcome. Agents make these inferences and adjust their voting strategy as a function of the margin of victory: tighter outcomes at an earlier stage signal that a compromise is both preferred, and more likely to be elected.

A main result is that, under a policy that reveals the margin of victory in each binary election, for any constellation of parameters, the amendment procedure with a convex agenda has an equilibrium that always selects the (complete-information) Condorcet winner. In contrast, the successive procedure does not always possess such an equilibrium even if convex agendas are used. Roughly speaking, the reason is that the amendment procedure—that considers two alternatives at a time—allows bidirectional learning about the preferences of both leftists and rightists, while the successive procedure only allows one-directional learning at each stage. Thus, these two procedures are **not** equivalent once preferences are interdependent, in stark contrast to the private values case (see Literature Review below).

Moreover, under the successive procedure, the two possible convex agendas (that are also equivalent under a private values assumption) are **not** anymore equivalent with interdependent values: the agenda that puts the more extreme alternative with ex ante higher support last elects the Condorcet winner with a higher probability than the agenda that puts that alternative first. The reason is, again, connected to the direction of learning: putting the alternative with ex ante higher support first on the ballot risks of "hastily" giving up that alternative in some cases, and rallying around the compromise before anything new has been learned about the number of opponents. Indeed, if the number of opponents is relatively

small, the Condorcet winner is the foregone extreme alternative, rather than the chosen compromise. Such an undesirable outcome is less likely if the first alternative on the ballot is the more extreme one with less ex ante support. This result fits well the well-documented practice to consider last on the agenda the Government's proposal—supposedly it has a higher ex ante chance of getting a majority (see also the description of agenda setting in the Weimar flag case below).

Finally, we analyze the optimal location of the compromise under the assumption that the majority party controls the agenda. This question is again motivated by the many real-life examples where traditional dichotomous options are often complemented by a third, more moderate one that emerges "synthetically" in the legislative process prior to voting. For example, as this paper was being written, the quest for a Brexit compromise continued via an "indicative voting" process, designed to establish which one out of at least 8 suggested compromises might get a majority (none did yet...) and also, for the first time in that process, via a cross-party consultation between Government and the main opposition party. Such cases are frequent in fragmented coalition governments, as in our case study below.

We identify the main forces motivating the location of the compromise: (i) to elect an alternative that is superior to those already on the table or (ii) to insure against the election of another, worse alternative. The optimal location of the compromise is shown to finely depend on several important parameters such as the size and ideology of the ex ante expected majority and the degree of interdependence in preferences, but also on the underlying voting procedure. The variation with respect to the voting procedure arises because in sequential voting the strategic equilibrium in both stages (and the resulting information disclosure) depends on the compromise location, while in amendment voting only the behavior at the second stage is affected by the compromise location.

1.1   Related Literature

Following the pioneering work by Farquharson [1969], almost the entire literature on binary, sequential voting assumed that agents are completely informed about the preferences of others (see Miller [1977], McKelvey and Niemi [1978] and Moulin [1979], among others, for early important contributions). Under complete information, the associated extensive form games are amenable to analysis by backward induction: voters can, at each stage, foresee which alternative will be finally elected, essentially reducing each decision to a vote among two

alternatives. If a simple majority is used at each stage, then, whenever it exists, a Condorcet winner is selected by sophisticated voters, independent of the particular structure of the binary voting tree, and independent of its agenda. If a Condorcet winner does not exist, then a member of the Condorcet cycle is elected. The influence of agenda manipulations on the outcome of binary, sequential voting under complete information has been emphasized by Ordeshook and Schwartz [1987], Austen-Smith [1987] and, more recently by Barbera and Gerber [2017].

An early analysis of strategic, sequential voting under incomplete information with private values is Ordeshook and Palfrey [1988]. They constructed Bayesian equilibria for an *amendment* procedure with three alternatives and with three possible preference profiles that potentially lead to a Condorcet paradox.

Gershkov, Moldovanu and Shi [2017] (GMS hereafter) analyzed voting by qualified majority in the successive procedure where agents are privately informed and where preferences are assumed to be single-peaked and following the private values paradigm.[2] In their study, the order in which alternatives are put to vote follows the order defining single-peakedness (or its inverse).

Kleiner and Moldovanu [2017] generalized the GMS result to the class of all sequential, binary procedures with a *convex* agenda. Recall that in a binary, sequential procedure each vote is taken by (possibly qualified) majority among two, not necessarily disjoint, subsets of alternatives. Convexity says that if two alternatives a and c belong to the left (right) subset at a given node, then any alternative b such that $a < b < c$ (in the order governing single-peakedness) also belongs to the left (right) subset.

Under single-peaked, private values preferences, Kleiner and Moldovanu showed that sincere voting constitutes an ex post perfect equilibrium in any voting game derived from a sequential, binary voting tree with any convex agenda.[3] In other words, voters cannot gain by manipulating their vote, regardless of their beliefs about others' preferences, and regardless of the information disclosure policy along the voting sequence. An important corollary is that, if simple majority is used at each stage of the voting tree, the equilibrium outcome of the incomplete information game induced by any binary, sequential voting procedure, by

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[2] Their focus was on finding the welfare maximizing procedure. This is achieved by varying the thresholds needed for the adoption of each alternative.

[3] Under a mild refinement, this equilibrium is unique.

any convex agenda and by any information disclosure policy is always the complete information Condorcet winner. Thus, all sequential binary voting trees with convex agendas and all information policies are **equivalent** under single-peaked, private values preferences, and this theory cannot discriminate among them.

In spite of an obvious need to understand voting with interdependent preferences, there are only a few papers that incorporate such a feature—this generalizes the more ubiquitous assumption of common values—into voting frameworks with more than two alternatives.

Gruener and Kiel [2004] and Rosar [2015] analyze static voting mechanisms in a setting where agents have interdependent preferences, focusing on the mean and the median mechanisms. Moldovanu and Shi [2013] analyze voting in a dynamic setting where multi-dimensional alternatives appear over time and where voters are only partially informed about some aspects of the alternative.

Piketty [2000] studies a two-period voting model where a large number of agents care about the decisions taken at both stages. As in our model, voting at the first stage reveals information about preferences that is relevant at the second stage. At the second stage, voters condition their behavior on the observed outcome at the first stage. Hence the preference communication motive must be traded-off against the short-term first stage strategic effects. Piketty concludes that electoral systems should be designed to facilitate efficient communication, e.g. by opting for two-round rather than one-round systems—this is congruent with the kind of multi-stage procedures observed in legislatures and discussed in this paper.

Dekel and Piccione [2000] analyzed sequential voting with interdependent values in a model with only two alternatives: sequentiality is with respect to individual voting, modeling procedures sometimes used in the US Congress. They showed that, although the history of the first votes should intuitively affect the behavior of the later voters, equilibrium conditioning on pivotality leads voters to ignore the revealed history. Ali and Kartik [2012] displayed other equilibria where voters do take into account the observed history. Such effects also play a certain role in the present study.

Enelow [1983] contains an early, basic model of optimal compromise location in a legislature that uses the amendment procedure with a non-convex agenda. His model is neither game-theoretic nor otherwise micro-founded: the (numerical) results depend on the agenda setter's exogenously given beliefs about the probabilities of various outcomes. Martin and Vanberg [2014] empirically test several models of legislative compromise in coalition govern-

ments, and conclude that these tend to be positions that average opinions in coalitions rather than representing, say, the view of the median coalition member.

Empirical evidence for interdependent preferences and their role in voting has been documented in the literature. For example, Ezrow et al. [2011] conducted an analysis of political parties in 15 Western European democracies from 1973 to 2003 and showed that the larger, mainstream parties tend to adjust their positions on the Left-Right spectrum in response to shifts in the position of the mean voter, while being less sensitive to policy shifts of their own supporters. The opposite pattern holds for smaller, niche parties. Chappel et al. [2004] studied the Federal Open Market Committee's detailed voting patterns on monetary policy, and test the hypothesis that the chairman's preferred policy is a weighted average of her own and the other members' signals – the same functional form as the one adopted here.[4,5]

Finally, our model and results are pertinent to voting in other committee settings. For example, Posner and Vermeulen [2016] note that a more or less evenly split decision by several judges, or by a jury, may be logically incompatible with a conviction based on guilt "beyond reasonable doubt". They propose a dynamic voting procedure where members learn about the positions of others and adjust their opinion, and also remark that a formal procedure where the revealed numbers of supporters for each option speak for themselves is better than an informal deliberation that cannot be quantified.

The present paper is organized as follows: In Section 2 we describe the model, the calculation of the Condorcet winner with interdependent preferences, and the main considered voting procedures. In Section 3 we analyze voting in the continental successive procedure, and compare the respective outcomes under various agendas and information disclosure policies. Section 4 considers the Anglo-Saxon amendment procedure, and compares its outcome to the one under the successive procedure. Section 5 studies the optimal location of the compromise alternative. In Section 6 we describe and analyze the historical background and voting process that determined the flag of the Weimar republic. Section 7 concludes.

In Appendix A we briefly describe two basic probabilistic tools employed in several arguments that involve a large number of voters. In Appendix B we gather several results that are

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[4]There are twelve members, and the chairman's weight on his own signal is estimated to be between 0.15 and 0.20. Chappel et al. take their cue from an earlier study by Yohe [1966] who writes "...there is also no evidence to refute the view that the chairman adroitly detects the consensus of the committee, with which he persistently, in the interests of Systems harmony, aligns himself."

[5]They also estimate the opposite influence of the chairman on members.

discussed, but not formally stated in the main text. Finally, Appendix C collects all proofs of results stated in the main text.

2 A Model of Compromise

2.1 The Basic Features

There are two parties, *Left* and *Right*, and $2n+1$ voters. The Left party has $n_L \geq 1$ members and the Right party has $n_R = (2n+1) - n_L$ members. There are three alternatives, L (left), C (compromise) and R (right). Let x_a denote the "location" of alternative a, $a \in \{L, C, R\}$, on a left-right ideological spectrum. The locations of alternatives L and R are exogenously given and normalized to be $x_L = -1$ and $x_R = +1$, while the location of the compromise, $x_C \in [-1, 1]$ may be chosen endogenously in order to maximize some goal.

Before voting, each agent i, $i = 1, ..., 2n+1$ obtains a signal $x_i \in \{-1, 1\}$. The signals of members of party $j \in \{L, R\}$ are independently drawn from a common distribution $\left(p_{-1}^j, p_1^j \right)$, $j \in \{L, R\}$ where p_{-1}^j, p_1^j denote the ex ante probability for a member of party j to draw signal -1 or signal $+1$, respectively. An interpretation is that party Right (Left) is traditionally associated with signal $+1(-1)$, but that some fraction within each party holds the respective opposite opinion.[6]

We denote by $\widetilde{n}_{-1}$ the random variable representing the number of voters with signals -1 and by n_{-1} its realization. Hence, the realized number of voters with signals $+1$ is $n_{+1} = 2n + 1 - n_{-1}$. For some arguments that involve a large number of voters, we define α by[7]

$$\alpha \equiv \lim_{n \to \infty} \frac{\mathbb{E}\left[\widetilde{n}_{-1}\right]}{n+1}. \tag{1}$$

Hence, voters with signal $+1$ hold an ex ante majority if and only if $\alpha < 1$.

Each voter, $i = 1, ..., 2n + 1$, has an "ideal" location y_i for the elected alternative. Voter i's ideal point depends on her own private signal x_i, and also on the mean of all other voters' private signals x_j, $j \neq i$. Let $\gamma_{-1}, \gamma_1 \in \left[\frac{1}{2n+1}, 1 \right]$ denote the weight that voters with signal -1 and $+1$ assign to their own signal, respectively. The ideal location $y_i (x_i, x_{-i})$ for voter i

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[6] In countries where many parties are represented in the parliament, the two groups, Right and Left, can be also interpreted as the governing coalition and the opposition, both possibly composed of fractions with diverging opinions. This is the case in our case study below.

[7] We assume that the limit exists which follows by assuming that, as n grows, the ratio $n_L/(n+1)$ (and hence also $n_R/(n+1)$) converges to a limit.

is

$$y_i\left(x_i, x_{-i}\right) = \gamma_i x_i + \frac{1 - \gamma_i}{2n} \sum_{j \neq i} x_j, \tag{2}$$

where $\gamma_i = \gamma_{-1}$ if $x_i = -1$ and $\gamma_i = \gamma_1$ if $x_i = +1$.

Thus, preferences are assumed here to be *interdependent*, and the weight γ_i on own signal x_i, captures the level of interdependence: $\gamma_1 = \gamma_{-1} = 1$ yields the private values case (no interdependence), while $\gamma_1 = \gamma_{-1} = \frac{1}{2n+1}$ yields the pure common values case where, ex post, all voters share the same ideal point. For simplicity, and in order to avoid a more tedious case distinction in the parameter space, we assumed above that the degree of interdependence in the preferences is determined by the obtained signal.[8]

If alternative $a \in \{L, C, R\}$ is elected, the utility of voter i with ideal point y_i is given by $u\left(x_a, y_i\right)$ where $u(\cdot, y_i)$ is single-peaked at, and symmetric around $x_a = y_i$. In particular, any utility function $u\left(x_a, y_i\right)$ that is monotonically decreasing in the absolute value of the difference between y_i and x_a is feasible. We use the cardinal representation for welfare comparisons, while all other results, such as equilibrium constructions, are solely based on ordinal information.

For some results we assume that the number of voters is large, and we use several basic probabilistic tools such as Hoeffding's Inequality and the Gärtner-Ellis Large Deviations Theorem (see Appendix A). Roughly speaking, these tools allow us to approximate the probabilities with which both typical and atypical realizations of random variables deviate from some central values (such as the mean). The used approximations are very precise for legislatures of large democracies that often have more than 500 members.[9]

2.2 The Condorcet Winner: Interdependence and Compromise

An alternative is the *complete information Condorcet winner* if it is the Condorcet winner when all voters' types are public information. For any given realization of signals, the assumed preferences are here single-peaked according to the left-right natural order L, C, R (or R, C, L). Therefore, the full information Condorcet winner always exists. Its identity depends

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[8] Other possible assumptions could be that it is determined instead by party membership, stochastically, etc.

[9] To get an idea about the involved numbers, consider $n = 250$ which yields 501 voters. Assume that $p_{-1}^L = p_{-1}^R = 0.45$ which gives an expected value for the number of -1 signals of 225, a minority. The probability of nevertheless having a majority – at least 251 voters – with this signal is then less than 1% !

on the realized numbers of the various signals, on the parameters governing the interdependence of preferences and, last but not least, on the location of the compromise alternative. An important question is whether various voting procedures, conducted under conditions of incomplete information, are able to dynamically discover and to elect the Condorcet winner.

To compute the Condorcet winner, note first that the ideal point of a voter i with signal $+1$ can be written as

$$y_i(+1, x_{-i}) = \gamma_1 + \frac{1 - \gamma_1}{2n}(-n_{-1} + (2n - n_{-1})) = 1 - (1 - \gamma_1)\frac{n_{-1}}{n}$$

In the private values case where $\gamma_1 = 1$, such a voter has a peak on alternative R. If $\gamma_1 < 1$, the peak monotonically shifts to the left as the number of voters with the opposite signal increases. Let

$$k = \frac{n}{2}\frac{1 - x_C}{1 - \gamma_1} \tag{3}$$

and observe that, if k is an integer and if $n_{-1} = k$, then voters with signal $+1$ are indifferent between alternatives C and R, because their peak is then given by

$$1 - (1 - \gamma_1)\frac{n}{2}\frac{1 - x_C}{1 - \gamma_1}\frac{1}{n} = \frac{1}{2}(1 + x_C),$$

exactly half-way between 1 and x_C. Thus, if $n_{-1} \leq n$, the voters with signal $+1$ form a majority, and the Condorcet winner is given by

$$CW = \begin{cases} R & \text{if} \quad n_{-1} \leq \lceil k \rceil - 1, \\ C & \text{if} \quad n_{-1} > \lceil k \rceil - 1 \end{cases} \tag{4}$$

where $\lceil z \rceil$ denotes the smallest integer no less than a real number z. In this case, C can be the Condorcet winner only if $k = \frac{n}{2}\frac{1-x_C}{1-\gamma_1} \leq n$ which is equivalent to $\gamma_1 \leq \frac{1}{2}(1 + x_C)$. If the majority group n_{+1} is relatively selfish, the Condorcet winner is always R.

Similarly, the ideal point of a voter i with signal -1 can be written as

$$y_i(-1, x_{-i}) = -\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}(n_{+1} - (2n - n_{+1})) = -1 + (1 - \gamma_{-1})\frac{n_{+1}}{n}$$

In the private values case where $\gamma_{-1} = 1$, such a voter has a peak on alternative L. If $\gamma_{-1} < 1$, then the peak monotonically shifts to the right as the number of voters with the opposite signal increases. Let

$$\kappa = \frac{n}{2}\frac{1 + x_C}{1 - \gamma_{-1}} \tag{5}$$

and observe that, if κ is an integer and if $n_{+1} = \kappa$, then voters with signal -1 are indifferent between L and C, because their peak is given by

$$-1 + (1 - \gamma_{-1})\frac{n_{+1}}{n} = -1 + (1 - \gamma_{-1})\frac{n}{2}\frac{1 + x_C}{1 - \gamma_{-1}}\frac{1}{n} = \frac{1}{2}(-1 + x_C),$$

exactly half way between -1 and x_C. If $n_{-1} \geq n + 1$, then voters with signal -1 form a majority, and the Condorcet winner is then given by

$$CW = \begin{cases} L & \text{if } n_{+1} \leq \lceil\kappa\rceil - 1, \\ C & \text{if } n_{+1} > \lceil\kappa\rceil - 1 \end{cases} \tag{6}$$

In this case, C can be the Condorcet winner only if $\kappa = \frac{n}{2}\frac{1 + x_C}{1 - \gamma_{-1}} \leq n$ which is equvalent to $\gamma_{-1} \leq \frac{1}{2}(1 - x_C)$. If the majority group n_{-1} is relatively selfish, then the Condorcet winner is always L.

To conclude, if the members of the signal majority are not too selfish (relative to the compromise location x_C), they will prefer the compromise alternative if and only if the number of voters with the opposite signal exceeds a certain threshold. As we shall see below, the cutoffs κ and k defined above play an important role also in the construction of strategic equilibria.

2.3 The Voting Procedures

We shall study and compare equilibria of two main voting procedures. A strategy profile is an *ex post equilibrium* if, given that all other agents follow their equilibrium strategies, each voter plays a best-response for all signal realizations.

1. Successive voting (SV). In this procedure alternatives are ordered according to an agenda, say $[L, \{C, R\}]$. With this agenda, voters vote in the first step by simple majority to accept, or to reject alternative L. If L is accepted, voting ends. Otherwise, voters vote by simple majority to decide whether to accept alternative C. Alternative C is accepted if it has majority support and R is accepted otherwise. The general results of GMS and Kleiner and Moldovanu (2017) for **private values** and single-peaked preferences imply that this procedure yields the Condorcet winner in an ex post and sincere voting equilibrium if the agenda is either $[L, \{C, R\}]$ or $[R, \{C, L\}]$. We focus here on these "convex" agendas, where the alternative put to vote in the first stage is one of the two extreme alternatives. Sincere voting need not be an equilibrium for the

agenda that starts by voting on the compromise C, and this agenda may not elect the Condorcet winner even under private values.

2. Voting by amendment (AV). In this procedure alternatives are ordered according to an agenda, say $[\{L, R\}, C]$. At the first step, voters vote by simple majority between alternatives L and R. If $L(R)$ is chosen, then at the second stage voters decide by simple majority between C and $L(R)$. The general results of Kleiner and Moldovanu (2017) for **private values** imply that this procedure also yields a Condorcet winner in a sincere equilibrium, while this is not necessarily the case for an agenda where the middle alternative C is one of the alternatives considered at the first step.

As we show below, the introduction of interdependent values yields quite different insights. First, different information disclosure policies along the voting sequence (while keeping a fixed agenda) may lead to different outcomes. Moreover, for a fixed information policy and a fixed voting procedure, different agendas that are otherwise equivalent in the private values case may lead to different outcomes. Finally, for a fixed information policy, different equilibrium outcomes may arise under different voting procedures.

3   Successive Voting: One-Directional Vote Shifting

We start with the successive voting (SV) procedure, and focus first on the policy that reveals the margin of victory at the first stage. The derived strategies remain an equilibrium even if individual voting behavior is reported, as long as we focus on type-symmetric equilibria where all voters with the same signal behave in the same way. Later, we provide a comparison with the minimal information disclosure environment where voters only get to know whether the first stage alternative was elected or not.

3.1   Equilibrium with Vote Shifting

We first introduce an important phenomenon, *vote shifting*, as a response to information disclosure and interdependent values: at the second stage, some voters may want to condition their behavior on the voting outcome of the first stage since this past result conveys valuable information about the signals of other agents (that directly affect their own preferences here).

Consider agenda $[L, \{C, R\}]$ and the following strategy profile: Voters with signal -1 vote in favor of L in the first stage and in favor of C at the second stage; Voters with signal

+1 vote against L in the first stage, and in the second stage vote for C if L received at least $\lceil k \rceil$ votes in the first stage, and vote against C otherwise. We denote this profile by $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$, where the first component denotes the strategy of voters with signal -1 at stage 1 (L_1) and at stage 2 (C_2), and the second component analogously denotes the strategy of voters with signal $+1$ at the two stages. The symbol $\neg$ denotes voting against the respective alternative.

Remarkably, the same cutoff

$$k = \frac{n}{2} \frac{1 - x_C}{1 - \gamma_1}$$

that appeared in the non-strategic determination of a Condorcet winner plays a role in the strategic analysis below: it is chosen such that, when there are k voters with signal -1, voters with signal $+1$ are indifferent between C and R. Intuitively, k is increasing in γ_1 and decreasing in x_C. That is, vote-shifting will be more likely when voters with signal $+1$ care more about other voters' private information (lower γ_1), or when the compromise is located closer to R.

Recall that, in the private values setting, the Condorcet winner may not be chosen under the successive voting procedure with a non-convex agenda say $[C, \{L, R\}]$, but is always chosen under a convex agenda such as $[L, \{C, R\}]$. In contrast, with interdependent values, the successive voting procedure with convex agenda $[L, \{C, R\}]$ critically relies on vote shifting to dynamically discover the Condorcet winner, and this discovery process need not be always successful.

Proposition 1 *Consider SV with agenda $[L, \{C, R\}]$.*

(i) If $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$, then the strategy profile $(L_1C_2, \neg L_1C_2 \text{ if } \geq k)$ constitutes an equilibrium that always selects the complete information Condorcet winner. If, in addition, $\gamma_1 \leq \frac{1}{2}(1 + x_C)$, actual vote-shifting may occur in equilibrium.

(ii) If $\gamma_{-1} < \frac{1}{2}(1 - x_C)$, then there is no type-symmetric equilibrium that always results in the selection of the full information Condorcet winner.

The main reason for the failure to select the Condorcet winner is the possibility that voting stops after the first vote on L. Under private values, this would imply that a majority indeed prefers this alternative. In contrast, here the relatively unselfish voters with signal -1 may actually prefer the compromise if they knew that there are sufficiently many voters with signal $+1$. But, this information has no chance to get revealed, and the Condorcet

winner cannot be discovered and elected. As we shall see below this defect cannot occur in the amendment procedure that reveals information about votes received by **both** L and R at the first stage.

The results for agenda $[R, \{C, L\}]$ are analogous: consider the strategy profile

$$(\neg R_1 C_2 \text{ if} \geq \kappa, \ R_1 C_2)$$

where, if alternative R receives at least $\lceil \kappa \rceil$ votes in the first stage, voters with signal -1 shift and vote for C in the second stage. The cutoff κ in a vote-shifting equilibrium is

$$\kappa \equiv \frac{n}{2} \frac{1 + x_C}{1 - \gamma_{-1}},$$

the same cutoff used to determine the complete information Condorcet winner. In order to have effective vote shifting in equilibrium, we need $\kappa \leq n$ which is equivalent to $\gamma_{-1} \leq \frac{1}{2}(1 - x_C)$.

3.2 Sincere Equilibria

We now proceed to compare the two convex agendas, $[L, \{C, R\}]$ and $[R, \{C, L\}]$. For this, we need an intuitive and consistent method of selecting equilibria for each possible parameter constellation: the multiplicity of voting equilibria is a standard problem in voting games, and pivotality considerations alone are not sufficient for equilibrium selection. We base our selection criterion, and hence our comparison, on the following concept:

Definition 1 *A voter's strategy in SV is sincere if, at each stage in the process, and conditional on all available information, the voter approves the current alternative if it yields the highest expected payoff among the remaining ones, and rejects it otherwise.*

If the equilibrium strategy is not sincere, legislators may have difficulties explaining their behavior to constituents. This feature often constrains opportunistic equilibrium behavior and is the subject of a large literature in Political Science (see Fenno [1978]). While in the private values case the strategy profile where each agent votes sincerely constitutes an ex post equilibrium for any convex voting procedure (and, in particular, is always consistent with pivotality considerations!), here the situation is more delicate.

In order to avoid too many case distinctions, we assume below that the number of voters is sufficiently large, so that, conditional on their own signal, both voters with signal -1 and

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voters with signal $+1$ believe that the total expected number of voters with signal -1 is well approximated by $\mathbb{E}\left[\widetilde{n}_{-1}\right] = n_L p^L_{-1} + n_R p^R_{-1}$. Otherwise, one needs to consider the conditional expected values $\mathbb{E}\left[\widetilde{n}_{-1}|x_i = -1\right]$ and $\mathbb{E}\left[\widetilde{n}_{-1}|x_i = +1\right]$ that may deviate from $\mathbb{E}\left[\widetilde{n}_{-1}\right]$ by at most one.

In this subsection (and some of the later subsections), we also assume the following:

Assumption A Voters with signal -1 have an ex ante minority (i.e., $\mathbb{E}\left[\widetilde{n}_{-1}\right] \leq n$), and they ex ante weakly prefer L to R.

Assumption A has two parts. The first part, $\mathbb{E}\left[\widetilde{n}_{-1}\right] \leq n$, assumes that voters with signal $+1$ have an ex ante majority. This is without loss of generality. The second part assumes that, ex ante, there is indeed a conflict of interest between the two types of voters. Formally, it requires that

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{n}\left(n + 1 - \mathbb{E}\left[\widetilde{n}_{-1}\right]\right) \leq 0 \Leftrightarrow \mathbb{E}\left[\widetilde{n}_{-1}\right] \geq n(1 - \frac{\gamma_{-1}}{1 - \gamma_{-1}}) + 1.$$

Let us define cutoffs γ^*_1 and γ^*_{-1} as follows:

$$\gamma^*_1 \equiv \frac{1}{2}\left(1 + x_C\right) - \frac{n - \mathbb{E}\left[\widetilde{n}_{-1}\right]}{\mathbb{E}\left[\widetilde{n}_{-1}\right]}\frac{1}{2}\left(1 - x_C\right),$$

$$\gamma^*_{-1} \equiv \frac{1}{2}\left(1 - x_C\right) + \frac{n - \mathbb{E}\left[\widetilde{n}_{-1}\right] + 1}{2n - \mathbb{E}\left[\widetilde{n}_{-1}\right] + 1}\frac{1}{2}\left(1 + x_C\right).$$

Then, voters with signal -1 ex ante prefer L to C if and only if $\gamma_{-1} \geq \gamma^*_{-1}$ and voters with signal $+1$ ex ante prefer R to C if and only if $\gamma_1 \geq \gamma^*_1$. We combine sincerity and pivotality considerations (the latter is of course necessary for equilibrium) to obtain:

Proposition 2 *Consider SV with agenda $[L, \{C, R\}]$ and suppose that Assumption A holds.*

(i) The profile $(\neg L_1 C_2, \neg L_1 C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma^*_{-1}$ and $\gamma_1 \leq \gamma^*_1$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma^*_{-1}$ and $\gamma_1 \neq \gamma^*_1$.

(ii) The profile $(\neg L_1 C_2, \neg L_1 \neg C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma^*_{-1}$ and $\gamma_1 \geq \frac{1}{2}\left(1 + x_C\right)$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma^*_{-1}$.

(iii) The profile $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ is a sincere equilibrium if $\gamma_{-1} \geq \gamma^*_{-1}$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma^*_{-1}$, and such that k is not an integer.

(iv) There is no sincere equilibrium if $\gamma_{-1} \in [\frac{1}{2n+1}, \gamma^*_{-1})$ and $\gamma_1 \in (\gamma^*_1, \frac{1}{2}\left(1 + x_C\right))$.

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In order to deal with the non-existence of a sincere equilibrium in above case (iv), we relax our selection criterion: we call a strategy profile **semi-sincere** if one type of voters votes sincerely, but not both. For the case where $\gamma_{-1} \in [\frac{1}{2n+1}, \gamma_{-1}^*)$ and $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1 + x_C))$, the profile $(\neg L_1 C_2, \neg L_1 C_2)$ forms a semi-sincere equilibrium: no voter is ever pivotal, and voters with signal -1 use a sincere strategy.

The table below summarizes sincere/semi-sincere equilibria for agenda $[L, \{C, R\}]$, where the strategy profile in quotation marks is semi-sincere, while all other strategy profiles are fully sincere.[10]

	$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1+x_C))$	$\gamma_1 \in (\frac{1}{2}(1+x_C), 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \gamma_{-1}^*)$	$(\neg L_1 C_2, \neg L_1 C_2)$	"$(\neg L_1 C_2, \neg L_1 C_2)$"	$(\neg L_1 C_2, \neg L_1 \neg C_2)$
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$	$(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$	$(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$

Table 1: Sincere/Semi-sincere equilibria for agenda $[L, \{C, R\}]$

For the alternative convex agenda $[R, \{C, L\}]$, we can follow an analogous procedure to derive sincere/semi-sincere equilibria.[11] The following table summarizes sincere/semi-sincere equilibria for agenda $[R, \{C, L\}]$:

	$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1+x_C))$	$\gamma_1 \in (\frac{1}{2}(1+x_C), 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \gamma_{-1}^*)$	$(\neg R_1 C_2, \neg R_1 C_2)$	"$(\neg R_1 C_2, \neg R_1 C_2)$"	$(\neg R_1 C_2 \text{ if } \geq \kappa, R_1 C_2)$
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$(\neg R_1 \neg C_2, \neg R_1 C_2)$	"$(\neg R_1 \neg C_2, \neg R_1 C_2)$"	$(\neg R_1 C_2 \text{ if } \geq \kappa, R_1 C_2)$

Table 2: Sincere/Semi-sincere equilibria for agenda $[R, \{C, L\}]$

Again, the profiles within quotation marks are semi-sincere equilibria for the given the parameter values: $(\neg R_1 C_2, \neg R_1 C_2)$ is an equilibrium because no voter is ever pivotal and voters with signal -1 play a sincere strategy. Profile $(\neg R_1 \neg C_2, \neg R_1 C_2)$ is an equilibrium because no one is pivotal in the first stage, and conditional on n other voters with signal -1, a voter with signal -1 prefers L to C and a voter with signal $+1$ prefers C to R. Voters with signal -1 play a sincere strategy.

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[10]We take (half-)open intervals to exclude the cutoff points $\gamma_1^*, \gamma_{-1}^*, (1 + x_C)/2$ and $(1 - x_C)/2$ so that the respective sincere/semi-sincere equilibrium is unique.

[11]See Proposition 9 in Appendix B for a formal characterization.

3.3 Agendas and Their Likelihood of Selecting the Condorcet Winner

Since Proposition 1 shows that we cannot always guarantee the selection of the complete information Condorcet winner, we compare now the two convex agendas $[L, \{C, R\}]$ and $[R, \{C, L\}]$ according to their respective likelihood of selecting the Condorcet winner (while keeping the information policy fixed).

Recall that effective vote shifting occurs in equilibrium under $[L, \{C, R\}]$ only if $\gamma_1 \leq \frac{1}{2}(1 + x_C)$ and occurs under $[R, \{C, L\}]$ only if $\gamma_{-1} \leq \frac{1}{2}(1 - x_C)$. The possible outcomes under the two agendas are:

		$\gamma_1 \in [\frac{1}{2n+1}, \gamma_1^*)$	$\gamma_1 \in (\gamma_1^*, \frac{1+x_C}{2})$	$\gamma_1 \in (\frac{1+x_C}{2}, 1]$
$\gamma_{-1} \in [\frac{1}{2n+1}, \frac{1-x_C}{2})$	$n_{-1} \geq n+1$	(C, C)	(C, C)	$\left(C, \left\{\begin{smallmatrix} C \text{ if } n_{+1} \geq \lceil \kappa \rceil \\ L \text{ if } n_{+1} < \lceil \kappa \rceil \end{smallmatrix}\right\}\right)$
	$n_{-1} \leq n$	(C, C)	(C, C)	(R, R)
$\gamma_{-1} \in \left(\frac{1-x_C}{2}, \gamma_{-1}^*\right)$	$n_{-1} \geq n+1$	(C, C)	(C, C)	(C, L)
	$n_{-1} \leq n$	(C, C)	(C, C)	(R, R)
$\gamma_{-1} \in (\gamma_{-1}^*, 1]$	$n_{-1} \geq n+1$	(L, L)	(L, L)	(L, L)
	$n_{-1} \leq n$	$\left(\left\{\begin{smallmatrix} C \text{ if } n_{-1} \geq \lceil k \rceil \\ R \text{ if } n_{-1} < \lceil k \rceil \end{smallmatrix}\right\}, C\right)$	$\left(\left\{\begin{smallmatrix} C \text{ if } n_{-1} \geq \lceil k \rceil \\ R \text{ if } n_{-1} < \lceil k \rceil \end{smallmatrix}\right\}, C\right)$	(R, R)

Table 3: Equilibrium outcomes under the two agendas

The first and second component in each cell denote the potential outcomes under agendas $[L, \{C, R\}]$ and $[R, \{C, L\}]$, respectively. The slight asymmetry between the agendas is due to the assumption that $\mathbb{E}[\widetilde{n}_{-1}] \leq n$: while this is without loss of generality per-se, it obviously implies that $\mathbb{E}[\widetilde{n}_{+1}] > n$.

Intuitively, when both types of voters care a lot about other voters' signals (i.e., $\gamma_{-1} < \gamma_{-1}^*$ and $\gamma_1 < \frac{1}{2}(1 + x_C)$), they all vote in the sincere/semi-sincere equilibrium against the first extreme alternative on the ballot, and vote for the compromise afterwards. When both types of voters are sufficiently selfish (i.e., $\gamma_{-1} > \gamma_{-1}^*$ and $\gamma_1 > \frac{1}{2}(1 + x_C)$), no vote shifting occurs in equilibrium, and the same extreme alternative (either L or R) is chosen under both agendas.

The equilibrium outcomes of the two agendas may differ only when the voter type that has the ex post majority is not too selfish ($n_{-1} \geq n+1$ and $\gamma_{-1} \leq \gamma_{-1}^*$, or $n_{-1} \leq n$ and $\gamma_1 < \frac{1}{2}(1 + x_C)$). In those cases, the agenda where the extreme alternative with ex post less support is put to vote first induces the ex post majority voters to condition their second stage votes on the first stage outcome, and thus always selects the Condorcet winner. It

dominates the agenda where the alternative with ex post more support is put to vote first and is elected before any information is revealed. As the number of voters grows, ex ante and ex post majorities coincide with probability approaching unity, and we obtain:

Proposition 3 *Suppose that Assumption A holds.*

(i) If $\gamma_1 > \frac{1+x_C}{2}$ and $\gamma_{-1} < \gamma^*_{-1}$, then $[R, \{C, L\}]$ selects the Condorcet winner with a higher probability than $[L, \{C, R\}]$, but this probability decays exponentially to zero as n grows.

(ii) If $\gamma_1 < \frac{1+x_C}{2}$ and $\gamma_{-1} > \gamma^*_{-1}$, then $[L, \{C, R\}]$ selects the Condorcet winner with a higher probability than $[R, \{C, L\}]$. Moreover, this probability remains significantly different from zero as n grows.

(iii) If $\gamma_1 > \frac{1+x_C}{2}$ and $\gamma_{-1} > \gamma^*_{-1}$, or if $\gamma_1 < \frac{1+x_C}{2}$ and $\gamma_{-1} < \gamma^*_{-1}$, the two agendas are outcome equivalent.

Since, by Assumption A, $\mathbb{E}[\widetilde{n}_{-1}] \leq n$, the "extreme" alternative R has more ex ante support than the other "extreme" alternative L. The condition $\gamma_{-1} > \gamma^*_{-1}$ ensures that, under agenda $[L, \{C, R\}]$, voters with signal -1 will vote for L in the first stage so that the first stage voting outcome is informative: this allows voters with signal $+1$ to condition on it when they choose between C and R at the second stage (if reached). Interestingly, $[L, \{C, R\}]$ dominates $[R, \{C, L\}]$ not because voters with signal $+1$ shift their votes to C in the second stage when there are sufficiently strong support for L in the first stage (in this case, the outcomes are identical under the two agendas), but because these voters choose **not** to shift to C when there is not enough support for L in the first stage.

3.4 The Minimal Information Policy: Equilibria without Vote Shifting

In this subsection we briefly analyze SV where no information is revealed, i.e., if the second stage is reached the voters know only that the first stage alternative did not get the support of a majority.[12] We then compare its outcome to the one obtained under the policy that reveals the margin of defeat.

Proposition 4 *Consider SV with agenda $[L, \{C, R\}]$. Suppose that $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$ and that at the second stage (if reached) voters only know that alternative L was rejected. The*

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[12] Here we do not impose the sincerity restriction.

profile $(L_1C_2, \neg L_1 \neg C_2)$ is an equilibrium if $\gamma_1 > \frac{1}{2}(1 + x_C)$, and the profile $(L_1C_2, \neg L_1C_2)$ is an equilibrium if $\gamma_1 \leq \frac{1}{2}(1 + x_C)$.

Proposition 4 allows us to compare the equilibrium outcomes under the two information policies from a utilitarian welfare perspective. We focus on the case of $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$ (so that not all voters will vote against L) and compare strategy profiles $(L_1C_2, \neg L_1 \neg C_2)$ and $(L_1C_2, \neg L_1C_2$ if $\geq k)$. The former profile is the outcome in SV with the minimal information policy, while the latter is the outcome in SV with disclosure of the defeat margin at the first stage. The policy of revealing the margin of defeat at the first stage is either outcome equivalent to the minimal information policy (for any number of agents), or unambiguously better if the number of agents is sufficiently large. (See Proposition 8 in Appendix B for a formal statement.)

4   The Amendment Procedure: Bidirectional Vote Shifting

We now proceed to AV, where voters choose between L and R at the first stage, and where, at the second stage, they choose between C and the winner of the first stage. This is the only possible convex agenda for AV with three alternatives. We first assume that the information regarding the margin of victory at the first round is disclosed before the second-stage vote. Recall that a voter with signal $+1$ prefers C to R if at least $\lceil k \rceil$ voters have signal -1, and that a voter with signal -1 prefers C to L if at least $\lceil \kappa \rceil$ voters have signal $+1$. Consider the following strategy profile denoted by Π:

1. Voters with signal -1 vote for L in the first stage. In the second stage they vote for C in a vote R vs. C; in a vote L vs. C they vote for C if and only if at least $\lceil \kappa \rceil$ voters voted for R in the first stage;

2. Voters with signal $+1$ vote for R in the first stage. In the second stage they vote for C in a vote L vs. C; in a vote R vs. C they vote for C if and only if at least $\lceil k \rceil$ voters voted for L in the first stage.

Proposition 5 *Consider AV with a convex agenda and assume that, at the second stage, voters know the margin of victory at the first stage. The strategy profile Π described above is an equilibrium, and the complete information Condorcet winner is always elected.*[13]

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[13]Note that Π is the unique sincere equilibrium if $n(1 - \frac{\gamma_{-1}}{1 - \gamma_{-1}}) + 1 \leq \mathbb{E}[\tilde{n}_{-1}] \leq \frac{n}{1 - \gamma_1}$. Then, at the first

Now suppose that information regarding the margin of decision in the first stage is not revealed: in the second stage voters only know which of the two alternatives (L or R) won the first round. Then the following strategy profile, denoted by Π', constitutes an equilibrium:

1. Voters with signal -1 vote for L in the first stage. In the second stage they vote for C in a vote R vs. C; in a vote L vs. C they vote for C if and only if $\gamma_{-1} \leq \frac{1}{2}(1 - x_C)$;

2. Voters with signal $+1$ vote for R in the first stage. In the second stage they vote for C in a vote L vs. C; in a vote R vs. C they vote for C if and only if $\gamma_1 \leq \frac{1}{2}(1 + x_C)$.

The strategy profile Π' is similar to Π. But, without information about the election margin at the first stage, it is not possible for Π' to always select the Condorcet winner. The advantage of a dynamic voting procedure over a static one is the gradual information revelation and aggregation. With minimal information disclosure, a dynamic voting procedure cannot do much better than a static one that asks agents to report their peaks and picks an alternative depending on the reported peaks.

4.1   The Advantage of AV over SV

We can now compare successive voting and voting by amendment. In both cases we consider the disclosure policies that reveal the margin of victory in the first stage. As we showed earlier (Propositions 1 and 5), SV does not always result in the election of the complete information Condorcet winner, but, for **any** parameter values, there is an equilibrium under AV with bidirectional potential shifting that always elects the Condorcet winner .

Let us next turn to welfare comparison. In AV, the first stage voting fully reveals the voters' signals, so the second stage voting is under complete information. For SV, we focus on agenda $[L, \{C, R\}]$.[14] As we argue below, AV clearly dominates SV if the first stage voting under $[L, \{C, R\}]$ reveals n_{-1}, or the first stage voting is not revealing but R is elected. The comparison is ambiguous and depends on n_{-1} only if the first stage voting of SV is not revealing and C is elected.

(i) If alternative L is elected, then AV elects either L or C; in the latter case, AV improves welfare relative to SV because it allows voters with signal -1 to shift their votes to C when

stage, voters with signal -1 ex ante prefer L to R, and voters with signal $+1$ ex ante prefer R to L. Voting at the second stage is under complete information, and hence also sincere.

[14] The comparison for the other convex agenda $[R, \{C, L\}]$ is analogous.

they prefer C to L (voters with signal $+1$ are also better off since they prefer C to L).

(ii) If alternative L is rejected by a split vote, then the second stage equilibrium play is also under complete information in SV. In this case, vote shifting is always welfare improving under both procedures. Let us then compare the respective incidences of effective vote shifting:

SV : -1 voters shift if $\#L \in [\lceil k \rceil, n]$

AV : $+1$ voters shift if $\#L \in [\lceil k \rceil, n]$ and -1 voters shift if $\#R \in [\lceil \kappa \rceil, n]$

Therefore, AV is welfare superior because it allows both types of voters to shift their votes at the second stage as a response to the information revealed at the first stage.

(iii) If alternative L is unanimously rejected, and if R is elected under $[L, \{C, R\}]$, then AV improves welfare because it allows voters with signal $+1$ to shift to the compromise alternative, and because vote shifting benefits both types of voters when it occurs.

(iv) If alternative L is unanimously rejected and if C is elected under $[L, \{C, R\}]$, the comparison is ambiguous, and it depends on the realization of n_{-1}. If $n_{-1} \in [\lceil k \rceil, 2n+1-\lceil \kappa \rceil]$ so that C is the Condorcet winner, then the outcomes of SV and AV coincide. When C is not the Condorcet winner, SV dominates AV if n_{-1} is close to either $\lceil k \rceil$ or $2n+1-\lceil \kappa \rceil$, and the reverse is true otherwise.[15]

The above findings – strong rationales in favor of AV – starkly contrast the results under complete information or under private values, where the two procedures are always equivalent under a convex agenda and single-peaked preferences.

5 The Location of the Optimal Compromise

So far we have assumed that x_C, the location of the compromise alternative C on the ideological scale, is exogenous. We now assume that the larger party (that, for example, forms the Government) controls the agenda, and that it can determine, prior to voting, the location of C. The decision where to place the compromise is not trivial because this party has

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[15] To see this, suppose that $n_{-1} \geq n+1$ and $n_{-1} > 2n+2-\lceil \kappa \rceil$. Then L is the Condorcet winner, and it is always elected under AV. On the one hand, SV is welfare superior to AV if n_{-1} is close to $2n+2-\lceil \kappa \rceil$, because voters with signal -1 are almost indifferent between L and C, but voters with signal $+1$ strictly prefer C to L. On the other hand, AV is welfare superior to SV if n_{-1} is close to $2n+1$, because then almost all voters strictly prefer L to C. As a result, there must exist a cutoff $n^*_{-1} \in (2n+2-\lceil \kappa \rceil, 2n+1)$ such that SV dominates AV if $n_{-1} \in (2n+2-\lceil \kappa \rceil, n^*_{-1})$ and the reverse is true if $n_{-1} \in (n^*_{-1}, 2n+1]$. The case where $n_{-1} < \lceil k \rceil$ is analogous.

here two factions with two different signals, and hence it incorporates diverse preferences. In general, the location of the optimal compromise will depend on the underlying goal, on the used voting procedure, on the expected numbers of voters with the various signals and on the interdependence parameters. Whatever the underlying goal is, the main constraint on the optimal compromise location is that, in order to be effective, it must also get, at least sometimes, elected! In this context, recall Theresa May's frustration from her continued failure to pass a negotiated Brexit compromise through Parliament, although her Government party, the Tories, possessed, together with an allied, small North-Irish party, a theoretical majority. That failure was followed by a process of "indicative voting" and negotiations with the opposition whose purpose was to find out other possible locations of a compromise that could be eventually elected by majority.

We first prove a key Lemma that identifies the compromise locations effectively leading to its election under SV and AV, respectively. For a given n and for a given realization of signals, let us define the compromise location such that, **ex post**, voters with signal -1 are indifferent between L and C:

$$x_C^L(n, n_{+1}) = -1 + 2(1 - \gamma_{-1})\frac{n_{+1}}{n}.$$

Analogously, for a given n, voters with signal -1 are **ex ante** indifferent between L and C if the compromise is located at

$$x_C^L(n) = -1 + 2\left(1 - \gamma_{-1}\right)\frac{2n + 1 - \mathbb{E}\left[\widetilde{n}_{-1}\right]}{n}$$

Finally, let

$$\overline{x}_C^L \equiv \lim_{n\to\infty} x_C^L(n) = 2\left(1 - \gamma_{-1}\right)\left(2 - \alpha\right) - 1.$$

Voters with signal $+1$ are ex post indifferent between R and C if the compromise is located at

$$x_C^R(n, n_{-1}) = 1 - 2(1 - \gamma_1)\frac{n_{-1}}{n}.$$

and are ex ante indifferent if the compromise is located at

$$x_C^R(n) = 1 - 2(1 - \gamma_1)\frac{\mathbb{E}\left[\widetilde{n}_{-1}\right]}{n}.$$

We denote

$$\overline{x}_C^R \equiv \lim_{n\to\infty} x_C^R(n) = 1 - 2(1 - \gamma_1)\alpha.$$

Lemma 1 *(i) Consider SV with agenda $[L, \{C, R\}]$, and suppose that Assumption A holds.*

   (a) If $n_{-1} \geq n + 1$, then the compromise C is elected if and only if $x_C \leq x_C^L(n)$. Otherwise, alternative L gets elected.

   (b) If $n_{-1} \leq n$, then the compromise C is elected if and only if $x_C \in \left[-1 + 2\gamma_1 \;,\; x_C^L(n)\right] \cup \left[x_C^R(n, n_{-1}), 1\right]$. Otherwise, alternative R gets elected.

(ii) Consider AV.

   (a) If $n_{-1} \geq n + 1$, then the compromise C is elected if and only if $x_C \leq x_C^L(n, n_{+1})$. Otherwise, alternative L gets elected.

   (b) If $n_{-1} \leq n$, then the compromise C is elected if and only if $x_C \geq x_C^R(n, n_{-1})$. Otherwise, alternative R gets elected.

If the voters with signal -1 form an ex post majority, C will be elected under both voting procedures if it is so close to L (i.e., if $x_C \leq x_C^L(n)$ under SV, and if $x_C \leq x_C^L(n, n_{+1})$ under AV) that even those voters find it more attractive than L (from an ex ante perspective under SV, and from an ex post perspective under AV). Such a logic seems to have escaped Theresa May for a long time: instead of seeking a compromise that is closer to the position of most of Labor's members (who, together with moderates in the Conservative party and in several smaller parties, favored a softer Brexit) she tried instead to appease the "Brexiteers" in her own party whose opinion was in minority.[16]

If voters with signal $+1$ form an ex post majority, C will be elected under AV only if it is sufficiently close to R (i.e., if $x_C \geq x_C^R(n, n_{-1})$) so that voters with signal $+1$ ex post prefer C to R. In contrast, C will be elected under SV in more cases: it will be elected if $x_C \geq x_C^R(n, n_{-1})$, but it can also get elected even if $x_C < x_C^R(n, n_{-1})$. In particular, C is elected under SV if $-1 + 2\gamma_1 \leq x_C \leq x_C^L(n) < x_C^R(n, n_{-1})$. In this case, the equilibrium profile in Table 1 is $(\neg L_1 C_2, \neg L_1 C_2)$, and C is elected with unanimous support because no information is released after the first stage voting. The difference arises because the strategic voting at both stages of SV (and the resulting information disclosure) depends on

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[16] Of course, other factors, such as inner-party considerations of leadership may have constituted a main obstacle for her.

the compromise location, while in AV only the second stage voting behavior is affected by the compromise location.

We can now analyze the location of optimal compromise. Throughout the sequel, we assume that the majority party has a single-peaked utility function that is maximized at some alternative $x^* \in [-1, 1]$, and that it locates the compromise in order to maximize the expected utility it derives from the elected alternative[17].

Suppose first that $x^* = 1$. That is, the goal of the majority party (assumed here to be the **Right** party) is to elect its "traditional" ideological position R, or at least an alternative that is as close as possible to R.[18] Since $x^* = 1$ is already on the table and coincides with alternative R, the only reason for the majority party to propose a compromise in this case is to prevent alternative L from being elected. From the point of view of this party, the compromise would ideally be elected if voters with signal -1 have a majority (it prevents then outcome L), but not be elected if voters with signal $+1$ have a majority (so that their ideal policy R gets elected then). The location of the compromise must therefore take into account the precise conditions under which the compromise will be elected.

Proposition 6 *Suppose that the majority party R chooses the location of the compromise $x_C(n)$ in order to maximize the expected location of the elected alternative, i.e., $x^* = 1$.*

(i) *Consider SV with agenda $[L, \{C, R\}]$ and assume that Assumption A holds. The optimal compromise C is located at*

$$
x_C(n) = \begin{cases} just\ below\ -1 + 2\gamma_1\ or\ at\ x_C^L(n) & if\ x_C^L(n) \geq -1 + 2\gamma_1 \\ at\ x_C^L(n) & if\ x_C^L(n) < -1 + 2\gamma_1 \end{cases}
$$

(ii) *Under AV, the optimal compromise location satisfies*

$$
\lim_{n \to \infty} x_C(n) = \begin{cases} \min\{-1 + 2\gamma_1,\ 1 - 2\gamma_{-1}\} & if\ \alpha < 1 \\ \overline{x}_C^L & if\ \alpha > 1 \end{cases}
$$

Under SV, the optimal compromise location for a traditionalist majority party is often determined by the position $x_C = x_C^L(n)$ that makes voters with the **opposite** signal ex ante

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[17] The empirical analysis of Martin and Vanberg [2014] suggests that, at least in coalition governments, the most likely compromise is an average of the positions of the represented parties.

[18] This may be the case, for example, if its constituent base supports that position more strongly than the legislators themselves.

indifferent between the compromise and their own traditional position: this is the highest compromise that will still be elected if those voters do have a majority. Sometimes it is even better to choose a location further to the left: rather than appealing to voters with the opposite signal, such a move makes the compromise less attractive for voters with signal $+1$ and increases the chance that R will be elected when they have a majority.

Under AV, the second stage voting is under complete information, so that the realization rather than the expectation of n_{-1} determines the compromise actually gets elected. The ex ante optimal compromise location will therefore depend on the exact probability distribution. But, we can explicitly determine the optimal compromise location for large populations. In that case, if $\alpha > 1$ then voters with signal -1 have a majority with high probability, and their population share is approximately $\alpha/2 \geq 1/2$. The largest compromise they are willing to elect is then close to $\overline{x}_C^L$, which is therefore optimal. The situation is more subtle if $\alpha < 1$. The optimal location must then take into account the value provided by the compromise (conditional on being elected), the likelihood that it gets elected if voters with signal -1 have a majority (in which case it protects against the election of L), and the likelihood that it gets elected if voters with signal $+1$ have a majority (in which case it prevents R from being elected). Using tools from large deviation theory, we show that the last channel dominates for large n: the optimal compromise satisfies $x_C(n) \leq -1 + 2\gamma_1$ so that it will never be elected by a majority of voters with signal $+1$, who instead elect R. For the compromise to be elected (at least sometimes) when voters with signal -1 have a majority, it must satisfy $x_C(n) \leq 1 - 2\gamma_{-1}$. We show that, conditional on $n_{-1} \geq n + 1$, $x_C = 1 - 2\gamma_{-1}$ gets elected with probability approaching 1, and the limit optimal compromise is therefore $x_C = \min\{-1 + 2\gamma_1 , 1 - 2\gamma_{-1}\}$.

Remark 1 *For large populations we can now compare the optimal compromise in AV with the optimal compromise in SV. Assumption A becomes $\frac{1-2\gamma_{-1}}{1-\gamma_{-1}} \leq \alpha < 1$ in the limit. Given that $x^* = 1$, in both procedures the optimal compromise is chosen low enough ($x_C \leq -1+2\gamma_1$) so that it will not be elected if voters with signal $+1$ have a majority, but it is likely to be elected if voters with signal -1 have a majority. For the latter, $x_C \leq \overline{x}_C^L$ is sufficient in SV, but $x_C \leq 1 - 2\gamma_{-1}$ is required in AV. Note that $\overline{x}_C^L = 2\left(1 - \gamma_{-1}\right)\left(2 - \alpha\right) - 1 > 1 - 2\gamma_{-1}$ if $\alpha < 1$ and $\gamma_{-1} < 1$. Therefore, if $\alpha < 1$ and $\gamma_{-1} < 1$, the majority party will choose a larger compromise under SV, and it would strictly prefer SV to AV whenever $1 - 2\gamma_{-1} < -1 + 2\gamma_1$ or equivalently whenever $\gamma_{-1} + \gamma_1 > 1$.*

We next assume that $x^* \in (-1, 1)$. In contrast to the case where $x^* = 1$, the agenda setter will propose here a policy because her first-best alternative hasn't been put forward yet. For example, x^* could be the policy position that maximizes the expected utility of the members of the majority party, or the expected utility of all voters, etc.

Whether it is optimal to actually introduce x^* depends on its chances of being elected. The key observation is that the location must be chosen such that the compromise is elected with high probability. Therefore, the agenda setter should set $x_C = x^*$ if x^* belongs to the set of electable compromises (characterized in Lemma 1), and otherwise choose among the electable compromise locations the one that is closest to x^*.

Proposition 7 *Assume that* $\max\left\{\gamma_{-1}, \gamma_1\right\} < 1$ *and that the majority party has a utility function that is symmetric around its peak* $x^* \in (-1, 1)$.

(i) *Suppose that Assumption A holds in the limit, i.e.,* $\frac{1-2\gamma_{-1}}{1-\gamma_{-1}} < \alpha < 1$, *and that* $-1+2\gamma_1 \neq \overline{x}_C^L$. *Let* $X_C \equiv \left[-1 + 2\gamma_1 \, , \, \overline{x}_C^L\right] \cup \left[\overline{x}_C^R, 1\right]$ *denote the set of compromise locations that get C elected under SV with agenda* $[L, \{C, R\}]$ *and suppose that* $\min_{x \in X_C} |x - x^*|$ *has a unique solution. Then, the optimal compromise location satisfies*

$$\lim_{n \to \infty} x_C(n) = \arg\min_{x \in X_C} |x - x^*|.$$

(ii) *Under AV, the optimal compromise location satisfies*

$$\lim_{n \to \infty} x_C(n) = \begin{cases} \max\left\{x^*, \overline{x}_C^R\right\} & \text{if} \quad \alpha < 1 \\ \min\left\{x^*, \overline{x}_C^L\right\} & \text{if} \quad \alpha > 1 \end{cases}$$

6 Case Study: The Flag of the Weimar Republic

We now describe and analyze an interesting historical case study where successive voting according to a convex agenda was used, and where vote-shifting as in our theoretical model with interdependent values has most probably occurred. Moreover, the location of the synthetic compromise was endogenous.

The flag was one of the most contested issues during the Weimar Republic. Article 3, defining it, was the only article of the Constitution (out of 181!) whose outcome was determined by open roll-calls where individual votes were registered. The flag controversy reflected, in compressed form, the entire century preceding Weimar, that included liberation

from Napoleon, failed revolutionary and unification attempts, the subsequent German uni-
fication under Prussian hegemony, empire and expansion, and finally the humiliating defeat
in WWI[19].

The principal argument was between the supporters of the Black-Red-Gold (BRG) flag
and those supporting the Black-White-Red (BWR) flag. The BRG flag first emerged during
the anti-Napoleonic wars of 1813-1815. It was mainly associated with the progressive, anti-
monarchist ideas that accompanied the 1832 patriotic revival, the 1848 revolution, and the
subsequent (failed) unification attempt.[20] In contrast, BWR were the official colors of the
Reich in the period 1871-1919, and, significantly, already from 1867, the flag adorning the
ships of the commercial fleet of the North German Confederation (the precursor of the Reich),
potent symbols and objects of pride that sailed all over the world from the havens of the North
Sea.[21]

A compromise alternative was elected by the Weimar National Assembly on July 3, 1919:
BRG replaced BWR as national colors, but the fleet was allowed to retain a BWR flag,
adjusted to contain a BRG *canton*.[22] The attempt to bridge deep societal fractures via
a compromise on the flag was ultimately unsuccessful, and the consequences continued to
plague Germany, and indeed the world, for many years to come.

6.1 The Assembly's Composition

The National Assembly's party composition was determined via free elections on January 19,
1919.[23] The 421 seats were divided among various parties as follows:

Party	SPD	Z	DDP	DNVP	USPD	DVP	BBB	DHP	SHBLD	BL
Seats	163	91	75	44	22	19	4	1	1	1

Table 1: The division of seats among parties

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[19] See Winkler[1993]

[20] It was also the flag of the Austrian-South German alliance in the Austro-Prussian war that ended the
German Confederation, and that paved the way for the creation of a unified German Reich under Prussian
hegemony (separate from Austria).

[21] The BWR flag emerged as a combination of the colors found in the bi-colored flags of (Nordic) Prussia and
the (Nordic) Hansa cities, the dominant powers - militarily and commercially - of the newly unified German
Reich.

[22] A canton is a small flag within a flag, usually at the NW corner.

[23] The elections to that assembly were also the first where women could vote.

SPD, the social democrats, were left-leaning, and constituted the main party of the ruling coalition. Z(entrum) and DDP were centrist parties, also members of the government coalition (in **bold**). Note that their combined weight was about the same as that of the SPD.

DNVP and DVP were right-leaning conservative parties, both in the opposition. USPD, the independent social democrats were to the left of the SPD, and were also in opposition. The other 4 very small parties in opposition, BBB, DHP, SHBLD and BL mostly represented regional interests.

6.2   The Proposed Flags

As the ruling coalition SPD-Z-DDP controlled more than 75% of the seats in the Assembly, the government should have been able to impose its will. But, because of existing deep divisions within the coalition itself, the situation was complex. The Assembly had to consider 4 alternative proposals:

1. BRG. This was the government's proposal, considered the "main" alternative. It was submitted to the Constitutional Committee on February 21, 1919 but, given the bitter controversy within the committee, it was adjusted, at the initiative of the SPD, to include a possible later determination of a different flag for the fleet (via standard rather than constitutional legislation). The colors of that future flag were left unspecified.

2. BWR. This was supported by the two opposition right-conservative parties DNVP and DVP, and by conservative factions of the centrist parties in the ruling coalition, Z and DDP.

3. R. Red, the color of revolution and of the Socialist International, was supported by the more radical left, the USPD[24].

4. BRG/BWR. This was the compromise arrangement: BRG as national colors, together with an adjusted BWR flag with BRG canton for the fleet. The compromise was formally proposed by members of the centrist parties in the Constitutional Committee since both Z and DDP were internally split on the flag question. Recall that both these parties were part of the ruling coalition.

---

[24]This proposal was also adjusted to include a provision about a future, possibly different flag for the fleet.

It is clear then that support for the various alternatives crossed several party lines and was present within the government coalition. There was genuine uncertainty about the outcome of a vote, and the only generally anticipated event was the rejection of R, which was supported by the relatively small USPD only.

6.3 Agenda Formation

The standard decision making procedure of the Weimar National Assembly was *successive voting,* as studied above. Agenda formation was under the jurisdiction of the *Elders' Council* - a group of senior Assembly members representing all major parties.

The left-right ideological order of the 4 flags was:

$$R - BRG - BRG/BWR - BWR$$

and the Council suggested agenda

$$\mathbf{A} : 1) \ R \longrightarrow 2) \ BWR \longrightarrow 3) \ BRG/BWR \longrightarrow 4) \ BRG$$

The "voting on the farthest alternative first" logic is explicitly mentioned as the guiding principle behind agenda $\mathbf{A}$ [25]: starting from the government's proposal BRG, R was subjectively considered to be the furthest deviation, BWR less far away, while the compromise proposal BRG/BWR was, obviously, located between BRG and BWR.

A conservative member proposed, with a somewhat dubious argument, a different agenda

$$\mathbf{B} : 1) \ R \longrightarrow 2) \ BRG/BWR \longrightarrow 3) \ BWR \longrightarrow 4) \ BRG$$

Note that under $\mathbf{B}$, the compromise BRG/BWR is voted upon *before* BWR and BRG, the more extreme remaining alternatives: assuming private values, sincere voting is not an ex post equilibrium if such an agenda is used, and the Condorcet winner need not be elected even under the private values assumption. After the vote on R, this agenda basically induces a decision between the compromise, and an uncertain lottery among the remaining, more extreme alternatives.

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[25] This is the method proposed by Trendelenburg, 1850, and Tecklenburg, 1914, for cases where the proposals are on both sides of the "main" alternative, taken here (and by those writers) to be the Government's position. See Protocols [1919]

6.4 The Voting Outcome and its Analysis

Agenda **A** prevailed by a simple majority in a first, procedural vote (this was not a roll-call). The first substantial vote was thus on R. It was also not a roll-call, and thus we do not have a detailed description of how it came about.

Starting with the vote on BWR, voting was by roll-call, where each individual vote was carefully registered. BWR was also defeated: 111 members voted in favor, 190 members voted against, and 6 abstained. Finally, BRG/BWR was accepted: 211 voted in favor, 90 against and 1 abstained. According to the rules of the successive procedure, the remaining proposal BRG was not put to vote anymore.

6.4.1 The Outcome in Light of the Theory

We assume below that voters had single-peaked preferences according to the left-right ideological order

$$R - BRG - BRG/BWR - BWR$$

USPD voters had an initial a peak on R. Conservative voters from the opposition parties DVP and DNVP and also right-leaning factions of the coalition parties Z and DDP clearly had an initial peak on BWR. Left-leaning factions within Z and DDP, and the SPD had initial peaks on either BRG/BWR or BRG. While these assertions cannot be "proved", they seem the only common-sense ones given the ideology of these parties, the preceding plenary debate, and the mass of historical evidence surrounding this case.

The outcome of the vote on R was widely anticipated, and this also explains why it was not deemed necessary to conduct a roll-call in that case. It is probable that only members of USPD - who had proposed R and who presumably had their peak on it - voted in its favor, while all other parties - who had peaks to the right - voted against.

We now focus on the remaining agenda

$$\mathbf{A}' : 2)\ BWR \longrightarrow 3)\ BRG/BWR \longrightarrow 4)\ BRG$$

and analyze below the outcome of the roll-calls, first the vote on BWR, and then the vote on BRG/BWR.

This situation precisely fits the set-up analyzed in the theoretical part: we show below how vote-shifting can explain the observed outcome. Note also that the government's proposal

BRG , that presumably had the highest ex ante support is put to here to vote last, as suggested by our theory.

The following table displays the results in disaggregated form[26]:

	$Y - Y$	$Y - N$	$N - N$	$N - Y$
SPD	0	0	0	106
Z	10	1	0	49
DDP	21	19	0	14
DNVP	0	33	0	0
USPD	0	0	18	0
DVP	1	16	0	0
BBB	0	0	0	1
DHP	0	0	1	1
BVP	0	0	0	1
Total	**32**	**69**	**19**	**172**

Table 3: The outcome of the roll-calls

1. 106 out of the 107 present SPD members voted N-Y.[27] This behavior is consistent with either a peak on the compromise BRG/BWR, or with an initial peak on BRG together with a subsequent shift to BRG/BWR caused by the relatively large and vocal support for BWR[28]. Fixing the behavior of all other actors, adding 106 No votes of SPD members would have led to a clear rejection of the compromise and the likely election of BRG. The omission to do so therefore suggests that these voters shifted their vote from BRG (recall that this was the government's proposal with presumably the highest ex ante support!) to the compromise BRG/BWR. The interdependent component of their preferences was clearly expressed, already before the vote, in the Government's willingness to compromise and to adjust its initial proposal to allow for a later determination of a fleet flag via regular legislation. Hence, we conclude that,

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[26] 111 members missed both votes, most of them from the ruling coalition. For simplicity, we do not list here the few incomplete profiles where one roll-call was missed, nor the few profiles that contained abstentions. Adding all these, a total of 17 (out of which 8 voted Yes on the compromise), does not change the result or its interpretation.

[27] One member missed the first vote, and voted Y on the second.

[28] Note that, given the chosen agenda, this shift is immaterial for the behavior in the first two votes on R and BWR.

after observing more than 100 votes in favor of BWR - about one third of the total -, the members of the SPD most likely shifted their votes from an initial peak BRG to the compromise BRG/BWR.

2. All 18 present members of USPD voted N-N, consistent with sincere voting given their presumed peak on R: after the defeat of their first-best alternative R, the members of USPD rejected the next two alternatives because they were both worse from their point of view than the last alternative on the agenda, BRG (that was, eventually, not reached anymore).

3. A large majority of members of Z voted N-Y (49) , while 7 abstained/missed the first vote and voted Y at the second. This is, again, consistent with either a peak on BRG/BWR, or with a peak on BRG and a subsequent shift after the BWR vote. But, the fact that BRG/BWR was formally proposed by this party, points to the first alternative. 10 other members of Z voted Y-Y, which is consistent with sincere voting and a peak on BWR: after the defeat of their first-best alternative BWR, these members voted for BRG/BWR because it was the best remaining on the ballot from their point of view.

4. The DDP party was also split: 21 members voted Y-Y, consistent with a peak on BWR,[29] while 14 of its members voted N-Y, which, as we saw above, is consistent with a peak on BRG/BWR.

5. Practically all members of the right-wing, conservative parties DNVP and DVP, 49 out of the 50 present, voted Y-N.[30] They were joined by 20 members of the coalition parties DDP (19) and Z (1). All these voters obviously had a peak on BWR: thus, they were expected to vote Y-Y, since the compromise BRG/BWR was their best remaining alternative after the defeat of BWR. But they didn't, and their behavior at the last stage of the game -the choice of seemingly **dominated** action - cannot be explained by only looking at the voting game in isolation. Therefore, we advance here a more speculative explanation: the radical conservative voters wanted to signal (to other parliament members and to the electorate) their complete unwillingness to compromise on the flag, thus lending credibility to their announced threat of rejecting

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[29] Another 2 members of this party voted Y-Miss.

[30] The remaining member voted Y and then missed the second vote.

the entire constitution because of it[31]. This explanation is a twist on Fenno's "home-style" hypothesis.[32] Home-style -the need to justify behavior to constituents- is invoked to explain seemingly sub-optimal behavior - such as sincere voting - instead of behavior that exploits each strategic opportunity (see Fenno [1978], Denzau et al. [1985], and Austen-Smith [1992]). Here sincere voting was in fact optimal but, at the last binary vote, it delivered the wrong signal and was hence avoided. It is also very likely that the conservatives anticipated the vote shifting by the SPD, and hence believed that the compromise will be adopted anyhow, even without their own support. Thus, they reckoned that their otherwise risky signaling behavior will remain costless in this case.

Remark 2 *The voting outcome can be checked for consistency in light of the theoretical considerations. Consider SV with agenda $[R, \{C, L\}]$ and the profile $(\neg R_1 C_2 \text{ if } \geq \kappa, R_1 C_2)$ which is an equilibrium if γ_1 is relatively high, i.e., when right-leaning members of parliament are close to weighing solely their own signal. This assumption certainly fits well their total unwillingness to compromise (recall that some of them actually chose a dominated option by voting against the compromise flag!). We obtain an estimate of γ_{-1}, the weight on own signal of the left leaning members. For the shifting parameter, we obtain that $\kappa = \frac{n}{2}\frac{1+x_C}{1-\gamma_{-1}} \leq 111$ (since 111 voters voted in favor of BGR). Observing that $n \approx 159$ (since about 319 voters participated in the vote), this yields $\gamma_{-1} \leq 0.3 - 0.7 x_C$. Note that the compromise location x_C of the cantoned flag BRG/BWR is here best thought to satisfy $-1 < x_C \leq 0$: it was definitely closer in spirit to the left alternative BRG (main flag) than to right alternative BWR (canton). The weighted average compromise in the spirit of the coalitional analysis of Martin and Vanberg [2014] would be about -0.52 in this case. Setting $x_C = 0.5$ yields $\gamma_{-1} \leq 0.65$, a high degree of interdependence, explaining the willingness to compromise of the governing coalition unter SPD leadership. Finally, the cantoned flag was most probably the Condorcet winner CW since*

$$n + 1 = 160 < n_{-1} \simeq 201 < 208 \leq 2n + 1 - \lceil \kappa \rceil,$$

where we estimated n_{-1} by counting the legislators that voted agains BWR at the first stage.

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[31]The main speaker for the conservative opposition suggested that a defeat of BWR may lead to a rejection of the entire new constitution by many of his fellows, and, implicitly, by large parts of the population.

[32]See Fenno [1978], Denzau, Riker and Shepsle [1985], and Austen-Smith [1992].

6.5 Epilogue

The BRG/BWR compromise became Article 3 of the new Weimar constitution, but large parts of the bourgeois-conservative electorate never came to terms with it. After the National Assembly completed its constitutional work, a regular parliament was elected on June 6, 1920. The SPD-Z-DDP coalition suffered catastrophic losses, while the more radical parties both from the left and from the right, who were all against the flag compromise, got much stronger. The new coalition government shifted to the right, and the SPD remained outside. In 1926 Chancellor Luther agreed to a request by the arch-conservative president Hindenburg, and allowed German commercial representatives and embassies outside Europe the use of BWR on a more equal footing along BRG. The social democrats and large parts of the public saw this as a direct attack on the fragile constitution, and Luther was forced to resign.

The compromise of 1919 was overturned by the Nazi regime, only a week after taking power in 1933: the BWR national colors were restored, and, later, another official flag, personally designed by Hitler, combining the BWR colors and the Swastika was added. This discredited use of the BWR colors was the main reason behind both East and West Germany's decisions to return to the exactly same (!) BRG flag after WWII.[33]

BRG remain the flag colors of the re-united Germany, while the BWR colors are mostly associated with extreme-right parties that use it as a surrogate for illegal Nazi symbols.

7 Conclusion

Under complete information and single-peaked preferences all sequential binary procedures and all agendas are equivalent: if simple majority is used at each step, the Condorcet winner is always elected. The situation changes under incomplete information and a private values assumption: the Condorcet winner is elected by any sequential, binary procedure under any convex agenda, but this may not be true if the agenda is not convex. Assuming interdependent preferences, our present results allowed us to differentiate among various voting procedures and among convex agendas pertaining to the same procedure. Our results explain the emergence of compromises and describes the forces that determine their location on the ideological spectrum.

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[33] At least in West Germany this decision was also controversial, and many pleaded for a complete new start (see Die Zeit, 1949). East Germany added a distinctively communist emblem only in 1959.

These insights may be used to explain a variety of observed phenomena in real-life voting situations. For example, in a very recent case form 2019, the German Bundestag considered a refom of Paragraph 219a, the law governing the advertising of abortion procedures. The "extreme" alternatives were: 1) keeping the status quo that forbids any such advertising, and includes criminal charges against doctors that do so, and 2) scraping this paragraph altogether. The ruling coalition (and also the opposition, which contains parties both on the left and on the right) was bitterly split on this question. A compromise was finally forged that allows doctors and hospitals to advertise that they perform abortions, but does not allow them to provide further information about the methods, etc... The sequential voting agenda started with the two motions that wanted to scrap the law altogether. After these were defeated, the compromise was elected by a large majority. On the other hand, Theresa May's inability to get her Brexit selected by the UK parliament points to a non-optimal choice of compromise, one that does not respect the majority opinion in that divided house. Even if a compromise would be finally achieved, it is not clear that the deeply divided public will learn to accept it in the following years.

8   Appendix A: Probabilistic Tools

For several comparison results we consider the case where the number of voters is large.We need then to bound and compare the probability of various events, e.g., the distribution of tails of the normalized random variable $(\widetilde{n}_{-1} - \mathbb{E}\left[\widetilde{n}_{-1}\right])/(2n + 1)$. For these purposes, we use two well-known probabilistic tools, the *Hoeffding inequality* and the *Gärtner-Ellis Large Deviation Principle*. Let us start with Hoeffding's inequality.

Definition 2 *A random variable X is σ-subgaussian if for all $t \in \mathbb{R}$ there is $\sigma > 0$ such that its moment generation function $\mathbb{E}(e^{tX})$ satisfies $\mathbb{E}\left[e^{t(X-\mathbb{E}[X])}\right] \leq e^{\sigma^2 t^2/2}$.*

A Bernoulli random variable $X \sim$ Bernoulli(p) is σ-subgaussian with $\sigma = 1/2$. If X_1 is σ_1-subgaussian, X_2 is σ_2-subgaussian and if X_1 and X_2 are independent, then $(X_1 + X_2)$ is $\sqrt{\sigma_1^2 + \sigma_2^2}$-subgaussian. Therefore, a binomial random variable $X \sim B\left(N, p\right)$, the sum of N independent Bernoulli random variables, is $\sqrt{N}/2$-subgaussian.

Any σ-subgaussian random variable X satisfies the *Hoeffding bounds*: for all $t \geq 0$,

$$\Pr\left\{X - \mathbb{E}\left[X\right] \geq t\right\} \leq e^{-t^2/\left(2\sigma^2\right)}, \tag{7}$$

and

$$\Pr\{X - \mathbb{E}[X] \leq -t\} \leq e^{-t^2/(2\sigma^2)}. \tag{8}$$

We shall repeatedly use (7) or (8) to bound (tail) probabilities: the random variable $\widetilde{n}_{-1}$, the number of agents with signal -1, is the sum of the two independent binomial random variables, $B(n_L, p^L_{-1})$ and $B(n_R, p^R_{-1})$, so that it is $\sqrt{2n+1}/2$-subgaussian.

For example, applying inequality (7) to $\widetilde{n}_{-1}$ yields

$$\Pr\{\widetilde{n}_{-1} - \mathbb{E}[\widetilde{n}_{-1}] \geq t\} \leq e^{-2t^2/(2n+1)}. \tag{9}$$

By setting $t = n + 1 - \mathbb{E}[\widetilde{n}_{-1}]$, we can rewrite the above inequality as

$$\Pr\{\widetilde{n}_{-1} \geq n + 1\} \leq e^{-2(n+1-\mathbb{E}[\widetilde{n}_{-1}])^2/(2n+1)} \simeq e^{-(n+\frac{3}{2})(1-\alpha)^2} \tag{10}$$

where $\alpha = \lim_{n\to\infty} \frac{\mathbb{E}[\widetilde{n}_{-1}]}{n+1}$. In other words, if voters with signal -1 are in an ex ante minority (i.e., $\alpha < 1$), the probability that they are in an ex post majority (i.e., $n_{-1} \geq n + 1$) decays exponentially to zero as n grows.

We now the state the *Large Deviation Principle* due to Gärtner and Ellis (see for example Ellis [2006]) that will be used in the proof of Proposition 6 below:[34]

Theorem 1 *Suppose that X_n , $n \in \mathbb{N}$, is a family of real-valued random variables such that*

$$\Lambda(t) := \lim_{n\to\infty} \left(\frac{1}{n} \log \mathbb{E}\left[e^{ntX_n}\right]\right)$$

exists, and is finite for all $t \in \mathbb{R}$. If Λ is differentiable, then, for all Borel sets A such that the closure of A equals the closure of its interior, it holds that

$$\lim_{n\to\infty} \frac{1}{n} \log \Pr(X_n \in A) = -\inf_{x\in A} I(x) \tag{11}$$

where

$$I(x) = \sup_{t\in\mathbb{R}}[xt - \Lambda(t)].$$

is the Fenchel-Legendre transform of Λ.[35]

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[34] This is a well-known generalization of a classical Theorem due to Cramer where an I.I.D. assumption is imposed.

[35] The function Λ, the logarithm of the moment generating function, is also known as the *cumulant generating function*.

9 Appendix B: Results discussed, but not formally stated in the main text

This appendix contains formal statements and proofs of the Propositions 8 and 9 that were referenced, but not formally stated in the main text.

Proposition 8 *Suppose that $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$.*

(i) If $\gamma_1 > \frac{1}{2}(1 + x_C)$, then the equilibrium profiles $(L_1 C_2, \neg L_1 \neg C_2)$ and $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ are outcome-equivalent.

(ii) Suppose that $\gamma_1 \leq \frac{1}{2}(1 + x_C)$. If $\alpha > \frac{1}{2}\frac{1 - x_C}{1 - \gamma_1}$, then the equilibrium profiles $(L_1 C_2, \neg L_1 C_2)$ and $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ are equivalent with a probability exponentially approaching 1 as n grows. If

$$\alpha < \min\left\{\frac{1}{2}\frac{1 - x_C}{1 - \gamma_1}, \frac{1 - 2\gamma_{-1}}{1 - \gamma_{-1}} - \frac{1}{2}\frac{1 + x_C}{1 - \gamma_{-1}}\right\},$$

then the equilibrium profile $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ provides a higher expected utility than $(L_1 C_2, \neg L_1 C_2)$ with a probability exponentially approaching 1 as n grows.

Proof of Proposition 8. **(i)**. If $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$ and if $\gamma_1 > \frac{1}{2}(1 + x_C)$, then both profiles $(L_1 C_2, \neg L_1 \neg C_2)$ and $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ form an equilibrium, and the two equilibria are outcome-equivalent: if $n_{-1} \geq n + 1$, then L gets elected in both equilibria; if $n_{-1} < n + 1$, then R gets elected in both equilibria because voters with signal $+1$ vote for R in the second stage under both profiles.

(ii). Suppose now that $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$ and $\gamma_1 \leq \frac{1}{2}(1 + x_C)$. If $n_{-1} \geq \lceil k \rceil$, the equilibrium outcomes associated with $(L_1 C_2, \neg L_1 C_2)$ and $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ coincide, but, if $n_{-1} \leq \lceil k \rceil - 1$, $(L_1 C_2, \neg L_1 C_2)$ selects C while $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ selects R. If $\alpha > \frac{1}{2}\frac{1 - x_C}{1 - \gamma_1}$ we can set $X = \widetilde{n}_{-1}$ and $t = \mathbb{E}[\widetilde{n}_{-1}] - (\lceil k \rceil - 1)$ in inequality (8) to obtain that

$$\Pr\{\widetilde{n}_{-1} \leq \lceil k \rceil - 1\} \simeq \Pr\{\widetilde{n}_{-1} \leq k - 1\} \leq e^{-2(\mathbb{E}[\widetilde{n}_{-1}] - (k-1))^2/(2n+1)} \simeq e^{-\left(n - \frac{1}{2}\right)\left(\frac{1}{2}\frac{1 - x_C}{1 - \gamma_1} - \alpha\right)^2}.$$

That is, the probability of the event $\{n_{-1} \leq \lceil k \rceil - 1\}$ decays exponentially as n grows. On the other hand, if

$$\alpha < \min\left\{\frac{1}{2}\frac{1 - x_C}{1 - \gamma_1}, \frac{1 - 2\gamma_{-1}}{1 - \gamma_{-1}} - \frac{1}{2}\frac{1 + x_C}{1 - \gamma_{-1}}\right\},$$

we can set $X = \widetilde{n}_{-1}$ and $t = k - \mathbb{E}[\widetilde{n}_{-1}]$ in inequality (7) to obtain that

$$\Pr\{\widetilde{n}_{-1} \geq \lceil k \rceil\} \simeq \Pr\{\widetilde{n}_{-1} \geq k\} \leq e^{-2(k - \mathbb{E}[\widetilde{n}_{-1}])^2/(2n+1)} \simeq e^{-\left(n - \frac{1}{2}\right)\left(\frac{1}{2}\frac{1 - x_C}{1 - \gamma_1} - \alpha\right)^2}.$$

By the definition of the cutoff k, voters with signal $+1$ prefers R to C. Voters with signal -1 will also prefer R to C if their peak satisfies

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}(-n_{-1} + (2n - n_{-1})) \geq \frac{1}{2}(1 + x_C)$$

which is equivalent to

$$\frac{n_{-1}}{n} \leq \frac{1 - 2\gamma_{-1}}{1 - \gamma_{-1}} - \frac{1}{2}\frac{1 + x_C}{1 - \gamma_{-1}}.$$

The claim follows since $\alpha = \lim_{n \to \infty} \mathbb{E}\left[\widetilde{n}_{-1}\right] / (n + 1)$. ∎

Proposition 9 *Consider SV with agenda $[R, \{C, L\}]$ and suppose that Assumption A holds.*

(i) The profile $(\neg R_1 C_2, \neg R_1 C_2)$ is a sincere equilibrium if $\gamma_1 \leq \gamma_1^*$ and $\gamma_{-1} \leq \gamma_{-1}^*$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_1 \neq \gamma_1^*$ and $\gamma_{-1} \neq \gamma_{-1}^*$.

(ii) The profile $(\neg R_1 \neg C_2, \neg R_1 C_2)$ is a sincere equilibrium if $\gamma_1 \leq \gamma_1^*$ and $\gamma_{-1} \geq \gamma_{-1}^*$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma_{-1}^*$ and $\gamma_1 \neq \gamma_1^*$.

(iii) The profile $(\neg R_1 C_2$ if $\geq \kappa$, $R_1 C_2)$ is a sincere equilibrium if $\gamma_1 \geq \frac{1}{2}(1 + x_C)$. This is the unique such equilibrium for any γ_{-1} and γ_1 in these ranges such that $\gamma_{-1} \neq \gamma_{-1}^*$, $\gamma_1 \neq \gamma_1^*$ and κ is not an integer.

(iv) There is no sincere equilibrium if $\gamma_1 \in \left(\gamma_1^*, \frac{1}{2}(1 + x_C)\right)$.

Proof of Proposition 9. Sincerity Considerations: Consider voting in the first stage. Voters with signal $+1$ ex ante prefer R to C if $\gamma_1 \geq \gamma_1^*$, and they always prefer R to L since their peak satisfies

$$1 - (1 - \gamma_1)\frac{\mathbb{E}\left[\widetilde{n}_{-1}\right]}{n} \geq 0.$$

By Assumption A, voters with signal -1 ex ante prefer L to R, and thus prefer C to R for all γ_{-1}. Therefore, sincerity requires that voters with signal -1 always vote against R and that voters with signal $+1$ vote for R if and only if $\gamma_1 \geq \gamma_1^*$.

For voting in the second stage, we consider two possible cases:

(a) $\gamma_1 \leq \gamma_1^*$: all voters vote against R in the first stage, and no information about n_{+1} is revealed. Voters with signal -1 ex ante prefer C to L if and only if $\gamma_{-1} \leq \gamma_{-1}^*$. Voters with signal $+1$ ex ante prefer C to L if

$$1 - (1 - \gamma_1)\frac{\mathbb{E}\left[\widetilde{n}_{-1}\right]}{n} \geq \frac{1}{2}(-1 + x_C)$$

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which is always satisfied given that $\mathbb{E}[\widetilde{n}_{-1}] \leq n$. In this case, the sincere strategy profile is $(\neg R_1 C_2, \neg R_1 C_2)$ if $\gamma_{-1} \leq \gamma^*_{-1}$ and $(\neg R_1 \neg C_2, \neg R_1 C_2)$ if $\gamma_{-1} \geq \gamma^*_{-1}$. They are unique if $\gamma_1 \neq \gamma^*_1$ and $\gamma_{-1} \neq \gamma^*_{-1}$.

(b) $\gamma_1 \geq \gamma^*_1$: voters with signal $+1$ sincerely vote for R, voters with signal -1 sincerely vote against R in the first stage, and thus n_{+1} is revealed. Voters with signal $+1$ always prefer C to L, because $\gamma^*_1 \geq \frac{1}{4}(1 + x_C)$ and because a voter with signal $+1$ prefers C to L in the situation where she is the lone $+1$ voter if

$$\gamma_1 + \frac{1 - \gamma_1}{2n}(-2n) \geq \frac{1}{2}(-1 + x_C) \Leftrightarrow \gamma_1 \geq \frac{1}{4}(1 + x_C).$$

By the definition of κ, voters with signal -1 prefer C to L if $n_{+1} \geq \lceil \kappa \rceil$. In this case, the sincere strategy profile is $(\neg R_1 C_2$ if $\geq \kappa$, $R_1 C_2)$. It is unique if $\gamma_1 \neq \gamma^*_1$ and $\gamma_{-1} \neq \gamma^*_{-1}$ and κ is not an integer. We conclude that the strategies in (i) to (iii) are sincere given the parametric assumptions.

Equilibrium Considerations: Profile $(\neg R_1 C_2, \neg R_1 C_2)$ is an equilibrium since no voter is pivotal, establishing (i). In profile $(\neg R_1 \neg C_2, \neg R_1 C_2)$ all voters vote against R in the first stage, so that we only need to consider pivotality in the second stage. Conditional on being on pivotal between C and L, voters with signal -1 prefer C if and only if $\gamma_{-1} \leq \frac{1}{2}(1 - x_C)$, while voters with signal $+1$ prefer C for all γ_1 because their ideal point, conditional being pivotal, is $\gamma_1 \geq \frac{1}{2}(-1 + x_C)$. Note that $\gamma^*_{-1} \geq \frac{1}{2}(1 - x_C)$. Therefore, $(\neg R_1 \neg C_2, \neg R_1 C_2)$ is a sincere equilibrium if $\gamma_{-1} \geq \gamma^*_{-1}$, establishing (ii).

For profile $(\neg R_1 C_2$ if $\geq \kappa$, $R_1 C_2)$, we start with the first stage. Voter i with signal -1 is pivotal if (1) there are $\lceil \kappa \rceil - 1$ voters having signal $+1$ (in this case, voter i is pivotal between C and L), or if (2) there are exactly n voters having signal of $+1$ (in this case, voter i is pivotal between R and C). By the definition of κ, a voter with signal -1 prefers L to C in the first case and thus she plays a best response by voting against R in the first stage. For the second case, the ideal point $-\gamma_{-1}$ is smaller than $\frac{1}{2}(1 + x_C)$, so voters with signal -1 prefer C to R and thus they play a best response by voting against R in the first stage. Voter i with signal $+1$ is pivotal if (1) there are $\lceil \kappa \rceil - 1$ other voters having signal of $+1$ (in this case, voter i is pivotal between C and L), or if (2) there are exactly n other voters having signal of $+1$ (in this case, voter i is pivotal between R and C). In the first case, voter i prefers C to L and thus votes for R in the first stage if

$$\gamma_1 + \frac{1 - \gamma_1}{2n}(-(2n - \lceil \kappa \rceil + 1) + (\lceil \kappa \rceil - 1)) \geq \frac{1}{2}(-1 + x_C) \Leftrightarrow \gamma_1 \geq \frac{n - 2\lceil \kappa \rceil + 2 + nx_C}{4n - 2\lceil \kappa \rceil + 2}.$$

In the second case, voter i prefers R to C if $\gamma_1 \geq \frac{1}{2}(1 + x_C)$. Note that $-2\lceil \kappa \rceil + 2 \leq 0$, so that

$$\frac{1}{2}(1 + x_C) \geq \frac{n + nx_C - 2\lceil \kappa \rceil + 2}{2n - 2\lceil \kappa \rceil + 2} \geq \frac{n - 2\lceil \kappa \rceil + 2 + nx_C}{4n - 2\lceil \kappa \rceil + 2}.$$

Therefore, it is optimal to vote for R if $\gamma_1 \geq \frac{1}{2}(1 + x_C)$.

Consider now the second stage voting. If $\gamma_1 \geq \frac{1}{2}(1 + x_C)$, voters with signal $+1$ play a best response by voting for C, because their ideal point is at least

$$\gamma_1 + \frac{1 - \gamma_1}{2n}(-2n) = -1 + 2\gamma_1 \geq x_C.$$

By the definition of κ, voters with signal -1 play a best response by voting for C if and only if the number of votes received by R in the first stage is at least $\lceil \kappa \rceil$. Note that $\gamma_1^* \leq \frac{1}{2}(1 + x_C)$. Therefore, we conclude that $(\neg R_1 C_2 \text{ if } \geq \kappa, R_1 C_2)$ is a sincere equilibrium if $\gamma_1 \geq \frac{1}{2}(1 + x_C)$, establishing (iii).

Finally, the non-existence of a sincere equilibrium when $\gamma_1 \in \left(\gamma_1^*, \frac{1}{2}(1 + x_C)\right)$ is, again, due to a conflict between sincerity and pivotality. Sincerity requires that voters with signal $+1$ vote for R in the first stage and voters with signal -1 vote against R. Consider a voter i with signal $+1$ and fix a signal realization such that n others have signal $+1$. Then i prefers C to R because $\gamma_1 < \frac{1}{2}(1 + x_C)$, and following the sincere strategy is not a best response at this signal realization. Hence there is no sincere equilibrium. ∎

10   Appendix C: Proofs of all other results

We gather here the proofs of all other results that were formally stated in the main text.

Proof of Proposition 1.   (i) We first show that the profile $(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$ is an equilibrium.   We only need to consider signal realizations such that an individual voter is pivotal at a given stage.

Consider first voters with signal -1. They play a best response by voting for C in the second stage because their ideal point is at most

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}(2n) = 1 - 2\gamma_{-1} \leq x_C.$$

Voter i with signal -1 is pivotal in the first stage if (1) there are $\lceil k \rceil - 1$ other voters having signal of -1 (in this case, voter i is pivotal between C and R), or if (2) there are exactly n

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other voters having signal of -1 (in this case, voter i is pivotal between L and C). The ideal point of voter i for the first case is

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}\left(-\left(\lceil k \rceil - 1\right) + \left(2n - \lceil k \rceil + 1\right)\right) = -\gamma_{-1} + \frac{1 - \gamma_{-1}}{n}\left(n - \lceil k \rceil + 1\right).$$

The ideal point for the second case is $-\gamma_{-1}$. Therefore, in the first case, voter i with signal -1 will vote for L in the first stage if

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{n}\left(n - \lceil k \rceil + 1\right) \leq \frac{1}{2}\left(x_C + 1\right) \Leftrightarrow \gamma_{-1} \geq \frac{n + 2 - 2\lceil k \rceil - nx_C}{4n - 2\lceil k \rceil + 2} \qquad (12)$$

Since $-2\lceil k \rceil + 2 \leq 0$ and $\gamma_{-1} \geq \frac{1}{2}\left(1 - x_C\right)$ by assumption, we get

$$\gamma_{-1} \geq \frac{1 - x_C}{2} \geq \frac{n - nx_C - 2\lceil k \rceil + 2}{2n - 2\lceil k \rceil + 2} \geq \frac{n - 2\lceil k \rceil + 2 - nx_C}{4n - 2\lceil k \rceil + 2}.$$

Therefore, condition (12) is always satisfied. In the second case, since $\gamma_{-1} \geq \frac{1}{2}\left(1 - x_C\right)$, voter i with signal -1 will vote for L in the first stage.

Consider next voters with signal $+1$. By the definition of cutoff k, they play a best response in the second stage by voting for C if and only if at least $\lceil k \rceil$ voters support L in the first stage. Voter i with signal $+1$ is pivotal in the first stage if (1) there are $\lceil k \rceil - 1$ voters having signal of -1 (in this case, voter i is pivotal between C and R), or if (2) there are exactly n voters having signal of -1 (in this case, voter i is pivotal between L and C). Therefore, for the first case, voter i plays a best response by voting against L in the first stage if

$$\gamma_1 + \frac{1 - \gamma_1}{2n}\left(-\left(\lceil k \rceil - 1\right) + \left(2n - \lceil k \rceil + 1\right)\right) \geq \frac{1}{2}\left(1 + x_C\right),$$

which always holds since $\lceil k \rceil - 1 \leq k$ and k satisfies by definition

$$\gamma_1 + \frac{1 - \gamma_1}{2n}\left(-k + \left(2n - k\right)\right) = \frac{1}{2}\left(1 + x_C\right).$$

For the second case, since her ideal point $\gamma_1 \geq \frac{1}{2}\left(x_C - 1\right)$, voter i plays a best response by voting against L in the first stage.

The complete information Condorcet winner will be selected under this strategy profile. To see this, note that if at least $n + 1$ voters have signal -1, L will be selected and this is the most preferred alternative for voters with signal -1 since $\gamma_{-1} \geq \frac{1}{2}\left(1 - x_C\right)$. If at least $n + 1$ voters have signal $+1$, L will be rejected in the first stage. In the second stage, agents are, essentially, completely informed about the preferences of others, and C gets elected if and only if voters with signal $+1$ prefer C to R given the realized preferences.

Finally, note that if $k > n$, then, whenever L receives at least $\lceil k \rceil$ votes in the first stage, L is chosen and thus vote shifting from voters with signal $+1$ never occurs in equilibrium. In order for vote shifting to possibly occur in equilibrium, we must have $k \leq n$, which is equivalent to $\gamma_1 \leq \frac{1}{2}(1 + x_C)$.

(ii). Consider a type-symmetric strategy profile that always results in the selection of the Condorcet winner. If at the first stage voters with signal -1 vote for L with a probability that is strictly below 1, then L will be rejected with positive probability even if all voters have signal -1. Therefore, in an equilibrium that always selects the Condorcet winner, voters with signal -1 must vote for L with probability 1, and, by an analogous argument, voters with signal $+1$ must vote against L with probability 1.

Now suppose that there are $n + 1$ voters with signal -1. Then L will get elected in equilibrium under the above strategy profile, but L is not the Condorcet winner! To see this, note that the ideal point of voters with signal -1 is $-\gamma_{-1}$, which is closer to x_C than to -1 because $\gamma_{-1} < \frac{1}{2}(1 - x_C)$. Therefore, voters with signal -1, who form a majority, prefer C to L, contradicting the assumption that the Condorcet winner is elected. ∎

Proof of Proposition 2. **Sincerity Considerations**: Consider the first stage voting. Ex ante, voters with signal -1 prefer C to L if

$$-1 + (1 - \gamma_{-1})\frac{\mathbb{E}[n_{+1}]}{n} \geq \frac{1}{2}(-1 + x_C)$$

which is equivalent to $\gamma_{-1} \leq \gamma_{-1}^*$. They prefer R to L if

$$-1 + (1 - \gamma_{-1})\frac{\mathbb{E}[n_{+1}]}{n} \geq 0$$

which is equivalent to

$$\gamma_{-1} \leq \gamma_{-1}^{**} \equiv \frac{n + 1 - \mathbb{E}[\widetilde{n}_{-1}]}{2n + 1 - \mathbb{E}[\widetilde{n}_{-1}]}.$$

Since

$$\gamma_{-1}^* - \gamma_{-1}^{**} = \frac{n}{2n + 1 - \mathbb{E}[\widetilde{n}_{-1}]}\frac{1}{2}(1 - x_C) > 0,$$

alternative L does not yield the highest payoff for voters with signal -1 as long as $\gamma_{-1} \leq \gamma_{-1}^*$. Ex ante, voters with signal $+1$ prefer R to L if

$$1 - (1 - \gamma_1)\frac{\mathbb{E}[n_{-1}]}{n} \geq 0$$

which is always satisfied. Therefore, it is a sincere strategy for voters with signal $+1$ to always vote against L, and it is a sincere strategy for voters with signal -1 to vote for L if and only if $\gamma_{-1} \geq \gamma_{-1}^*$.

For the analysis of voting in the second stage, we need to consider two possible cases:

(a) $\gamma_{-1} \leq \gamma_{-1}^*$: all voters vote against L in the first stage and no information about n_{-1} is revealed. In this case, voters with signal -1 ex ante prefer C to R by Assumption A, and voters with signal $+1$ ex ante prefer R to C if

$$1 - (1 - \gamma_1)\frac{\mathbb{E}[n_{-1}]}{n} \geq \frac{1}{2}(1 + x_C)$$

which is equivalent to $\gamma_1 \geq \gamma_1^*$. Therefore, if $\gamma_{-1} \leq \gamma_{-1}^*$, it is a sincere strategy for voters with signal -1 to always vote for C, while for voters with signal $+1$ it is sincere to vote against C if and only if $\gamma_1 \geq \gamma_1^*$.

(b) $\gamma_{-1} \geq \gamma_{-1}^*$: not all voters vote against L and n_{-1} is revealed. In this case, voters with signal -1 always prefer C to R, because $\gamma_{-1}^* \geq \frac{1}{4}(1 - x_C)$ and because a voter with signal -1 prefers C to R in the situation where she is the lone -1 voter if

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n}(2n) \leq \frac{1}{2}(1 + x_C) \Leftrightarrow \gamma_{-1} \geq \frac{1}{4}(1 - x_C).$$

By the definition of the cutoff k, voters with signal $+1$ prefer R to C in the second stage if and only if $n_{-1} \geq k$. Therefore, if $\gamma_{-1} \geq \gamma_{-1}^*$, it is a sincere strategy for voters with signal -1 to always vote for C, while voters with signal $+1$ vote sincerely against C if and only if $n_{-1} \geq k$.

To summarize, in the first stage, it is sincere for voters with signal -1 to vote in favor of L if and only if $\gamma_{-1} \geq \gamma_{-1}^*$, and for voters with signal $+1$ to always vote against L. In the second stage, sincerity requires that (1) if $\gamma_{-1} \leq \gamma_{-1}^*$, voters with signal $+1$ vote for C if and only if $\gamma_1 \leq \gamma_1^*$ while voters with signal -1 always vote for C, and that (2) if $\gamma_{-1} \geq \gamma_{-1}^*$, voters with signal $+1$ vote for C if and only if $n_{-1} \geq k$ while voters with signal -1 always vote for C. As a result, the profiles $(\neg L_1 C_2, \neg L_1 C_2)$, $(\neg L_1 C_2, \neg L_1 \neg C_2)$, and $(L_1 C_2, \neg L_1 C_2$ if $\geq k)$ are the unique sincere profiles correponding to the cases $(\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \gamma_1^*)$, $(\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \geq \gamma_1^*)$, and $(\gamma_{-1} \geq \gamma_{-1}^*)$, respectively. Moreover, if k is not an integer, and if $\gamma_{-1} \neq \gamma_{-1}^*$, and $\gamma_1 \neq \gamma_1^*$ the sincere strategies are uniquely defined.

Equilibrium Considerations: In profiles $(\neg L_1 C_2, \neg L_1 C_2)$ and $(\neg L_1 C_2, \neg L_1 \neg C_2)$, all voters vote against L in the first stage, so that we only need to consider pivotality in the

second stage voting. Conditional being on pivotal between R and C, voters with signal $+1$ prefer C if and only if $\gamma_1 \leq \frac{1}{2}(1 + x_C)$, while voters with signal -1 prefer C for all γ_{-1}. It follows from the proof of Proposition 1 that profile $(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$, this strategy profile is an equilibrium if $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$. Note that, by Assumption A we obtain $\gamma_1^* \leq \frac{1}{2}(1 + x_C)$ and $\gamma_{-1}^* \geq \frac{1}{2}(1 - x_C)$. Therefore, $(\neg L_1 C_2, \neg L_1 C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \gamma_1^*$, $(\neg L_1 C_2, \neg L_1 \neg C_2)$ is a sincere equilibrium if $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \geq \frac{1}{2}(1 + x_C)$, and $(L_1 C_2, \neg L_1 C_2 \text{ if } \geq k)$ is a sincere equilibrium if $\gamma_{-1} \geq \gamma_{-1}^*$.

Finally, when $\gamma_{-1} < \gamma_{-1}^*$ and $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1 + x_C)]$, the non-existence of sincere equilibrium is due to a conflict between sincerity and pivotality. Sincerity requires that both type of voters vote against L at the first stage and voters with signal -1 vote for C at the second stage. Therefore, no new information would be revealed by the vote at the first stage. If $\gamma_1 \in (\gamma_1^*, \frac{1}{2}(1 + x_C)]$, then, based on ex ante information, voters with signal $+1$ prefer R to C. But, conditional on pivotality, this preference is reversed. Hence, sincere voting suggests that voters with signal $+1$ vote against C, but pivotality requires that they vote in favor of C. Thus, sincere voting by voters with signal $+1$ cannot be part of an equilibrium in this case. ∎

Proof of Proposition 3. If $\gamma_1 > \frac{1 + x_C}{2}$ and $\gamma_{-1} < \gamma_{-1}^*$, we see from Table 3 that the outcomes of the two agendas differ only when (1) $\gamma_{-1} \in [\frac{1}{2n+1}, \frac{1 - x_C}{2})$ and $n_{-1} > 2n + 1 - \lceil \kappa \rceil$, or when (2) $\gamma_{-1} \in \left(\frac{1 - x_C}{2}, \gamma_{-1}^*\right)$ and $n_{-1} \geq n + 1$. In both cases, C is chosen under $[L, \{C, R\}]$ and L is chosen under $[R, \{C, L\}]$. Moreover, whenever L is chosen under $[R, \{C, L\}]$, it is the Condorcet winner. In both cases, $n_{-1} \geq n + 1$ and $[R, \{C, L\}]$ selects the Condorcet winner while $[L, \{C, R\}]$ does not. The event $\{\widetilde{n}_{-1} \geq n + 1\}$, however, has a vanishing probability as n grows. To see this, note that, by assumption A, we must have $\alpha < 1$. It follows from (10) that the probability of the event $\{\widetilde{n}_{-1} \geq n + 1\}$ decays exponentially to zero as n grows:

$$\Pr\{\widetilde{n}_{-1} \geq n + 1\} \leq e^{-2(n + 1 - \mathbb{E}[\widetilde{n}_{-1}])^2 / (2n+1)} \simeq e^{-(n + \frac{3}{2})(1 - \alpha)^2}. \tag{13}$$

Therefore, as n grows, the advantage of $[R, \{C, L\}]$ becomes **negligible**.

If $\gamma_1 < \frac{1 + x_C}{2}$ and $\gamma_{-1} > \gamma_{-1}^*$, the outcomes of the two agendas differ only if $n_{-1} \leq \lceil k \rceil - 1$. In this case, $[L, \{C, R\}]$ elects R while $[R, \{C, L\}]$ elects C. Whenever R is chosen under $[L, \{C, R\}]$, it is the Condorcet winner. Therefore, agenda $[L, \{C, R\}]$ selects the Condorcet winner with a higher probability than agenda $[R, \{C, L\}]$. We can set $t = \lceil k \rceil - \mathbb{E}[\widetilde{n}_{-1}]$ in

(9) to obtain

$$\Pr\left\{\widetilde{n}_{-1} \geq \lceil k \rceil\right\} \leq e^{-2(\lceil k \rceil - \mathbb{E}[\widetilde{n}_{-1}])^2/(2n+1)} \simeq e^{-(n+\frac{3}{2})\left(\frac{1}{2}\frac{1-x_C}{1-\gamma_1} - \alpha\right)^2}. \tag{14}$$

Therefore, if $\mathbb{E}\left[\widetilde{n}_{-1}\right] \leq \lceil k \rceil - 1$ (and thus $\alpha < \frac{1}{2}\frac{1-x_C}{1-\gamma_1}$ in the limit), then the event $\{\widetilde{n}_{-1} \geq \lceil k \rceil\}$ has a probability exponentially decaying to 0 as n grows. Equivalently, the event $\{\widetilde{n}_{-1} \leq \lceil k \rceil - 1\}$ has a probability exponentially approaching one, and the advantage of $[L, \{C, R\}]$ can be **significant**.

For the other parameter values of γ_1 and γ_{-1}, the two convex agendas are outcome equivalent. ∎

Proof of Proposition 4. A voter with signal -1 is pivotal at the second stage only if there are n voters with signal $+1$. In that case her ideal point is $-\gamma_{-1}$, which given that $\gamma_{-1} \geq \frac{1}{2}(1 - x_C)$ is closer to C than to R. This voter is pivotal in the first stage (between L and C) if there are n voters with signal $+1$, in which case her ideal point is again $-\gamma_{-1} \leq \frac{1}{2}(-1 + x_C)$. Hence, she prefers L over C in the first stage.

A voter with signal $+1$ is pivotal in the first stage if there are n voters with signal -1, in which case her ideal point is γ_1. Therefore, she prefers C over L. In the second stage, her ideal point conditional on being pivotal is again γ_1. If $\gamma_1 > \frac{1}{2}(1 + x_C)$ she prefers R over C, otherwise she prefers C over R. ∎

Proof of Proposition 5. We focus on voters with signal -1. The arguments for voters with signal $+1$ are analogous.

We start with the second stage. If L wins the first stage, a voter with signal -1 prefer C to L if at least $\lceil \kappa \rceil$ voters have signal $+1$; otherwise she prefers L to C. Given the strategy profile Π, all first-stage votes for R come from voters with signal $+1$. Therefore, voters with signal -1 play a best response. If R wins the first stage, voters with signal -1 are not pivotal in the second stage and, therefore play a best response.

In the first stage, voter i with signal -1 is pivotal if (1) there are exactly n other voters with signal -1, if (2) there are exactly $\lceil k \rceil - 1$ other voters with signal -1 and $\lceil k \rceil \leq n$, or if (3) there are exactly $\lceil \kappa \rceil - 1$ others with signal $+1$ and $\lceil \kappa \rceil \leq n$.

In case (1), voter i likes alternative R the least. If she prefers L to C, then the worst feasible alternative when L wins the first stage is weakly better than the best possible alter-

native when R wins the first stage, so voting for L is a best response. If she prefers C to L, she will get her most preferred alternative by voting for L in the first stage and by voting for C in the second stage.

In case (2), voter i is pivotal between C and R, and voting L is a best response if her ideal point satisfies

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n} \left(-\lceil k \rceil + 1 + 2n - \lceil k \rceil + 1 \right) \leq \frac{1}{2} \left(1 + x_C \right).$$

Using the definition of $\lceil k \rceil$, a sufficient condition for this inequality is

$$-\gamma_{-1} + \frac{1 - \gamma_{-1}}{2n} \left(-\frac{n(1 - x_C)}{1 - \gamma_1} + 2n + 2 \right) \leq \frac{1}{2} \left(1 + x_C \right).$$

Note that $k \leq n$ implies $\gamma_1 \leq \frac{1}{2} \left(1 + x_C \right)$. The left-hand side is therefore decreasing in both γ_1 and γ_{-1}, and is equal to $\frac{1}{2} \left(1 + x_C \right)$ when $\gamma_{-1} = \gamma_1 = \frac{1}{2n+1}$. Since $\gamma_{-1} \geq \frac{1}{2n+1}$ and $\gamma_1 \geq \frac{1}{2n+1}$, the above inequality always holds. We conclude that voting for L is a best response.

In case (3), voter i is pivotal between L and C. By the definition of κ, she prefers L if there are $\lceil \kappa \rceil - 1$ others with signal $+1$. Voting for L is therefore a best response.

For the Condorcet claim, suppose that alternative R wins in the first stage. Then the number of voters with signal $+1$ is at least $n + 1$. Hence, once all private information becomes public, any voter with signal $+1$ prefers R over L, and therefore L is not the full information Condorcet winner. At the second stage, the number of voters with signal -1 is public information, and hence the full information Condorcet winner gets elected. The argument is analogous if L wins in the first stage. $\blacksquare$

Proof of Lemma 1. **(i).** We first prove the claim for SV. Suppose that $n_{-1} \geq n + 1$. Then C gets elected if and only if $\gamma_{-1} \leq \gamma_{-1}^*$ (see Table 3). Recall that

$$\gamma_{-1}^* = \frac{1}{2} \left(1 - x_C \right) + \frac{n - \mathbb{E}\left[\tilde{n}_{-1} \right] + 1}{2n - \mathbb{E}\left[\tilde{n}_{-1} \right] + 1} \frac{1}{2} \left(1 + x_C \right).$$

We obtain then that $\gamma_{-1} \leq \gamma_{-1}^*$ is equivalent to $x_C \leq x_C^L(n)$:

$$\begin{aligned} \gamma_{-1} &\leq \frac{1}{2} \left(1 - x_C \right) + \frac{n - \mathbb{E}\left[\tilde{n}_{-1} \right] + 1}{2n - \mathbb{E}\left[\tilde{n}_{-1} \right] + 1} \frac{1}{2} \left(1 + x_C \right) \\ &\Leftrightarrow\ x_C \leq 2 \left(1 - \gamma_{-1} \right) \frac{2n + 1 - \mathbb{E}\left[\tilde{n}_{-1} \right]}{n} - 1 \Leftrightarrow x_C \leq x_C^L(n). \end{aligned}$$

Suppose next that $n_{-1} \leq n$. Then C gets elected either if (1) $\gamma_{-1} \leq \gamma_{-1}^*$ and $\gamma_1 \leq \frac{1 + x_C}{2}$ or if (2) $\gamma_{-1} \geq \gamma_{-1}^*$, $\gamma_1 \leq \frac{1 + x_C}{2}$ and $n_{-1} \geq k$ (see Table 3). As shown above, $\gamma_{-1} \leq \gamma_{-1}^*$ is

equivalent to $x_C \leq x_C^L(n)$. Also, $\gamma_1 \leq \frac{1+x_C}{2}$ is equivalent to $x_C \geq 2\gamma_1 - 1$. Hence, case (1) applies if and only if $2\gamma_1 - 1 \leq x_C \leq x_C^L(n)$. Case (2) applies if and only if $x_C \geq x_C^L(n)$, $x_C \geq 2\gamma_1 - 1$, and $n_{-1} \geq k$ (which is equivalent to $x_C \geq x_C^R(n, n_{-1}) = 1 - 2(1 - \gamma_1)\frac{n_{-1}}{n}$). Since $n_{-1} \leq n$, we have $2\gamma_1 - 1 \leq x_C^R(n, n_{-1})$, and therefore, alternative C is chosen in case (2) if $x_C \geq \max\left\{x_C^L(n), x_C^R(n, n_{-1})\right\}$. Finally, since $n_{-1} \leq n$, we have $x_C^R(n, n_{-1}) \geq 2\gamma_1 - 1$, and thus the set of implementable compromise locations

$$\left[-1 + 2\gamma_1, x_C^L(n)\right] \cup \left[\max\left\{x_C^L(n), x_C^R(n, n_{-1})\right\}, 1\right]$$

can be rewritten as

$$\left[-1 + 2\gamma_1, x_C^L(n)\right] \cup \left[x_C^R(n, n_{-1}), 1\right].$$

(ii). Consider now AV, and recall that it always selects the Condorcet winner. Alternative C is the Condorcet winner if either $n_{-1} \geq n + 1$ and $n_{+1} \geq \kappa(x_C)$, or if $n_{+1} \geq n + 1$ and $n_{-1} \geq k(x_C)$, where $\kappa(x_C) = \frac{n}{2}\frac{1+x_C}{1-\gamma_{-1}}$ and $k(x_C) = \frac{n}{2}\frac{1-x_C}{1-\gamma_1}$. Rearranging the terms, we obtain that C gets elected if $n_{-1} \geq n + 1$ and $x_C \leq x_C^L(n, n_{+1})$ or if $n_{-1} \leq n$ and $x_C \geq x_C^R(n, n_{-1})$.

∎

For the Proof of Proposition 6-(ii), we need the following Lemma:

Lemma 2 *Let* $\alpha < b < c$. *Then*

$$\lim_{n \to \infty} \left(\frac{\Pr(\frac{\tilde{n}_{-1}}{n} \geq c)}{\Pr(\frac{\tilde{n}_{-1}}{n} \geq b)}\right) = 0$$

Proof. Let X_i^L (X_i^R) be a Bernoulli random variable with success probability p_{-1}^L (p_{-1}^R), and assume all random variables are independently distributed. We apply the Gaertner-Ellis Theorem to the family of random variables of the form $\frac{1}{2n}[\sum_{i=1}^{n^L} X_i^L + \sum_{i=1}^{n^R} X_i^R]$, $n \in \mathbb{N}$. Note that the cumulant generating function is

$$\Lambda(t) = \lim_{n \to \infty} \frac{1}{2n} \log \mathbb{E}\left[e^{t2n\frac{1}{2n}[\sum X_i^L + \sum X_i^R]}\right] = \lim_{n \to \infty} \frac{1}{2n} \log \left\{\mathbb{E}\left[e^{tX_i^L}\right]^{n^L} \mathbb{E}\left[e^{tX_i^R}\right]^{n^R}\right\}$$

$$= \lim_{n \to \infty} \frac{n_L}{2n} \log\left\{1 - p_{-1}^L + p_{-1}^L e^t\right\} + \frac{n_R}{2n} \log\left\{1 - p_{-1}^R + p_{-1}^R e^t\right\}$$

$$= s_L \log\left\{1 - p_{-1}^L + p_{-1}^L e^t\right\} + s_R \log\left\{1 - p_{-1}^R + p_{-1}^R e^t\right\},$$

where $s_L = \lim_{n \to \infty} \frac{n_L}{2n}$ and $s_L = \lim_{n \to \infty} \frac{n_R}{2n}$. This shows that $\Lambda(t) < \infty$ for all t, and that Λ is convex and twice differentiable. The Gaertner-Ellis theorem implies then that the family of random variables $\frac{\tilde{n}_{-1}}{2n} = \frac{1}{2n}[\sum X_i^L + \sum X_i^R]$, $n \in \mathbb{N}$, satisfies the large deviation principle (11) with rate function $I(x) = \sup_{t \in \mathbb{R}} xt - \Lambda(t)$.

To understand the properties of $I(x)$, fix an arbitrary x and note that the function $xt - \Lambda(t)$ is concave and differentiable. For each real x, denote by t_x the maximizing t in the definition of I. The maximizer t_x must satisfy the first-order condition:

$$x = \Lambda'(t_x) = s_L \frac{p^L_{-1} e^{t_x}}{1 - p^L_{-1} + p^L_{-1} e^{t_x}} + s_R \frac{p^R_{-1} e^{t_x}}{1 - p^R_{-1} + p^R_{-1} e^{t_x}}$$

Note that

$$
\begin{aligned}
\Lambda'(0) &= s_L \frac{p^L_{-1}}{1 - p^L_{-1} + p^L_{-1}} + s_R \frac{p^R_{-1}}{1 - p^R_{-1} + p^R_{-1}} \\
&= s_L p^L_{-1} + s_R p^R_{-1} = \alpha/2 > 0
\end{aligned}
$$

Since $\Lambda''(t) > 0$, we obtain that $t_x > 0$ for $x > \alpha/2$. By the envelope theorem, we obtain that the rate function I is strictly increasing for $x > \alpha/2$ since $I'(x) = t_x > 0$.

We conclude that

$$\Pr(\tilde{n}_{-1} \geq an) = e^{-2nI(a/2) + o(n)}$$

for all $a > \alpha$ and that

$$\lim_{n \to \infty} \left(\frac{\Pr(\frac{\tilde{n}_{-1}}{n} \geq c)}{\Pr(\frac{\tilde{n}_{-1}}{n} \geq b)} \right) = \lim_{n \to \infty} \left(e^{-2n[I(c/2) - I(b/2)] + o(n)} \right) = 0$$

where the last equality follows because $\alpha < b < c$ and because $I(x)$ is strictly increasing for $x > \alpha/2$. ∎

Proof of Proposition 6. **(i). SV**. Note that Assumption A implies $x^L_C(n) \leq 1$. Also, observe that $n_{-1} \leq n$ implies $-1 + 2\gamma_1 1 \leq x^R_C(n, n_{-1})$.

(a). If $-1 + 2\gamma_1 \leq x^L_C(n)$, then, by Lemma 1, we can set x_C just below $-1 + 2\gamma_1$ to get C elected if $n_{-1} \geq n + 1$, and get R elected if $n_{-1} < n + 1$. On the other hand, setting $x_C = x^L_C(n)$ will always get C elected. No other compromise location can be optimal: Setting x_C substantially below $-1 + 2\gamma_1$ is dominated by setting it just below $-1 + 2\gamma_1$. Setting x_C between $-1 + 2\gamma_1$ and $x^L_C(n)$ is dominated by setting $x_C = x^L_C(n)$, while setting x_C above $x^L_C(n)$ is dominated by setting it just below $-1 + 2\gamma_1$.[36]

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[36] Setting x_C just below $2\gamma_1 - 1$ is not dominated by $x_C = x^L_C$, because R is elected when x_C is set just below $2\gamma_1 - 1$ and $n_{-1} \geq n + 1$. Hence, the optimal location depends on the probability of a realized majority of voters with signal -1. It is optimal to locate the compromise at x^L_C if

$$x^L_C \geq \Pr(\tilde{n}_{-1} \geq n + 1) \cdot (2\gamma_1 - 1) + \Pr(\tilde{n}_{-1} \leq n) \cdot 1.$$

(b). If $x_C^L(n) < -1 + 2\gamma_1$, then Lemma 1 implies that if we set $x_C = x_C^L(n)$, then if $n_{-1} \geq n + 1$ alternative C gets elected, while if $n_{-1} < n + 1$ alternative R gets elected. Any higher compromise location is worse because such a compromise will never be elected if $n_{-1} \geq n + 1$. Any lower compromise location is worse because it will be elected in the same instances, but will provide lower utility conditional on being elected.

(ii). AV

(a) Assume first that $\alpha > 1$. Suppose, by contradiction, that there exists $\varepsilon > 0$ such that the optimal compromise satisfies $x_C(n) < \overline{x}_C^L - \varepsilon$ for infinitely many n, and consider the corresponding subsequence. We argue that, for n large enough, the compromise $\overline{x}_C^L - \varepsilon/2$ is strictly better than $x_C(n)$ because its location is further to the right, and because it still gets elected with probability approaching 1. Note that $\lim_{n\to\infty} \Pr\{n_{+1} \geq n + 1\} = 0$ since $\alpha > 1$. Observe also that

$$\lim_{n\to\infty} \Pr\{\kappa(\overline{x}_C^L - \frac{\varepsilon}{2}) \leq \tilde{n}_{+1} \leq n\} = 1$$

since $\kappa(\overline{x}_C^L - \frac{\varepsilon}{2}) = n\left[(2 - \alpha) - \frac{\varepsilon}{4(1-\gamma_{-1})}\right]$. This yields

$$\Pr\{\kappa(\overline{x}_C^L - \frac{\varepsilon}{2}) \leq \tilde{n}_{+1}\} = 1 - \Pr\{-\frac{\varepsilon}{4(1 - \gamma_{-1})} > \frac{\tilde{n}_{+1}}{n} - (2 - \alpha)\}$$

By Hoeffding's inequality, the last expression converges to 1 as n grows The compromise $\overline{x}_C^L - \varepsilon/2$ will therefore be elected with probability approaching 1, and it dominates compromise $x_C(n)$ for n large enough. This contradicts the assumed optimality of $x_C(n)$.

If there exists $\varepsilon > 0$ such that $x_C(n) > \overline{x}_C^L + \varepsilon$ for infinitely many n, then the probability that L is elected converges to 1 along this subsequence, which is therefore dominated by choosing a compromise location just below $\overline{x}_C^L$ - such a compromise will be elected with probability approaching 1. We conclude that the optimal compromise converges to $\overline{x}_C^L$.

(b) Assume now that $\alpha < 1$, and let $x_C(n)$ denote the optimal compromise. The proof is divided in several steps:

Step 1: For all n, it holds that $x_C(n) \leq 1 - 2\gamma_{-1}$. If $x_C(n) > 1 - 2\gamma_{-1}$ then the compromise would never be elected if $n_{-1} \geq n + 1$. It is then strictly better to choose $x_C(n) < 1 - 2\gamma_{-1}$ instead.

Step 2: For any $\varepsilon > 0$, $x_C(n) \leq -1 + 2\gamma_1 + \varepsilon$ for all n large enough. Assume, by contradiction, that there exists $\varepsilon > 0$ such that $x_C(n) > -1 + 2\gamma_1 + \varepsilon$ for infinitely many n, and consider the corresponding subsequence of compromise locations. We compare below

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any compromise $x_C \in (-1 + 2\gamma_1 + \varepsilon \ , \ 1 - 2\gamma_{-1})$ with the compromise location $-1 + 2\gamma_1$, and show that $-1 + 2\gamma_1$ is superior, which contradicts the optimality of $x_C(n)$.

The resulting outcomes differ in the following events: (a) x_C gets elected but a compromise located at $-1 + 2\gamma_1$ does not; (b) both compromises get elected; (c) compromise $-1 + 2\gamma_1$ gets elected but x_C does not.

By Lemma 1, event (a) can only occur if $n_{+1} \geq n + 1$, in which case R gets elected if the compromise is located at $-1 + 2\gamma_1$. Therefore, compromise $-1 + 2\gamma_1$ is strictly better than x_C in event (a). Event (c) can only occur if $n_{-1} \geq n + 1$, hence L gets elected in event (c) if the compromise is located at x_C. Therefore, compromise $-1 + 2\gamma_1$ is also strictly better than x_C in event (c). In event (b) both compromises get elected; hence, compromise x_C is strictly better in event (b) because $x_C > -1 + 2\gamma_1$. To show that, in expectation, compromise $-1 + 2\gamma_1$ is better than compromise x_C it therefore suffices to show that the probability of event (a) divided by the probability of event (b) grows without bound as n grows. The probability of event (a) is

$$\Pr\{n + 1 \leq \tilde{n}_{+1} \leq 2n + 1 - k(x_C)\} \tag{15}$$

and that the probability of event (b) is

$$\Pr\{\kappa(x_C) \leq \tilde{n}_{+1} \leq n\}$$

Let $\beta = \frac{k(2\gamma_1 - 1 + \varepsilon)}{n}$ where $k(x) = \frac{n}{2} \frac{1-x}{1-\gamma_1}$ and note that β does not depend on n. Since the function k is decreasing, $k(-1 + 2\gamma_1) = n$ and $x_C > -1 + 2\gamma_1 + \varepsilon$ imply that $\frac{k(x_C)}{n} \leq \beta < 1$. Since $\tilde{n}_{-1} + \tilde{n}_{+1} = 2n + 1$, the probability of event (i) satisfies

$$\Pr\{n + 1 \leq \tilde{n}_{+1} \leq 2n + 1 - k(x_C)\} \geq \Pr\{\tilde{n}_{-1} \geq \beta n\} - \Pr\{\tilde{n}_{-1} \geq n + 1\}.$$

Also, the probability of event (b) satisfies

$$\Pr\{\kappa(x_C) \leq \tilde{n}_{+1} \leq n\} \leq \Pr\{\tilde{n}_{-1} \geq n + 1\}.$$

Hence,

$$\frac{\Pr\{\text{event (a)}\}}{\Pr\{\text{event (b)}\}} \geq \frac{\Pr\{\tilde{n}_{-1} \geq \beta n\}}{\Pr\{\tilde{n}_{-1} \geq n + 1\}} - 1,$$

Since $\alpha < 1$, Lemma 2 implies that the term on the left side grows without bound as n goes to infinity.

Step 3: $x_C(n)$ converges to $\min\{-1 + 2\gamma_1 \ , \ 1 - 2\gamma_{-1}\}$.

Steps 1 and 2 imply then that $\limsup_n\{x_C(n)\} \leq \min\{-1 + 2\gamma_1 \ , \ 1 - 2\gamma_{-1}\}$. Suppose now that there exists $\varepsilon > 0$ such that $x_C(n) < \min\{-1+2\gamma_1 \ , \ 1-2\gamma_{-1}\}-\varepsilon$ for infinitely many n, and consider the corresponding subsequence. Let $x_1 := \min\{-1 + 2\gamma_1 \ , \ 1 - 2\gamma_{-1}\} - \frac{\varepsilon}{2}$. Neither $x_C(n)$ nor x_1 gets elected if $n_{-1} \leq n$, and we can focus on the event $n_{-1} \geq n + 1$. Note that x_1 gets elected whenever $n_{-1} \geq n + 1$ and n_{-1} is sufficiently close to n.

Since $\alpha < 1$ it follows from Lemma 2 that for all $\delta \in (0, 1)$, $\lim_{n\to\infty} \Pr\{\frac{\tilde{n}_{-1}}{n} > 1+\delta | \frac{\tilde{n}_{-1}}{n} > 1\} = 0$. This allows us to conclude that the probability with which x_1 gets elected, conditional on $n_{-1} \geq n + 1$, approaches 1, and the same holds for $x_C(n)$. Since x_1 is strictly better conditional on being elected, the compromise location x_1 is strictly superior to $x_C(n)$, which yields a contradiction. Therefore, $\liminf_n\{x_C\} \geq \min\{-1+2\gamma_1 \ , \ 1-2\gamma_{-1}\}$, which concludes the proof. ∎

Proof of Proposition 7. **(i).** Consider SV and suppose $\frac{1-2\gamma_{-1}}{1-\gamma_{-1}} \leq \alpha < 1$. For any x in the interior of X_C, the probability that x gets elected converges to 1. To see this, note that, for any $t > 0$, it follows from (8) that

$$\Pr\left\{x_C^R(n, \tilde{n}_{-1}) - \overline{x}_C^R \geq t\right\} = \Pr\left\{2(1 - \gamma_1)[\alpha - \frac{\tilde{n}_{-1}}{n}] \geq t\right\}$$

$$= \Pr\left\{\frac{\tilde{n}_{-1}}{n} - \alpha \leq -\frac{t}{2(1 - \gamma_1)}\right\} \leq e^{-\frac{2t^2}{4(2n+1)(1-\gamma_1)^2}}.$$

Therefore, any compromise $x > \overline{x}_C^R$ is elected with probability approaching 1 as n grows. By analogous arguments, any compromise $x \in (-1 + 2\gamma_1 \ , \ \overline{x}_C^L)$ is elected with probability approaching 1. For $x \notin X_C$, however, the probability of x being elected converges to 0.

Let $x_C(n)$ denote the optimal compromise location for a population of $2n + 1$ voters. We assume by contradiction that $\lim_{n\to\infty} x_C(n) \neq \arg\min_{x\in X_C} |x - x^*|$.

Suppose first that $x^* \in X_C$, but there exists $\varepsilon > 0$ such that $x_C(n) > x^* + \varepsilon$ for infinitely many n (the argument is analogous if $x_C(n) < x^* - \varepsilon$ for infinitely many n). Then, there is x in the interior of X_C that is sufficiently close to x^* such that the utility of the majority party if x gets elected is strictly higher than the one when $x^* + \varepsilon$ is elected, and also strictly higher than the one when R is elected. Since the probability that x gets elected converges to 1, and since the majority party's utility is single-peaked, we conclude that, for n large enough, it is strictly better to propose x than to propose any compromise above $x^* + \varepsilon$. Since $x_C(n)$ is optimal by assumption, this yields a contradiction.

Suppose now that $x^* \notin X_C$ and that $\arg\min_{x\in X_C} |x - x^*|$ is a singleton, denoted by x'.

To obtain a contradiction, suppose $x_C(n) > x' + \varepsilon$ or $x_C(n) < x' - \varepsilon$ for infinitely many n, and let x be in the interior of X_C and sufficiently close to x'. Then x gets elected with probability approaching 1 and, conditional on being elected, provides strictly greater utility compared to $x' + \varepsilon$ and compared to R. It follows that, for n large enough, it is strictly better to propose compromise x than to propose compromise $x_C(n)$, a contradiction.

(ii). Consider now AV, and observe that, if $\alpha < 1$, any $x > \overline{x}_C^R$ gets elected with probability approaching 1, and any $x < \overline{x}_C^R$ gets elected with probability approaching 0. Similarly, if $\alpha > 1$, any $x < \overline{x}_C^L$ gets elected with probability approaching 1, and any $x > \overline{x}_C^L$ gets elected with probability approaching 0.

Assume that $\alpha < 1$, and suppose that there exists $\varepsilon > 0$ such that $x_C(n) < \max\{x^*, \overline{x}_C^R\} - \varepsilon$ for infinitely many n. The probability that the compromise $x \equiv \max\{x^*, \overline{x}_C^R\} + \delta$ with $\delta > 0$ gets elected approaches 1. If $x^* \geq \overline{x}_C^R$ then, conditional on being elected, the compromise x provides strictly higher utility than either $\max\{x^*, \overline{x}_C^R\} - \varepsilon$ or R if δ is small enough. For n large enough, it is therefore strictly better to propose compromise x than to propose $x_C(n)$. If $x^* < \overline{x}_C^R$, the probability that $x_C(n)$ gets elected approaches 0, and it is again better to propose compromise x for sufficiently small δ, a contradiction.

Suppose now that there exists $\varepsilon > 0$ such that $x_C(n) > \max\{x^*, \overline{x}_C^R\} + \varepsilon$ for infinitely many n. The probability that the compromise gets elected along this subsequence approaches 1. Similarly, the probability that the compromise $\max\{x^*, \overline{x}_C^R\} + \frac{\varepsilon}{2}$ gets elected also approaches 1. Since the utility gain of compromise $\max\{x^*, \overline{x}_C^R\} + \frac{\varepsilon}{2}$ compared to compromise $x_C(n)$ is strictly positive and remains bounded away from 0, we conclude that compromise $\max\{x^*, \overline{x}_C^R\} + \frac{\varepsilon}{2}$ is strictly better than $x_C(n)$ for n large enough, a contradiction.

We conclude that $\lim_{n\to\infty} \max\{x^*, \overline{x}_C^R\}$ if $\alpha < 1$. The arguments are similar for $\alpha > 1$. ∎

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