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Heterogeneity within communities: A stochastic model with tenure choice

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Abstract

Standard explanations for the observed income heterogeneity within US communities rely on heterogeneity of the housing stock and differences of preferences across households. We propose a dynamic stochastic model of location and tenure choice where homes are identical within locations and households initially differ according to income only. The model highlights how differences in the timing of moves generate income heterogeneity across homeowners, in particular within communities that experience strong positive demand shocks. Using US Census data, we provide evidence of the relevance of this income mixing mechanism. Our empirical findings suggest that incorporating information on time since moved and tenure choice may be useful in the estimation of equilibrium sorting models and of households' willingness to pay for local amenities, especially so in communities with a history of strong housing price growth.

JEL classification: D31; R12; R21

Keywords: Equilibrium sorting; Income mixing; Housing demand; Tenure choice

1. Introduction

There is considerable income heterogeneity within neighborhoods. Epple and Sieg (1999) estimate that 89 percent of the income variance in the Boston metropolitan area in 1980 could be explained by within-community variance. Davidoff (2005) finds that only 6 percent of the variation of household incomes within US metropolitan areas could be explained by differences across jurisdictions in 1990. Hardman and Ioannides (2004) report that in 1993 more than two thirds of US metropolitan neighborhoods in the American Housing Survey included at least one household with income in the bottom quintile of the metropolitan income distribution; more than half the neighborhoods had at least one household with income in the top quintile.[1]

Standard explanations for income mixing rely on dimensions of heterogeneity beyond that of household income, e.g., heterogeneity of households' place of work

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[1] For further evidence of the same sort, see Wheeler and La Jeunesse (2006) and the references therein.

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(Brueckner, 1994), heterogeneity of preferences for local amenities (Epple and Platt, 1998), or heterogeneity of the housing stock (Nechyba, 2000).

We propose an alternative explanation that is motivated by the following observation. In central London, there are taxicab drivers who live in neighborhoods that are so expensive that today only investment bankers can afford to move there. What distinguishes these taxicab drivers from their peers is that they bought their homes several years ago, at a time when housing in these neighborhoods was much more affordable. Had these taxicab drivers chosen to rent, they would have moved out due to a local rent growth that far outpaced their own income growth. But since they own their home, they have enjoyed substantial capital gains. This wealth increase more than makes up for the income gap with the young investment bankers who have become their neighbors in recent years.

We show that income mixing of this nature arises as the equilibrium outcome of a model where tenure choice is driven by concerns over housing expenditure risk and housing prices fluctuate in response to stochastic population growth. The insights from the model then lead us to uncover new empirical regularities concerning the drivers of income heterogeneity within neighborhoods. For example, in locations with a history of strong housing price growth, we find a significant positive correlation between the heterogeneity of homeowners' incomes and the heterogeneity of the times since they bought their homes. Our results suggest that incorporating information on time since moved and tenure choice may be useful in the estimation of equilibrium sorting models and the inference of households' willingness to pay for neighborhood attributes.

Our model has two locations and two periods. The locations differ in the amenities they provide to their residents and in their housing supply elasticities. Native households populate the model from the start. With some probability, newcomers appear in the second period. In equilibrium, second-period housing rents in the more desirable location depend on whether or not the city population grows.

The model features a single dimension of exogenous household heterogeneity: the size of a lump-sum endowment that we interpret as life-time income or human capital. In the second period, once the uncertainty about the arrival of newcomers is resolved, the equilibrium problem reduces to a standard static sorting problem à la Tiebout (1956) with total wealth as the relevant state variable. For native households, wealth is human capital minus the cost of first-period housing. As this cost depends on the location and tenure choice in the first period, there is a second dimension of (endogenous) household heterogeneity: households are differentiated by their previous housing market experience, and especially by the past enjoyment of capital gains. For newcomers, whom we interpret to be young households who have not accumulated any assets yet, total wealth simply coincides with human capital.

We assume that native households choose whether to own or rent a home in the first period motivated by concerns over housing expenditure risk. In equilibrium, the only natives who buy a home in the more desirable location in the first period are those who plan to remain there independently of the population shock. This makes their second-period wealth risky, but perfectly insures them against the shock: wealth is higher precisely when newcomers move in and rents rise.

For both natives and newcomers, there is a common critical level of second-period wealth such that households with wealth above that level choose the more desirable location, while all others choose the other location. When the arrival of newcomers generates a sufficiently large rent rise in the more desirable location, some native homeowners realize capital gains that lift their wealth above the critical level even though their human capital is below it. By contrast, the newcomers who become their neighbors all have human capital above this threshold, and so their human capital exceeds that of the poorest native homeowners.

This result suggests that locations with a history of strong housing price growth should exhibit a positive correlation between the heterogeneity of homeowners' incomes and the heterogeneity of the times when they bought their homes. But the model neglects the complication that the newcomers may have financial wealth in addition to human capital just as natives homeowners do at the start of period 2. For example, if newcomers have moved from markets with strong housing price growth, they may have as much home equity on top of their incomes as the natives. The model further neglects the life-cycle profile of earnings and the heterogeneity of properties within locations.

This raises a number of empirical questions: Does the heterogeneity in times since moved predict income heterogeneity among homeowners in locations with a history of strong housing price growth? Is it the case that in such locations, homeowners who bought more recently tend to have higher incomes? Are locations that have not experienced housing price growth different in this respect? Does the income heterogeneity of renters follow different patterns?

Starting from the 5 percent sample of the 2000 US Census, we focus our empirical analysis on the 1351 urban Public Use Micro Areas (PUMAs) that we can match to metropolitan areas covered by the Freddie Mac housing price index. The housing price data starts in 1975; as an indicator of housing price growth, we take the growth over the period 1975–1999. We split the metropolitan areas into four quartiles according to the price growth they experienced. This gives us four sub-samples of PUMAs differentiated along the dimension relevant to the model.

We first study the correlation between income heterogeneity and heterogeneity of time since moved among homeowners. We regress income heterogeneity on heterogeneity of time since moved and control variables that capture the heterogeneity of the age of the heads of households and the heterogeneity of property values. We find a significant positive correlation between income heterogeneity and heterogeneity of time since moved in the sub-sample of PUMAs whose price growth over the period 1975–1999 lies in the top quartile.

In the model, the relationship between heterogeneity of income and of time since moved arises because the most recent movers to a location with strong housing price growth have higher incomes than their neighbors. In the data, we find that households who bought a home more recently than their neighbors have a higher income relative to their neighbors, holding constant the age of the head of the household and the property value. Furthermore, the regression coefficient on differences in time since moved is larger the greater the local housing price growth over the past 25 years.

We do not find the same empirical patterns for renters. In particular, the renter households who move in more recently have *lower* incomes, holding constant the age of the head of the household and the monthly rent. Moreover, the history of local housing price growth does not seem to matter for any of our estimates on renters.

We abstract from transaction costs in the model. Transactions costs would also generate hysteresis in the allocation of properties across households. Absent any wealth effect, if transaction costs were the drivers of the income heterogeneity we observe, we would expect a positive relationship between income heterogeneity and heterogeneity of time since moved in places that have experienced *weak* housing price growth. In places that have experienced strong housing price growth, we would expect transaction costs to be irrelevant, and hence no relationship between income heterogeneity and heterogeneity of time since moved. This goes counter to our empirical finding that the relationship between the heterogeneity of homeowners' incomes and the heterogeneity of time since moved is strongest in the locations that have experienced the largest price growth.

We are not the first to study a dynamic sorting model. Bénabou (1996a, 1996b), Durlauf (1996) and Fernandez and Rogerson (1996, 1998) propose dynamic sorting models to analyze macroeconomic and policy issues. They assume that the benefits of living in a community depend on the make-up of the community and are therefore determined endogenously. The same is true in static models that determine the benefits of each community by a political equilibrium; see, for example, Epple et al. (1984, 1993). Common to all these models is that households make only one location decision in equilibrium, either by assumption or because of a focus on stationary environments. We instead take the amenities of a community as given, but we allow households to relocate and to choose whether to own or rent their property in the face of endogenous fluctuations in housing costs.

What distinguishes the more expensive community in our model is a combination of greater desirability and a more inelastic housing supply. Gyourko et al. (2004) find that the households that move to desirable cities with inelastic housing supplies tend to be richer than the households already living in these cities. Although our discussion is cast in terms of communities within the same urban area, our arguments seem to apply equally to cities within a country.

As in Ortalo-Magné and Rady (2002), we focus on tenure choice driven by concerns over future housing expenditure risk. Davidoff (2006), Diaz-Serrano (2005), Han (2006) and Hilber (2005) provide evidence of the relevance of this driver of tenure choice. Our two-period model captures the idea that, at short horizons, household concerns over period-to-period rent risk are dominated by concerns over end-of-holding-period price risk, and vice versa at long horizons (Sinai and Souleles, 2005). From a modeling standpoint, the innovation in the present paper relative to this literature is that we cast such tenure concerns within an equilibrium model of the housing market and we explore the implications for local income distributions.

Our results contribute to the empirical literature concerned with the analysis of housing demand and households' preferences, in particular the literature that builds on static equilibrium sorting models; e.g., Bajari and Kahn (2005), Bayer at al. (2005, 2007), Calabrese et al. (2006), Sieg et al. (2004). Data requirements with regards to housing consumption restrict these studies to cross-sectional data sets with no ability to track households over time. These data sets provide household income but not household wealth. This is the case with the widely used Census data, for example. As a result, it is common for researchers to approach the data through the lens of a static model of housing choice constrained by income. But a household's housing choice is the outcome of a dynamic optimization constrained by wealth, not income. The typical empirical approach therefore suffers from the fact that income is a poor predictor of a household's wealth; e.g., Kennickell (1999). This paper offers a partial remedy.

2. The model

There are two periods, 1 and 2, and two communities, 0 and 1. In community 0, the supply of homes is perfectly elastic at a constant rent normalized to zero. In community 1, there is a measure S of identical homes owned initially by absentee landlords. For simplicity, the landlords are assumed to be risk-neutral. They discount rents at the same exogenous interest rate at which households can borrow and save. Without loss of generality, we assume that this interest rate is zero.

Initially, the area is populated by a measure one of households that we call the natives. Natives derive additively separable utility from the consumption of housing and a numeraire good. There is no discounting of utility across periods. Community 1 is more desirable than community 0: housing utility derived from a home in community 0 is normalized to zero, whereas a home in community 1 yields an additive utility premium of $\mu > 0$ per period, whether the home is owned or rented.

The numeraire good is enjoyed at the end of period 2 only. The utility derived from consumption of c units of the numeraire good is described by the constant absolute risk aversion function $U(c) = -e^{-c}$ where the coefficient of absolute risk aversion is assumed to be 1 to economize on notation. Within each period, trading takes place before consumption.

There is uncertainty in period 2. With probability $\pi \in (0, 1)$, state H occurs: a strictly positive measure ν of newcomer households moves to the area at the start of period 2. With probability $1-\pi$, state L occurs: nobody moves in. Although the shock is asymmetric by design, we will see later that from the point of view of the natives, it amounts to either a rent increase (state H), or a rent decline (state L). Our specific modeling choice for the shock is motivated by our interest in the allocation of homes between households that had the opportunity to buy their homes early and those who move in later.

Each native household is characterized by a total life-time income (or human capital) of $W \geqslant 0$ units of the numeraire. The distribution of these incomes has a strictly positive density on $(0, \infty)$. The corresponding cumulative distribution function is $F : [0, \infty] \to [0, 1]$. We assume perfect capital markets, so the household faces a single budget constraint: life-time expenditures on housing and numeraire consumption cannot exceed W.

The distribution of the incomes of newcomer households also has a strictly positive density on $(0, \infty)$. The corresponding cumulative distribution function is $\tilde{F} : [0, \infty] \to [0, 1]$. Newcomers do not possess any assets besides their human capital.

For ease of exposition, we assume $S < \frac{1}{2}$ throughout. This limits the number of cases we will have to consider without taking anything away from the results.

2.1. Tenure choice

Whether a household owns or rents a home in community 0, the cost is nil by assumption. Since we also assume that housing utility does not depend on tenure, all households are indifferent between renting and owning a home in community 0.

Tenure matters for homes in community 1. We denote R_1 their rent in period 1; R_H in period 2, state H; and R_L in period 2, state L. The expected second-period rent is $\bar{R}_2 = \pi R_H + (1-\pi) R_L$. The period 1 price of a home in location 1 is denoted p_1. Since period 2 is the last period of the economy, renting a home in period 2 is equivalent to buying it, so the price of a home in period 2 coincides with the rental cost of that home in period 2.

A native household's *location plan* is denoted by (h_1, h_H, h_L), where h_1, h_H and h_L take the value of 1 for community 1, and 0 for community 0. To indicate the tenure choice when $h_1 = 1$, we denote the combined *location-tenure plan* by $(1_B, h_H, h_L)$ if the household buys a home, and $(1_R, h_H, h_L)$ if it rents one. Fig. 1 summarizes the location-tenure choices available to a native household.

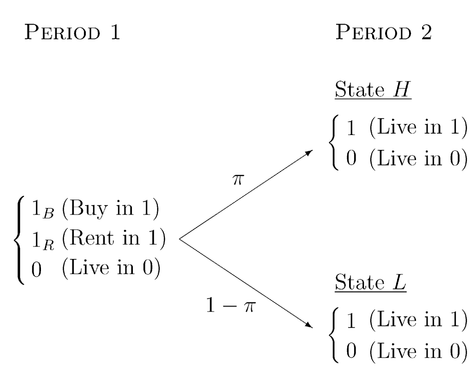

Fig. 1. Native households' housing choices.

There are eight location plans. For the four location plans that involve living in location 1 in period 1, native households must decide whether to buy or rent. So natives choose among twelve location-tenure plans.

We analyze this problem making two assumptions: $R_L < R_H$ and $p_1 = R_1 + \bar{R}_2$. The first reflects the price pressure that newcomers exert in state H. The second is an arbitrage condition for landlords when there are both renters and owners in location 1 in the first period.[2]

Tenure choice affects how shocks to the housing markets translate into shocks to the household's cost of housing and then through the budget constraint into shocks to non-housing consumption. It is easy to check that expected numeraire consumption is independent of tenure choice when $p_1 = R_1 + \bar{R}_2$. The tenure decision thus reduces to choosing the option that produces the smallest absolute difference between the numeraire consumption levels in the two states of the economy.

For the location plans with a deterministic horizon in the type 1 home, $(1, 0, 0)$ and $(1, 1, 1)$, tenure choice is obvious as one of the tenure modes provides full insurance and the other does not. A household that rents in period 1 and moves to location 0 in period 2 does not suffer any shock to its numeraire consumption. A household that buys in period 1 and remains in location 1 in period 2 does not face any numeraire consumption risk either. The plans $(1_R, 0, 0)$ and $(1_B, 1, 1)$ therefore dominate the plans $(1_B, 0, 0)$ and $(1_R, 1, 1)$, respectively.

Under the location plans $(1, 1, 0)$ and $(1, 0, 1)$, however, either tenure mode imposes some risk on the household. Under $(1, 1, 0)$, if the household rents, it pays the rent R_H in state H and no rent in state L; its numeraire consumption is lower by R_H in state H than in state L. If the household buys in the first period, it sells the home if the state L occurs. The price of a location 1 home in state L is R_L. The household's numeraire consumption is therefore lower by R_L in state H than in state L if it buys in period 1. Buying is thus less risky when R_L is lower than R_H. The location-tenure plan $(1_B, 1, 0)$ therefore dominates the plan $(1_R, 1, 0)$. Under $(1, 0, 1)$, the logic is reversed: the location-tenure plan $(1_R, 0, 1)$ dominates the plan $(1_B, 0, 1)$.

We summarize these findings in

Lemma 1. *If $R_L < R_H$ and $p_1 = R_1 + \bar{R}_2$, a native household wanting to live in location 1 in the first period prefers to own its home if and only if it plans to stay in location 1 should state H occur in the second period.*

[2] Lemma A.2 in Appendix A confirms that both assumptions are correct in equilibrium.

2.2. Location choice

We are left with eight plans to consider: $(0, 0, 0)$, $(0, 0, 1)$, $(0, 1, 0)$, $(0, 1, 1)$, $(1_R, 0, 0)$, $(1_R, 0, 1)$, $(1_B, 1, 0)$, and $(1_B, 1, 1)$. Each of these plans determines a curve in the plane with coordinates W (the household's life-time income) and EU (the expected overall utility level). Determining the optimal plan for every W amounts to characterizing the upper envelope of the expected utility curves.

First, using CARA utility, it is easy to verify that the preference ranking of the plans $(1_R, 0, 0)$ and $(0, 1, 1)$ does not depend on the household's income. In other words, the expected utility curves associated with these two plans are either identical or do not intersect. Both plans generate the same utility of housing, μ; their ranking is determined by the cost difference alone. We thus have

Lemma 2. *The plan $(1_R, 0, 0)$ weakly dominates $(0, 1, 1)$ if and only if*

$$e^{R_1} \leqslant \pi e^{R_H} + (1 - \pi) e^{R_L}, \qquad (*)$$

with a strict preference if the inequality is strict.

Second, we find that the plans $(0, 1, 0)$ and $(1_B, 1, 0)$ are not chosen by any native household.

Lemma 3. *If $R_L < R_H$ and $p_1 = R_1 + \bar{R}_2$, each native household chooses one of the location-tenure plans $(0, 0, 0)$, $(0, 0, 1)$, $(0, 1, 1)$, $(1_R, 0, 0)$, $(1_R, 0, 1)$ and $(1_B, 1, 1)$.*

To see this, note that the location choice in period 2 obeys a simple cutoff rule in terms of period 2 wealth, W', which is total income minus the cost of housing consumed in period 1. In state $s \in \{H, L\}$, a household with wealth W' strictly prefers location 1 if and only if $W' > W'_s$ where $-e^{-(W'_s - R_s)} + \mu = -e^{-W'_s}$ or $\mu e^{W'_s} = e^{R_s} - 1$. If $R_L < R_H$, then $W'_L < W'_H$.

Households that spend period 1 in location 0 have the same wealth at the start of period 2 in either state. If this wealth is such that they choose location 1 in state H, they obviously also choose location 1 in state L. This rules out the plan $(0, 1, 0)$.

Households that buy a location 1 home in period 1 enjoy gains or losses depending on the state in period 2. The difference between the corresponding second-period wealth levels is $R_H - R_L$. As

$$W'_H - W'_L = \ln\left(e^{R_H} - 1\right) - \ln\left(e^{R_L} - 1\right) > \ln\left(e^{R_H}\right) - \ln\left(e^{R_L}\right) = R_H - R_L$$

by the concavity of the logarithm, the inequality $W' > W'_H$ implies $W' > W'_L$. This rules out the plan $(1_B, 1, 0)$.

The newcomers appear in the second period only if state H occurs. Any newcomer with income above W'_H chooses location 1, all others choose location 0.

2.3. Equilibrium

An equilibrium is a triple of rents, (R_1, R_H, R_L), and a period 1 price, p_1, for homes in location 1, together with a location-tenure plan for each native household and a location choice for each newcomer, such that

- (i) each property in location 1 is either sold in period 1 or rented out in both periods, with all landlords choosing optimally between these alternatives,
- (ii) each household's utility is maximized given its budget constraint and the prices and rents of homes in location 1.

Proposition 1. *There is a unique equilibrium. The equilibrium rents satisfy $R_L < R_1 < R_H$. The equilibrium price of location 1 homes in the first period is $p_1 = R_1 + \bar{R}_2$, so landlords are indifferent between selling and renting out their properties. There is a unique size $\nu^* > 0$ of the newcomer cohort such that condition $(*)$ holds as an equality for all $\nu \leqslant \nu^*$, and as a strict inequality for all $\nu > \nu^*$. Native households' equilibrium choices are characterized by critical income levels $0 < W_1 < W_2 < W_3 < W_4$ such that*

- *all native households with income smaller than W_1 choose* $(0, 0, 0)$;
- *all native households with income between W_1 and W_2 choose* $(0, 0, 1)$;
- *if $(*)$ holds as a strict inequality, all native households with income between W_2 and W_3 choose* $(1_R, 0, 0)$;
- *if $(*)$ holds as an equality, more than half of all native households with income between W_2 and W_3 choose $(1_R, 0, 0)$, and the rest* $(0, 1, 1)$;
- *all native households with income between W_3 and W_4 choose* $(1_R, 0, 1)$;
- *all native households with income greater than W_4 choose* $(1_B, 1, 1)$.

All newcomers with income greater than W'_H choose community 1 in state H, all others community 0.

Proof. See Appendix A.1. The formulas for the income cutoffs are given in Appendix A.4. □

The inequality $R_L < R_H$ reflects the price pressure newcomers exert when they appear in state H. That the opportunity cost of choosing community 1 in the first period, R_1, lies strictly in between R_L and R_H is then dictated by market clearing. Intuitively, the cost of living in community 1 in period 1 cannot be too different from the cost of living in community 1 in period 2 for sure, a cost that lies in between R_L and R_H.

How much price pressure newcomers exert depends on the size of their cohort, ν. If it is large enough, location 1 is sufficiently expensive in state H for the plan $(1_R, 0, 0)$ to strictly dominate $(0, 1, 1)$.

Fig. 2 summarizes native households' choices in equilibrium for this case and graphs the mapping from total income to second-period wealth. A household's income, W, is on the horizontal axis. Wealth at the time when the housing market opens in period 2, W', is on the vertical axis; it equals W minus the cost of housing consumed in period 1. Up to $W = W_2$, W' equals W. From $W = W_2$ to W_3, W' equals W minus first-period rent R_1 for those households that choose $(1_R, 0, 0)$, and W for those that choose $(0, 1, 1)$. From $W = W_3$ to W_4, W' equals W minus first-period rent. For W above W_4, W' depends on the realization of the shock. In state H, $W' = W - p_1 + R_H$; in state L, $W' = W - p_1 + R_L$.

The figure also shows the critical wealth levels W'_H and W'_L that determine second-period location choice. By time consistency, the second-period wealth of the poorest native households that follow plan $(1_B, 1, 1)$ must be at least W'_H in state H. In fact, we have the strict inequality $W_4 - p_1 + R_H > W'_H$. To prove it, suppose that $W_4 - p_1 + R_H = W'_H$. The households with income W_4 are then indifferent between the plans $(1_B, 0, 1)$ and $(1_B, 1, 1)$. However, we know from Lemma 1 that the plan $(1_R, 0, 1)$ dominates the plan $(1_B, 0, 1)$. This is inconsistent with the definition of W_4.

Similarly, by time consistency, the second-period wealth of the poorest native households that follow plan $(1_B, 1, 1)$ must be at least W'_L in state L. Again, we have the strict inequality $W_4 - p_1 + R_L > W'_L$. To prove it, suppose that $W_4 - p_1 + R_L = W'_L$. The households with income W_4 are then indifferent between the plans $(1_B, 1, 0)$ and $(1_B, 1, 1)$. However, Lemma 3 implies that households that are indifferent between the plans $(1_B, 1, 0)$ and $(1_B, 1, 1)$ strictly prefer the plan $(1_R, 0, 1)$ to the plan $(1_B, 1, 1)$. This is inconsistent with the definition of W_4.

The second-period wealth of the poorest native households that buy a location 1 home in period 1 is thus strictly larger than the wealth of the poorest newcomers who move to location 1 in state H. While a newcomer's wealth consists entirely of human capital,

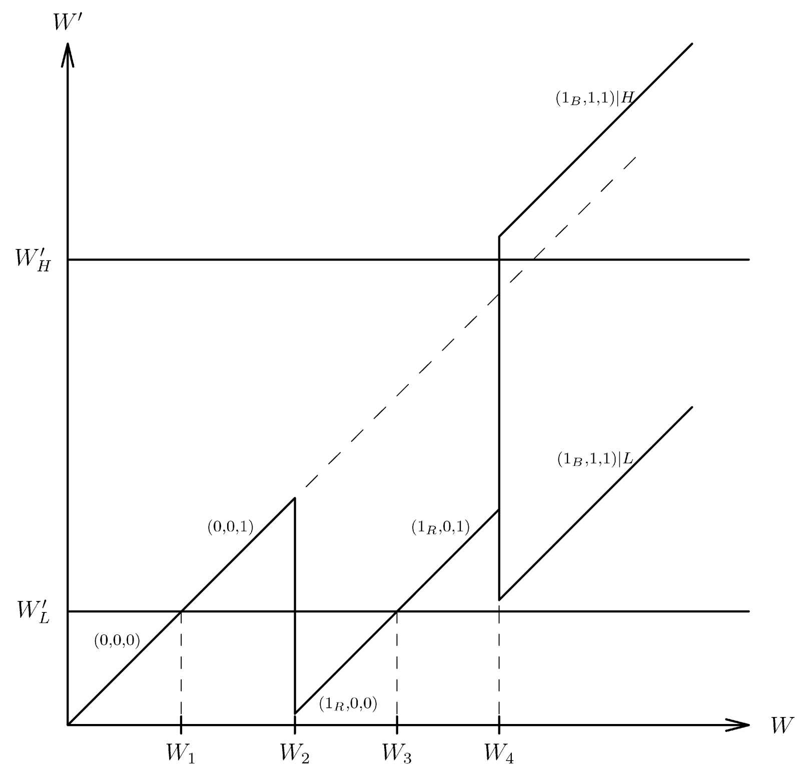

Fig. 2. Equilibrium location choices and second-period wealth of native households.

a native household's wealth is human capital minus the cost of housing occupied in period 1. As long as the newcomers exert only mild price pressure on the housing market in location 1 (and so there are no capital gains), this cost is strictly positive, implying the following result for the *income* of the poorest natives who buy in location 1.

Proposition 2. *For sufficiently small size ν of the newcomer cohort, the income of the poorest native households that buy a location 1 home in period 1 is larger than the income of the poorest newcomers that move to location 1 in state H; i.e., $W_4 > W'_H$.*

Proof. See Appendix A.5. □

When the wealth of the poorest natives who buy in location 1 is boosted by sufficiently large capital gains on their homes, however, their income can be strictly *smaller* than the income of the poorest newcomers who move to location 1 in state H. This is the situation depicted in Fig. 2. We find that it arises if the price pressure from newcomers is sufficiently large.

Proposition 3. *For sufficiently large size ν of the newcomer cohort, the income of the poorest native households that buy a location 1 home in period 1 is smaller than the income of the poorest newcomers that move to location 1 in state H; i.e., $W_4 < W'_H$.*

Proof. See Appendix A.6. □

Proposition 3 allows us to rationalize our motivating observation about central London. The taxicab drivers moved in when housing prices were low relative to their incomes. By buying a home, they insured themselves against their incomes not growing as fast as local rents. Those newcomers whose wealth consists primarily of their human capital need substantial earning power to move next to the old taxicab drivers. This explains why the young people who move next to the old taxicab drivers are investment bankers and not university professors.[3]

3. Empirical evidence

Our model is very stylized and abstracts from a number of considerations that are relevant to housing decisions (e.g., transactions costs) and income heterogene-

[3] Both authors struggled to find housing in central London at the start of their academic career.

ity (e.g., life-cycle effects). Nevertheless, it suggests a plausible channel through which housing price growth may increase local income heterogeneity. To address the empirical relevance of this channel, we now ask the following empirical questions: Does the heterogeneity in times since moved predict income heterogeneity among homeowners in locations with a history of strong housing price growth? Is it the case that in such locations, homeowners who bought more recently tend to have higher incomes? Are locations that have not experienced housing price growth different in this respect? Does the income heterogeneity of renters follow different patterns?

To answer these four questions, we turn to the household data from the 2000 5% Census sample (Ruggles et al., 2004). To identify locations, we use the smallest geographic unit that is identifiable in this data set, the Public Use Microdata Area (PUMA). We are able to match 1351 PUMAs to metropolitan areas for which Freddie Mac has been publishing quality-adjusted housing price indices since 1975. We compute real housing price growth between the first quarter of 1975 and the fourth quarter of 1999 using the CPI-US index for all urban consumers from the Bureau of Labor Statistics as the price deflator.

We group the PUMAs into four quartiles according to their history of housing price growth over the period 1975–1999. The price growth cutoff points are 4.2%, 19.5% and 58.2%. The highest price growth in the sample was 188% for a PUMA located in San Jose, CA.[4] Note that we classify PUMAs according to ex post housing price appreciation, ignoring potential differences in ex ante expectations about housing price growth. Households in each PUMA certainly formed different expectations about future housing price growth in their location. From the point of view of the model, it does not matter whether a history of weak housing price growth is a disappointing outcome within the set of anticipated future outcomes (a state L realization in the language of the model) or the strongest outcome in a set of anticipated outcomes that did not include any possibility of strong price growth (a weak state H with low ν in the language of the model). Unless past shocks brought about strong capital gains for homeowners, the mechanism highlighted in the model does not generate income heterogeneity correlated with heterogeneity of time since moved.

There are between 82 and 4467 homeowner households within each PUMA with a median of 1060. We measure income heterogeneity within each PUMA with the coefficient of variation of income in order to take out mean effects. From a theoretical point of view, it makes no difference whether households moved in one and two years ago or eleven and twelve years ago. We therefore use the standard deviation to measure the heterogeneity of the time since households moved to the neighborhood.

We control for two additional factors. First, income heterogeneity may arise as a function of time since moved because income varies systematically with age. Consider for example a world where all households buy their first home at the same age. Independent of any capital gains effect, a greater dispersion of times since moved is then likely to generate a greater dispersion of incomes simply because of a greater dispersion of ages. We therefore control for the standard deviation of the age of the head of household. Second, communities with more heterogeneous properties are likely to house more heterogeneous households. We therefore control for the coefficient of variation of property values. We choose the coefficient of variation in order to be consistent with our measure of income heterogeneity. Finally, we compute robust standard errors accounting for clustering at the MSA level.

Table 1 presents the results. We find no evidence of a positive relationship between income heterogeneity and heterogeneity of time since moved except for the PUMAs located in cities in the top price growth quartile. The greater the heterogeneity of the age of heads of households, the greater the income heterogeneity. The greater the heterogeneity of property values, the greater the income heterogeneity. An increase in the heterogeneity of property values by one standard deviation increases income heterogeneity by slightly more than half a standard deviation. The corresponding effect for the heterogeneity of time since moved in the top quartile is about half as big, and slightly larger than that for age heterogeneity.

The relationship between income heterogeneity and time-since-moved heterogeneity in the model is due to the fact that in markets with strong housing price growth, homeowners who moved in more recently have higher human capital than homeowners who moved in earlier. To check whether the same relationship holds in the data, we turn to household level data: we regress relative household income on the relative time since the household moved to the PUMA. We compute a household's relative income as the ratio of its income to the median household income for all homeowners who live in the same community. We compute a household's relative time since moved as the difference between the

[4] Appendix B reports definitions of the variables we use, sample sizes and summary statistics.

Table 1
Coefficient of variation of homeowners' income within PUMAs

	PUMAs			
	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
S.D. time since moved	−0.0046	−0.0116	0.0095	0.0184
	(0.0031)	(0.0101)	(0.0063)	(0.0046)*
S.D. age head	0.0237	0.0295	0.0184	0.0150
	(0.0030)*	(0.0103)*	(0.0065)*	(0.0059)*
C.V. home value	0.4549	0.4044	0.4127	0.4466
	(0.0401)*	(0.0347)*	(0.0382)*	(0.0373)*
Intercept	0.2097	0.2429	0.1824	0.1510
	(0.0392)*	(0.0677)*	(0.0879)*	(0.1032)
R^2	0.57	0.54	0.51	0.42

Robust standard errors in parentheses.
* Statistical significance at the 0.05 level.

Table 2
Homeowners' income relative to other homeowners in the same PUMA

	PUMAs			
	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Time since moved	−0.0011	−0.0021	−0.0023	−0.0056
	(0.0002)*	(0.0002)*	(0.0002)*	(0.0003)*
Age household head	−0.0074	−0.0079	−0.0082	−0.0084
	(0.0002)*	(0.0001)*	(0.0002)*	(0.0002)*
Home value	0.4924	0.5112	0.5349	0.6367
	(0.0077)*	(0.0107)*	(0.0105)*	(0.0134)*
Intercept	0.6750	0.6524	0.6328	0.5667
	(0.0080)*	(0.0109)*	(0.0106)*	(0.0118)*
R^2	0.20	0.19	0.18	0.18

Robust standard errors in parentheses.
* Statistical significance at the 0.05 level

time since the household bought its current home and the median time since homeowner households bought their current home in the community. Again, we control for age differences by including as covariates the difference between the age of the head of the household and the median age of household heads for the community. We also control for differences in property value by including the ratio of the value of the household's property to the median property value in the community. We compute robust standard errors, accounting for clustering at the PUMA level.

The regression results reported in Table 2 indicate that households who moved in more recently tend to have a higher income than households who moved in earlier. The coefficient on relative time since moved is larger in communities located in metropolitan areas that have experienced the strongest price growth. Households younger than their neighbors tend to have greater income. Households who own a more valuable property also tend to have greater income. As to economic significance, the effect of an increase of the relative property value by one standard deviation is about three times the corresponding effect for relative age, and about six times that for relative time since moved.

The insurance mechanism behind Proposition 3 applies to homeowners and not to renters. As a control, we therefore replicate the above regressions for renters. We compute the same income, time since moved and age measures for renters within each PUMA. We replace the coefficient of variation of property values and the relative property value with the coefficient of variation of gross monthly rents and relative gross monthly rent.

The results we obtain for the PUMA-level heterogeneity of income are reported in Table 3. We find a positive relationship between income heterogeneity and heterogeneity in the time since moved. However,

Table 3
Coefficient of variation of renters' income within PUMAs

	PUMAs			
	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
S.D. time since moved	0.0328	0.0213	0.0366	0.0337
	(0.0074)*	(0.0084)*	(0.0058)*	(0.0120)*
S.D. age head	−0.0095	−0.0162	−0.0032	−0.0130
	(0.0044)*	(0.0055)*	(0.0079)	(0.0090)
C.V. gross rent	0.3551	0.7928	0.3941	0.3249
	(0.0978)*	(0.1318)*	(0.1696)*	(0.1847)
Intercept	0.7654	0.7094	0.5711	0.7932
	(0.0646)*	(0.1112)*	(0.1202)*	(0.1447)*
R^2	0.14	0.18	0.26	0.13

Robust standard errors in parentheses.
* Statistical significance at the 0.05 level.

Table 4
Renters' income relative to other renters in the same PUMA

	PUMAs			
	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Time since moved	0.0104	0.0114	0.0113	0.0099
	(0.0006)*	(0.0005)*	(0.0006)*	(0.0006)*
Age household head	−0.0033	−0.0051	−0.0062	−0.0055
	(0.0003)*	(0.0003)*	(0.0003)*	(0.0003)*
Gross rent	0.7139	0.7060	0.7840	0.8361
	(0.0155)*	(0.0257)*	(0.0186)*	(0.0207)*
Intercept	0.5462	0.5622	0.4951	0.4133
	(0.0154)*	(0.0232)*	(0.0173)*	(0.0186)*
R^2	0.08	0.08	0.09	0.11

Robust standard errors in parentheses.
* Statistical significance at the 0.05 level.

the coefficients that we estimate are similar across all sub-samples. Differences in local housing price growth do not seem to affect the correlation between the heterogeneity of renters' incomes and that of time since moved. At the household-level, contrary to what we obtained for owners, Table 4 shows a positive relationship between differences in time since moved and relative income for renters. The renters who moved in more recently tend to have *lower* income than the renters who moved in earlier, ceteris paribus. Again, estimates for renters do not differ significantly across the four sub-samples.

PUMAs are large communities with the advantage of containing sufficiently many households for us to study the relationship between relative income and time since moved. Moreover, they are sufficiently large for metropolitan area housing prices to provide a good indicator of price growth over the long period we focused on. The neighborhood cluster samples of the American Housing Survey (AHS) offer an opportunity to examine the relationship between the heterogeneity of incomes and of time since moved at a much lower level of aggregation. For a description and detailed analysis of this survey data, see Ioannides (2004), who finds in particular that the coefficient of variation in neighborhood incomes increases with the *mean* time since moved. When we regress the coefficient of variation of incomes in AHS neighborhoods on the *standard deviation* of time since moved, the standard deviation of the age of heads of households and the coefficient of variation of property values, we find a significant positive coefficient on the standard deviation of time since moved for homeowners. For renters, this coefficient is not significant.[5]

[5] The results are available from the authors on request. Owing to lack of data, we cannot control for the heterogeneity of rents in the

4. Concluding remarks

The US Census data indicates a positive relationship between the heterogeneity of homeowners' incomes and the heterogeneity of the times since they bought their homes when the community is located in a city that has experienced strong price growth. The rationalization for this fact that is offered by the model is not rejected by the data: we find evidence that homeowners who moved in more recently have greater income than their neighbors, and more so in communities that have experienced strong price growth. This is all the more remarkable as the model abstracts from a number of phenomena that are likely to blur the relationship between income and time since moved in the data, such as the possibility that recent movers might also possess financial or housing wealth in addition to their human capital.

Our results contribute to the empirical literature concerned with the analysis of housing demand and households' preferences, in particular the literature that relies on data sets with no information on household wealth (e.g., US Census data). Some researchers restrict their samples to recent movers, usually motivated by concerns about housing consumption hysteresis because of moving costs. At the very least, our findings provide an additional justification for such a sample restriction, or for separating households according to time since moved.

More generally, our findings should encourage researchers to study the predictive power of differences in tenure choice and in time since moved. Time-since-moved information may indeed help in disentangling the contribution of wealth heterogeneity to observed housing and location choices from the contribution of preference heterogeneity. This point is particularly relevant for the numerous studies focused on coastal metropolitan areas that have experienced strong housing price growth over the past three decades (e.g., Boston, Los Angeles, San Francisco).

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Appendix A

A.1. Proof of Proposition 1

The proof draws on auxiliary results that are established in Sections A.2–A.4 of this appendix. Lemma A.2 shows that in equilibrium, $R_L < R_H$ and $p_1 = R_1 + \bar{R}_2$. Lemma A.3 shows that $(*)$ holds. Lemma A.4 shows that the equilibrium configuration must be as stated in the proposition. This implies that the relevant market clearing conditions are (A.10)–(A.13). Lemma A.5 shows that these conditions are equivalent to the system of Eqs. (A.14)–(A.17). Lemmas A.6 and A.7 show that this system admits a unique solution. Lemma A.8 shows that this solution yields an equilibrium. Lemma A.9 shows the existence of ν^*. $\square$

A.2. An auxiliary result on household behavior

To ease the notational burden, we define

$$e_1 = e^{R_1}, \quad e_H = e^{R_H}, \quad e_L = e^{R_L}, \quad e_2 = e^{\bar{R}_2}. \tag{A.1}$$

Lemma A.1. *Let $R_L < R_H$. Then:*

(i) *the plan $(1_R, 0, 1)$ is preferred over both $(1_R, 0, 0)$ and $(1_B, 1, 1)$ at all incomes in some set of positive measure*;
(ii) *at least one of the plans $(0, 0, 1)$ and $(1_R, 0, 0)$ is preferred over both $(0, 0, 0)$ and $(1_R, 0, 1)$ at all incomes in some set of positive measure*;
(iii) *the plan $(0, 0, 1)$ is preferred over both $(0, 0, 0)$ and $(0, 1, 1)$ at all incomes in some set of positive measure.*

Proof. Part (i): Let W_* be the income at which a native household would be indifferent between the plans $(1_R, 0, 0)$ and $(1_B, 1, 1)$, and $W_\dagger$ the income at which

regression for renters. The AHS data samples are too small to replicate the other regressions we carry out on the Census data, especially once we exclude neighborhoods for which we do not have metropolitan area price data.

it would be indifferent between the plans $(1_R, 0, 1)$ and $(1_B, 1, 1)$. To show that the plan $(1_R, 0, 1)$ is preferred to both $(1_R, 0, 0)$ and $(1_B, 1, 1)$ on a set of income levels of positive measure, it is enough to show that $W_* < W_\dagger$. To see this, note that if the expected utility curves of two plans cross, the curve associated with the plan that promises a larger amount of housing consumption in location 1 ex ante is steeper at all income levels. The curve associated with $(1_R, 0, 1)$ is above the curve associated with $(1_B, 1, 1)$ to the left of $W_\dagger$, and the latter is above the curve associated with $(1_R, 0, 0)$ to the right of W_*. If $W_* < W_\dagger$, therefore, $(1_R, 0, 1)$ is preferred to both $(1_R, 0, 0)$ and $(1_B, 1, 1)$ at all incomes strictly between W_* and $W_\dagger$.

It is straightforward to verify that the incomes W_* and $W_\dagger$ are defined by

$$\mu e^{W_*} = e_1(e_2 - 1), \tag{A.2}$$

$$\mu e^{W_\dagger} = e_1\left[e_2 - 1 + \frac{1-\pi}{\pi}(e_2 - e_1)\right]. \tag{A.3}$$

The inequality $W_* < W_\dagger$ is easily seen to be equivalent to $e_L < e_2$, which in turn is the same as $e_L < e_H$.

Part (ii): An argument similar to the one used for part (i) shows first that for $e_1 \leqslant e_L$, $(1_R, 0, 0)$ is preferred to $(0, 0, 0)$ and $(1_R, 0, 1)$ on some open interval of incomes; and second, that for $e_1 \geqslant e_L$, $(0, 0, 1)$ is preferred to $(0, 0, 0)$ and $(1_R, 0, 1)$ on some open interval of incomes.

Part (iii): Let $W_\#$ be the income level at which a native household would be indifferent between the plans $(0, 0, 0)$ and $(0, 1, 1)$, and $W_\flat$ the income level at which it would be indifferent between the plans $(0, 0, 1)$ and $(0, 1, 1)$:

$$\mu e^{W_\#} = \pi e_H + (1-\pi)e_L - 1, \tag{A.4}$$

$$\mu e^{W_\flat} = e_H - 1. \tag{A.5}$$

It suffices to show that $W_\# < W_\flat$. This is easily seen to be equivalent to $e_L < e_H$. □

A.3. Auxiliary results on equilibrium prices and configurations

In the following, we shall write D_1, D_H, and D_L for native households' aggregate demand for location 1 housing in period 1, period 2 state H and period 2 state L, respectively.

Lemma A.2. *In equilibrium,* $R_L < R_H$ *and* $p_1 = R_1 + \bar{R}_2$.

Proof. Suppose that $R_L \geqslant R_H$. Then the wealth cutoffs that determine second-period location choice satisfy $W_L' \geqslant W_H'$, and we have $D_L \leqslant D_H$ by backward induction as in the argument underlying Lemma 3. As a positive measure of newcomers demand housing in location 1, total demand for housing in location 1 is strictly higher in state H than in state L. This contradicts market clearing in at least one of these states.

Now suppose $p_1 < R_1 + \bar{R}_2$, so that no landlord is willing to sell a property in location 1. Owing to diminishing marginal utility of non-housing consumption, native households with sufficiently high income desire the location plan $(1, 1, 1)$, and since buying yields full insurance and is cheaper in expectation than renting for two periods, these households' optimum is $(1_B, 1, 1)$. This implies excess demand for owner-occupied homes in location 1 in the first period. So we must have $p_1 \geqslant R_1 + \bar{R}_2$.

Finally, suppose $p_1 > R_1 + \bar{R}_2$, so that no landlord is willing to rent out a property in location 1. Then there cannot be a positive measure of native households that desire the location plans $(1, 0, 0)$ or $(1, 0, 1)$, because for these households renting would be less risky and yield higher expected non-housing consumption than buying, so there would be excess demand for location 1 rental housing in the first period. As market clearing requires $D_1 = D_L$, the measure of native households that choose the location plans $(0, 0, 1)$ and $(0, 1, 1)$ must then also be zero. As the location plans $(0, 1, 0)$ and $(1, 1, 0)$ are ruled out by the backward induction argument underlying Lemma 3, only the plans $(0, 0, 0)$ and $(1, 1, 1)$ can be chosen by a positive measure of native households. This in turn implies $D_H = D_L$, which we have already shown to be incompatible with market clearing. □

Lemma A.3. *In equilibrium, the measure of native households that choose the plan* $(1_R, 0, 0)$ *is larger than the measure of native households that choose the plan* $(0, 1, 1)$. *As a consequence,* $(0, 1, 1)$ *cannot dominate* $(1_R, 0, 0)$, *so* $(*)$ *holds.*

Proof. If it were otherwise, Lemmas A.2 and 3 would imply $D_1 < D_L$, which is incompatible with market clearing. □

Lemma A.4. *In equilibrium, the location-tenure plans chosen by positive measures of native households are* $(0, 0, 0)$, $(0, 0, 1)$, $(1_R, 0, 0)$, $(1_R, 0, 1)$ *and* $(1_B, 1, 1)$ *plus possibly* $(0, 1, 1)$.

Proof. From Lemmas A.2 and 3, we know that the only plans that may be chosen by a positive measure of native households are $(0,0,0)$, $(0,0,1)$, $(0,1,1)$, $(1_R,0,0)$, $(1_R,0,1)$ and $(1_B,1,1)$. As the income distribution for native households has support $[0,\infty)$, we know that $(1_B,1,1)$ is chosen. (Here and in what follows, we interpret the word "chosen" to mean "chosen by a positive measure of native households.")

First, suppose $(0,0,0)$ is not chosen. Then, market clearing in period 1 implies $m_{001}+m_{011}=1-S$ where m_{001} denotes the measure of native households choosing $(0,0,1)$, and m_{011} the measure of native households choosing $(0,1,1)$. Market clearing in period 2 state L implies that the measure of native households choosing $(1_R,0,0)$ is $m_{100}=1-S$. Adding these two equations yields $m_{001}+m_{011}+m_{100}=2(1-S)>1$, which contradicts the fact that the total native population has size 1.

Second, suppose that $(1_R,0,1)$ is not chosen. By part (i) of Lemma A.1, $(1_R,0,0)$ is then not chosen either. By Lemma A.3, the same is true for $(0,1,1)$. Once these plans are eliminated, however, one either has $D_1=D_H<D_L$ or $D_1=D_H=D_L$ depending on whether $(0,0,1)$ is chosen or not. Both cases are incompatible with market clearing, which requires $D_1=D_L>D_H$. So $(1_R,0,1)$ must be chosen.

Third, suppose that $(1_R,0,0)$, and hence $(0,1,1)$, is not chosen. By part (ii) of Lemma A.1, $(0,0,1)$ is then chosen. However, this implies $D_1<D_L$, which is again incompatible with market clearing. So $(1_R,0,0)$ must be chosen.

Finally, suppose that $(0,0,1)$ is not chosen. By part (iii) of Lemma A.1, $(0,1,1)$ is then not chosen either. But then $D_1>D_L$, again in contradiction to market clearing. So $(0,0,1)$ must be chosen. □

A.4. Auxiliary results on equilibrium existence and uniqueness

Four critical income levels fully characterize native households' equilibrium choices. For indifference between $(0,0,0)$ and $(0,0,1)$, the critical income level is W_1 with

$$\mu e^{W_1}=e_L-1. \tag{A.6}$$

For indifference between $(0,0,1)$ and $(1_R,0,0)$, the critical income level is W_2 with

$$\mu e^{W_2}=\big[e_1-\pi-(1-\pi)e_L\big]/\pi. \tag{A.7}$$

For indifference between $(1_R,0,0)$ and $(1_R,0,1)$, the critical income level is W_3 with

$$\mu e^{W_3}=e_1(e_L-1). \tag{A.8}$$

For indifference between $(1_R,0,1)$ and $(1_B,1,1)$, the critical income level is W_4 with

$$\mu e^{W_4}=\big[e_1\big(e_2-\pi-(1-\pi)e_L\big)\big]/\pi. \tag{A.9}$$

Clearly, $W_1=W'_L$ and $W_3=W'_L+R_1$. Given our results on the set of possible equilibrium configurations, these critical income levels satisfy $0<W_1<W_2<W_3<W_4$.

For $k=1,\dots,4$, we let $i_k=F(W_k)$ denote the measure of native households with income lower than W_k. Thus, $0<i_1<i_2<i_3<i_4<1$. For the newcomers, $n_1=\tilde F(W'_H)$ denotes the measure of newcomer households with wealth lower than W'_H; it satisfies $0<n_1<1$. With this notation, the market clearing conditions for location 1 housing in period 1, period 2 state H and period 2 state L take the form

$$S=1-i_3+\rho(i_3-i_2), \tag{A.10}$$

$$S=1-i_4+(1-\rho)(i_3-i_2)+(1-n_1)\nu, \tag{A.11}$$

$$S=1-i_3+(1-\rho)(i_3-i_2)+i_2-i_1, \tag{A.12}$$

where ρ is the fraction of native households with income between W_2 and W_3 that choose $(1_R,0,0)$. By Lemma A.3, we have $\frac{1}{2}<\rho\leqslant 1$ and $(*)$. Moreover, Lemma 2 implies that

$$(1-\rho)\big[e_1-\pi e_H-(1-\pi)e_L\big]=0. \tag{A.13}$$

We write W_{1-S} for the $(1-S)$-quantile of the income distribution of native households; that is, $F(W_{1-S})=1-S$. We set $\psi=\mu e^{W_{1-S}}+1$.

Lemma A.5. *The system of Eqs.* (A.10)–(A.13) *is equivalent to the system of equations*

$$2(1-S)=i_1+i_3, \tag{A.14}$$

$$2(1-S)+\nu=i_2+i_4+\nu n_1, \tag{A.15}$$

$$e_1=\pi\min\{e_H,\psi\}+(1-\pi)e_L, \tag{A.16}$$

and

$$\rho=\frac{i_3-(1-S)}{i_3-i_2}. \tag{A.17}$$

Proof. Adding Eqs. (A.10) and (A.11), we obtain (A.15). Adding Eqs. (A.10) and (A.12), we obtain (A.14). Now, if $e_1<\pi e_H+(1-\pi)e_L$ then $\rho=1$ by (A.13). Eq. (A.10) then implies $i_2=1-S$ and $W_2=W_{1-S}$, which by the definition of W_2 yields

$$e_1=\pi\psi+(1-\pi)e_L<\pi e_H+(1-\pi)e_L. \tag{A.18}$$

If $e_1=\pi e_H+(1-\pi)e_L$ then $\rho\leqslant 1$ and the definition of W_2 becomes $\mu e^{W_2}=e_H-1$. Moreover, (A.10) implies $i_2 \leqslant 1 - S$, hence $W_2 \leqslant W_{1-S}$ and $e_H \leqslant \psi$. Therefore:

$$e_1 = \pi e_H + (1-\pi)e_L \leqslant \pi\psi + (1-\pi)e_L. \tag{A.19}$$

So Eq. (A.16) holds. Finally, rearranging (A.10) yields (A.17).

Conversely, Eq. (A.16) gives us two possible cases. First, if $\psi < e_H$, then (A.16) plus the definitions of W_2 and i_2 imply $i_2 = 1 - S$, which yields $\rho = 1$ by Eq. (A.17) and implies that Eqs. (A.10) and (A.13) hold. Then, replacing one term $1 - S$ by i_2 in Eqs. (A.15) and (A.14) yields Eqs. (A.11) and (A.12) for the case $\rho = 1$. Second, if $\psi \geqslant e_H$, then (A.16) implies that (A.13) holds. Using (A.17) to replace one term $1 - S$ in Eqs. (A.15) and (A.14) yields Eqs. (A.11) and (A.12). Rearranging (A.17) yields (A.10). □

For our next result, define $\underline{e}$ as the unique real number strictly exceeding 1 and satisfying the equality

$$2(1-S) = F\left(\ln\frac{\underline{e}-1}{\mu}\right) + F\left(\ln\frac{\underline{e}(\underline{e}-1)}{\mu}\right). \tag{A.20}$$

It is straightforward to see that $\mu + 1 < \underline{e} < \psi$. We write W_{1-2S} for the $(1-2S)$-quantile of the income distribution of native households and set $\phi = \mu \mathrm{e}^{W_{1-2S}} + 1$.

Lemma A.6. *Eqs.* (A.14) *and* (A.16) *yield e_1 and e_L as continuous functions of e_H on* $[1,\infty[$. *For $e_H < \psi$, e_1 is strictly increasing and e_L strictly decreasing in e_H, with $e_L = e_1 = e_H$ if and only if $e_H = \underline{e}$, and $\phi < e_L < \underline{e} < e_1 < e_H$ if $\underline{e} < e_H < \psi$. For $e_H \geqslant \psi$, e_1 and e_L do not vary with e_H, and $\phi < e_L < \underline{e} < e_1 < \psi$.*

Proof. By the definitions of i_1 and i_3, the right-hand side of (A.14) is strictly increasing in e_L and e_1. Eq. (A.14) thus defines e_L as a strictly decreasing function of e_1 which assumes the value ψ at $e_1 = 1$ and tends to ϕ as e_1 goes to infinity. Rearranging Eq. (A.16) into

$$(1-\pi)e_L = e_1 - \pi\min\{e_H, \psi\} \tag{A.21}$$

defines e_L as a strictly increasing function of e_1, given e_H. This function assumes a value strictly below 1 at $e_1 = 1$ and tends to infinity as e_1 does. This implies that for any given e_H, (A.14) and (A.16) determine unique values of e_1 and e_L with $\phi < e_L < \psi$. When $e_H < \psi$, an increase in e_H shifts the second function down and leaves the first unchanged; when $e_H \geqslant \psi$, an increase in e_H leaves both functions unchanged. Continuity is obvious.

Next, note that in the (e_1, e_L)-plane, the graph of the function defined by (A.21) cuts the 45 degree line from below at $e_1 = \min\{e_H, \psi\}$, while the graph of the function defined by (A.14) cuts the 45 degree line from above at $e_1 = \underline{e}$. Using these facts, it is easy to verify the statements about the ranking of e_1, e_H and e_L. □

Lemma A.7. *The system of Eqs.* (A.14)–(A.16) *has a unique solution (e_1, e_H, e_L) in* $[1,\infty[^3$. *This solution satisfies $e_H > \underline{e}$ and $e_L < e_1 < e_H$. Moreover, e_H is strictly increasing in ν with $e_H \to \underline{e}$ as $\nu \to 0$, and $e_H \to \infty$ as $\nu \to \infty$.*

Proof. We want to establish that Eq. (A.15) admits a unique solution e_H once e_1 and e_L are solved for as functions of e_H according to Lemma A.6. First, we note that i_2 is strictly increasing in e_1 and strictly decreasing in e_L. This implies that i_2 is weakly increasing in e_H. Second, n_1 is strictly increasing in e_H. Third, the definition of W_4 can be rearranged into

$$\mu e^{W_4} = e_1 e_L - e_1 + e_1 e_L z, \tag{A.22}$$

where $z = [(e_H/e_L)^\pi - 1]/\pi$ is strictly increasing in e_H and non-negative when $e_H \geqslant \underline{e}$. Note that $e_1 e_L - e_1 = \mu e^{W_3}$, which is weakly increasing in e_H by Eq. (A.14) and the fact that i_1 is weakly decreasing in e_H. As e_1 is weakly increasing in e_H, the product $e_1 e_L$ on its own is weakly increasing. So i_4 is strictly increasing in e_H. This establishes that the right-hand side of (A.15) is strictly increasing in e_H.

At $e_H = \underline{e}$, we have $i_2 = i_1$ and $i_4 = i_3$, so (A.14) implies that the right-hand side of (A.15) is smaller than the left-hand side. For $e_H \geqslant \psi$, Eq. (A.16) and the definition of i_2 imply that $i_2 = 1 - S$. As e_H tends to ∞, the right-hand side of (A.15) therefore converges to $2 - S + \nu$ which is greater than the left-hand side. This establishes existence and uniqueness of a solution to the system of Eqs. (A.14)–(A.16) with the stated properties.

As $n_1 < 1$, raising ν makes the left-hand side of (A.15) exceed the right-hand side. As the latter is strictly increasing in e_H once e_1 and e_L are solved for as functions of this variable, we have the claimed comparative statics and asymptotics for e_H. □

Lemma A.8. *The solution to the system of Eqs.* (A.14)–(A.16) *identified in Lemma* A.7 *constitutes an equilibrium.*

Proof. Lemma A.6 implies that $0 < i_1 < i_2 < i_3 < i_4$. Thus, the ranking of the measures i_1 through i_4 is the one that we assumed when formulating the market clearing conditions (A.10)–(A.13). The solution thus constitutes an equilibrium. □

Lemma A.9. *There is a unique $\nu^* > 0$ such that condition $(*)$ holds as an equality for all $\nu \leqslant \nu^*$, and as a strict inequality for all $\nu > \nu^*$.*

Proof. By the last part of Lemma A.7, there is a unique ν such that $e_H = \psi$; call this ν^*. The result then follows from Eq. (A.16). □

A.5. Proof of Proposition 2

The definitions of W_4 and W'_H yield

$$\mu e^{W_4 - W'_H} = \frac{e_1[e_2 - \pi - (1-\pi)e_L]}{\pi(e_H - 1)}. \tag{A.23}$$

For $\nu \to 0$, the right-hand side of (A.23) tends to $\underline{e}$ by Lemmas A.6 and A.7. As $\underline{e} > \mu$, continuity of W_4 and W'_H in ν thus implies $W_4 > W'_H$ for ν close to 0. □

A.6. Proof of Proposition 3

For $\nu \geqslant \nu^*$, e_1 and e_L are independent of e_H by Lemma A.6. Using the fact that $e_2 = e_H^{\pi} e_L^{1-\pi}$, we find that the derivative of the right-hand side of (A.23) with respect to e_H is

$$-\frac{e_1[\pi(e_2 - 1) + (1-\pi)(e_2 - e_L)]}{\pi(e_H - 1)^2}, \tag{A.24}$$

hence strictly negative and bounded away from zero. By the last part of Lemma A.7, there are thus two cases. Either $W_4 - W'_H < 0$ at $\nu = \nu^*$ and we can take $\nu^{**} = \nu^*$; or $W_4 - W'_H \geqslant 0$ at $\nu = \nu^*$ and there is a $\nu^{**} > \nu^*$ with the stated property. □

Appendix B

B.1. Data sources and summary statistics

Real housing prices are built from the MSA Conventional Mortgage Home Price Index produced by the Office of the Chief Economist at Freddie Mac. To obtain real housing prices, we use the CPI-US index (series cuur0000sa0) from the US Bureau of Labor Statistics.

We build all other variables from the Census data provided at www.ipums.org. The website provides detailed definitions for each variable. For each household in the sample, we download household income (HHINCOME), tenure (OWNERSHP), home value (VALUEH), gross monthly rent (RENTGRS) and location indicators (PUMA, STATEFIP, METAREA). The census questionnaire in 2000 did not ask households to explicitly identify the head of household. To compute the age of the head of household, we download the age of each person in the household (AGE) and its wage income (INCWAGE). We define the age of the head of household as the age of the person with the highest wage income in the household. If no person receives a wage in the household, we take the age of the oldest person in the household. To determine the number of years since the household moved into its current home, we use

Table 5
PUMAs: summary statistics

	PUMAs			
	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Number of PUMAs	349	327	398	277
	Homeowners			
Mean C.V. household income	0.8948	0.8670	0.8394	0.8524
Standard deviation	0.1162	0.1134	0.1112	0.0963
Mean S.D. time since moved	10.6521	11.4233	11.2733	11.5503
Standard deviation	1.7649	1.3880	1.3549	1.1108
Mean S.D. age head	16.0495	15.9543	15.7162	15.8541
Standard deviation	1.4446	1.2912	1.3724	1.1346
Mean C.V. home value	0.7785	0.7084	0.6328	0.5604
Standard deviation	0.1591	0.1693	0.1542	0.1220
	Renters			
Mean C.V. household income	0.9693	0.9628	0.9644	0.9480
Standard deviation	0.1526	0.1774	0.1661	0.1416
Mean S.D. time since moved	5.6055	6.4128	6.9176	6.2490
Standard deviation	1.3377	1.3223	1.9325	1.2618
Mean S.D. age head	16.9028	17.6863	16.8909	16.3005
Standard deviation	2.0836	2.0382	1.7669	1.6309
Mean C.V. gross rent	0.5095	0.5080	0.4909	0.4784
Standard deviation	0.0860	0.0870	0.0894	0.0757

Table 6
Households: summary statistics

	PUMAs			
	1st growth quartile	2nd growth quartile	3rd growth quartile	4th growth quartile
Number of Households	537,725	536,527	580,238	381,121
		Homeowners		
Median income	51,000	53,000	62,200	68,950
Median time since moved	8	8	8	8
Median age head	49	48	48	48
Median home value	95,000	112,500	162,500	225,000
S.D. relative income	1.1660	1.1034	1.0504	1.0760
S.D. diff. time since moved	10.8661	11.7818	11.5142	11.7488
S.D. diff. age head	16.1431	16.0068	11.5142	15.7554
S.D. relative home value	1.0287	0.9023	0.7856	0.6534
		Renters		
Median income	27,000	26,100	30,500	34,000
Median time since moved	4	4	4	4
Median age head	37	38	39	38
Median gross rent	573	560	667	767
S.D. relative income	1.3253	1.3337	1.3788	1.3033
S.D. diff. time since moved	5.8399	6.6317	7.9955	6.5087
S.D. diff. age head	16.8338	17.7254	17.0313	16.2931
S.D. relative gross rent	0.5272	0.5347	0.5243	0.5087

the number of years since our defined head of household moved into residence (MOVEDIN). The variables VALUEH and INCWAGE are coded in intervals. We replace each interval code with the median value of the interval.

We restrict the sample to households that live in the 1351 PUMAs located in one of the 164 MSAs for which we have real housing prices. We end up with 2,035,611 households that own their home and 1,084,878 households that rent their home.

We group PUMAs according to the housing price growth in the MSA where they are located. The groups vary in size because we have more than one PUMA for most MSAs (between 1 and 67 PUMAs, with a median of 4). Each group is computed including PUMAs with growth strictly greater than the low cutoff value and less than or equal to the high cutoff value. Note that the results we report are not sensitive to changes in the grouping rule.

Table 5 reports summary statistics at the PUMA level computed over owners and renters separately.

Table 6 reports summary statistics at the household level where again households are grouped according to the same PUMA groups as above. Recall that relative income, relative home value and relative gross rents are computed as ratios to the median of the PUMA. For time since moved and age, we use the difference between the value of the head of the household and the median of the PUMA.

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