## Cash Breeds Success:

# The Role of Financing Constraints in Patent Races* 

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## The Role of Financing Constraints in Patent Races


#### Abstract

This paper studies the impact of financing constraints on the equilibrium of a patent race. We develop a model where firms finance their R\&D expenditures with an investor who cannot verify their effort. We solve for the optimal financial contract of any firm along its best-response function. In equilibrium, any firm in the race is more likely to win the more cash and assets it holds prior to the race, and the less cash and assets its rivals hold prior to the race. We use NBER evidence from pharmaceutical patents awarded between 1975 and 1999 in the US, patent citations, and COMPUSTAT to measure the effect of all the racing firms' cash holdings on the equilibrium winning probabilities. The empirical findings support our theoretical predictions.


Keywords: Patent Race, optimal contract, innovation, financial constraints.
JEL Classification: G24, G32, L13

Do a firm's financing constraints affect its decisions to pursue innovation? Since Fazzari, Hubbard, and Petersen's (1988) seminal paper, economists have found that financing matters through various channels for total firm level investment in R\&D. For example, Hall (1992) shows that the source of financing matters and Himmelberg and Petersen (1994) show that internal finance predicts $\mathrm{R} \& \mathrm{D}$ expenditures of small high tech firms. But do a firm's financing constraints also affect its rivals' decisions to pursue innovations?

To our surprise, the role of financing constraints in patent races has not been comprehensively studied in the literature. Theorists have focused mainly on how firms' R\&D effort depends on technological standing and market structure. In this paper, we incorporate financing constraints explicitly into Reinganum's (1983) seminal model and test the model's comparative statics predictions empirically. In our model, firms finance their $\mathrm{R} \& \mathrm{D}$ expenditures with internal and external funds. The probability of making the discovery at a point in time depends on the effort exerted by the entrepreneur, which cannot be verified by the investor. In equilibrium, finance is costly for the entrepreneur and the marginal cost of innovative activity is increasing in the fraction of outside funds to the total investment, very much following the logic proposed by Jensen and Meckling (1976). An increase in the marginal cost of innovating shifts a firm's best response function downwards which in turn decreases the firm's equilibrium $R \& D$ expenditures. The practical upshot is that in a setting of strategic interactions, deep pockets are a source of comparative advantage. This prediction is testable and is at the core of our empirical investigation.

We face two major empirical challenges. First, we need data that combines financial information with a racing environment. We use the NBER Patent Citations Data File developed by Hall, Jaffe and Trajtenberg (2002), which records all utility patents granted in the United States between 1963 and 1999 . Every patent granted after 1975 is linked to all the patents it cites and to the CUSIP code of the assignee as it appears in COMPUSTAT. We merge the patent records with COMPUSTAT to obtain the financial data of the firms in the race before the patent was awarded. To make sure that the patent awards capture innovative success, we focus on the drug industry, where patents are crucial to reap the returns to R\&D investment (see Levin et al., 1987, and Cohen, Nelson and Walsh, 2000) and where firms use the exclusivity of the drug patent to block imitation during and after
the clinical trials phase of the development ${ }^{2}$ Second, we need to identify in the data which firms are effectively racing for each patent. We propose here a method to pre-select the firms most likely to race for a patent based on the model's prediction that firms with a very low expected probability of winning a race will rather drop out. This probability itself is predicted using the firm's ownership of the prior technology and the past record of winning patents of the same class.

Our model links the probability that any firm in the race wins to the characteristics of all the firms in the race, e.g., their financial resources and the value of their prior innovations. A firm is more likely to win a given race the higher its wealth and the lower its rivals' wealth. To test this prediction we fit a multinomial logistic model that selects the winner as a function of these variables. We find that a firm's probability of winning a race is increasing -on average- in its stock of cash and decreasing in its rivals' stock of cash. The predicted impacts are not only statistically significant but also economically meaningful: differences in stocks of cash imply large differences in the probability of winning.

Our empirical analysis distinguishes between the ability to finance R\&D internally and externally. Besides using its own generated cash to finance R\&D internally, the firm can also pledge its less liquid resources to reduce the cost of external finance. We find that the total asset value of a firm increases its probability of winning but decreases that of its rivals. Because we use only COMPUSTAT firms, it is not surprising that we find that innovation success is generally more sensitive to the value of assets than to cash holdings. Indeed, it is likely that these firms became public to have better access to external finance in the first place. Interestingly, though, we find that innovation success has become as sensitive to cash as it is to assets in the late 90s.

This paper is related to several strands of the literature but novel in its focus and comprehensiveness. The literature has devoted some attention to the commitment effects of financial structure on pricing, output, and investment strategies in oligopolistic product market games. A capital structure choice that is observed by rivals can make a firm reduce its prices or increase investment (see Brander and Lewis, 1986; Maksimovic, 1988, and Rotemberg and Scharfstein, 1990; Fudenberg and Tirole, 1986; and Bolton and Scharfstein,
1990). Chevalier (1995) shows that increased leverage in the supermarket industry softened competition, whereas Jensen and Showalter (2004) show that increased leverage decreases firm-level R\&D expenditures. We depart from this literature in two respects. First, we assume that financing choices are not observable to rivals, so that the commitment effects of financing choices play no role. We believe that our assumption is appropriate to analyze the interaction between large firms, where rivals find it difficult to disentangle the financing of individual projects from the overall financing of the concern. Second, we do not take the form of the contracts as given but work from first principles, i.e., we derive the equilibrium financing contracts for competitors given their financing gap. Thus, we focus on a different comparative statics exercise. Instead of varying the capital structure directly, we vary the firm's ability to finance herself internally and externally, which in turn induces changes in the capital structure.

Our empirical investigation explores a game theoretic setup with a comprehensive data base. Only few studies share these two features. Blundell, Griffith and Van Reenen (1999) study the relationship between market share and innovation using a panel of British pharmaceutical firms. They find that leading firms innovate more often. In contrast to their study, we incorporate financing explicitly into ours and show that financing matters even if we control for technological leadership and patenting experience.

Cockburn and Henderson (1994) address whether or not R\&D investments are strategic. Gathering detailed data at the individual project level for ten of the largest firms in the pharmaceutical industry, they find that research investments are only weakly correlated across firms. However, as they acknowledge, their study may miss correlations between investments of smaller potential entrants and the large firms by focusing only on the large players $3^{3}$ We identify strategic behavior from the outcome of the races and not the inputs firms devote to these races. We are thus able to use a much more comprehensive data base and show that the winning probabilities of firms are significantly affected by other firms' characteristics. Moreover, as mentioned above, we include measures of the firms' financial wealth in the empirical analysis.

Lerner (1997) finds evidence of strategic interaction in R\&D: the leaders in the disk drive industry between 1971 and 1988 were less likely to improve their disk drive density than the
laggards ${ }^{4}$ Lerner is able to identify this effect through the distance of a firms current drive density to the industry's maximum. In contrast to the drugs industry, not only the first but any firm that innovates is rewarded for its $R \& D$ in the disk drive industry. Therefore, he treats observation errors independently across firms. We cannot rely on such assumptions in the pharmaceutical industry because, in a race, the success of any firm is jointly determined by the characteristics of all the firms that race. Our approach identifies strategic behavior from the dependence of the outcome of races on all the competitors' characteristics. 5

Hellman and Puri (2000) also study the empirical relationship between product market strategies and finance. They find evidence that budding firms with innovative strategies are more likely to be funded by venture capitalists. Our results are consistent with theirs insofar as firms with a bigger expected probability of success at innovation are externally financed at smaller costs. However, in our setup, the expected probability of success is not taken as given but determined endogenously in a Nash Equilibrium, conditional on the technological standing of firms and the availability of cash before the race.

The remainder of this article is organized as follows. The next section develops the model and shows that wealthier firms are more likely to win patent races. Section 2 describes our data sources and discusses their relevance to test the comparative statics results of our model. Section 3 shows how our model's equilibrium innovation probabilities map directly into an estimable multinomial selection and section 4 discusses the econometric specification we use to select firms into the race. Section 5 presents the results from estimating the winner selection model and section 6 extends the analysis to the determination of firm-level R\&D. Section 7 summarizes our findings and concludes briefly. All proofs are gathered in an appendix.

## 1 Theory

We consider the financing of research in a version of the Reinganum (1983) model . There are $n$ firms, indexed $i=1, \ldots, n$, that obtain current flow profits $\pi_{i}$ from producing state-of-the-art products. The firms can enter a research race for a higher quality product. We model the uncertain success in this research race as the outcome of a Poisson process. The
state-of-the-art products and the innovation are protected by patents of infinite length. If firm $i$ innovates, then its flow profit increases to $\bar{\pi}_{i}>\pi_{i}$ and the flow profits of firms $j \neq i$ drop to $\underline{\pi}_{j} \leq \pi_{j}$. This formulation allows for the case where $\pi_{i}=0$ for some $i$ and/or $\underline{\pi}_{j}=0$ for some $j$. Hence, the model can capture both drastic and non-drastic innovations.

If a firm enters the research race, it has to spend a fixed cost $F$. Once this cost is sunk the entrepreneur running firm $i$ can exert a flow of effort $a_{i}$. If a firm spends a constant flow of effort $a_{i}$, then the conditional likelihood at any point in time to innovate within the next instant given that it has not innovated before is $a_{i}^{\alpha}$, where $\alpha<1$. The cost of effort is equal to $a_{i}$. Firms have limited financial resources, $W_{i}$. If $W_{i}<F$ the firm needs outside funds to finance the fixed cost ${ }^{6}$

We assume that many investors compete in Bertrand fashion for the right to finance a firm's investment. They make take-it-or-leave-it offers to firms and then firms decide whether or not to accept the contract $]^{7}$ A firm with $W_{i}<F$ that rejects its contract cannot innovate, i.e., has probability of innovation equal to zero for all $a_{i}$. After the firm has accepted a contract, it chooses its research intensity $a_{i}$.

We assume that contracts between investor and firm are not observable to other investors and firms. That is, we adopt the simultaneous move assumption from Reinganum (1983) and solve for the Nash Equilibrium. We do not consider sequential (Stackelberg) games where one firm can observe the financing of the other firm before it chooses its research intensity. This rules out commitment effects of finance. Our comparative statics results are not affected by this modeling choice.

We begin our analysis with the derivation of firms' best responses, first characterizing optimal contracts and then a firm's research intensity that results from accepting an optimal contract.

### 1.1 Optimal financing

The Poisson nature of research implies that there are $n$ classes of positive probability events, distinguished by the firm that innovates first. Within these classes, events differ only in the time of innovation. We consider stationary contracts where the repayment conditions depend
on whether a firm wins the race but not on when the firm wins. Moreover, since $\underline{\pi}_{i}$ does not depend on which firm $j \neq i$ innovates, the repayments of a losing firm do not depend on the identity of the winning firm. Hence, from the perspective of contracting within a firm-investor coalition, the research process has three relevant outcomes at any time $t$ : (i) some firm $j \neq i$ wins the race, (ii) firm $i$ wins the race, and (iii) no firm innovates. We place no further restrictions on the form of contracts. Contracts with any arbitrarily complex time-dependent repayments (in the sense of the length of time elapsed since the arrival of the innovation) have a simple equivalent representation where the firm commits to repay a constant share $s_{i}$ of $\pi_{i}$ from the start of the race until the innovation is found by some firm, and constant shares $s_{i}^{-}$and $s_{i}^{+}$of profits $\bar{\pi}_{i}$ and $\underline{\pi}_{i}$ thereafter, respectively. Since everybody is risk-neutral, all that matters is the present value of the repayment stream.

Our aim is to have a simple model to derive comparative statics predictions of equilibrium research intensities with respect to a firm's wealth $W_{i}$. By definition, such a dependency arises only in a second-best world, where $F-W_{i}$, the investment by the investor, is large relative to the values of $\underline{\pi}_{i}$ and $\pi_{i}$. Otherwise the firm becomes a safe investment, because it is able to repay the investor in every state of the world. For the remainder of this section, we focus only on the case where the first-best is not implementable.

An optimal contract specifies that a firm repays all its profits if either no firm or another firm innovates. We prove this result in Lemma 1, in the appendix. We now proceed to analyze optimal contracting by backwards induction. First, we characterize the best contracts that can be offered to a firm. Then, we discuss whether or not the firm will accept such a contract.

### 1.1.1 Characterization of second-best contracts

Let $h \equiv \sum_{j \neq i} a_{j}^{\alpha}$ and let $V_{i}\left(h, s_{i}^{+}\right)$denote the value of firm $i$ 's claim of future profits for given values of the other firms' aggregate research activity and the investor's repayment share $s_{i}^{+}$. Firm $i$ 's problem is to accept or reject a contract offered by the investor and to choose its research effort conditional on accepting. The second stage of firm $i$ 's problem can be described by the following asset equation:

$$
\begin{equation*}
r V_{i}\left(h, s_{i}^{+}\right) d t=\max _{a_{i}}\left\{a_{i}^{\alpha}\left(\left(1-s_{i}^{+}\right) V_{i}^{+}-V_{i}\left(h, s_{i}^{+}\right)\right)-h V_{i}\left(h, s_{i}^{+}\right)-a_{i}\right\} d t \tag{1}
\end{equation*}
$$

where $r$ is the risk-free interest rate and $V_{i}^{+} \equiv \frac{\bar{\pi}_{i}}{r}$, i.e., the net present value of the perpetual flow of profits, $\bar{\pi}_{i}$, starting at the time of innovation. We assume that $V_{i}^{+}>F$. In a short interval of time between $t$ and $t+d t$ firm $i$ innovates with probability $a_{i}^{\alpha} d t$ and any of the other firms innovates with probability $h d t$. In case firm $i$ innovates, the firm receives a share $\left(1-s_{i}^{+}\right)$of all future profits and thus a claim that is worth $\left(1-s_{i}^{+}\right) V_{i}^{+}$as of the time of innovation. If any firm innovates, firm $i$ loses the value of its current claim, $V_{i}\left(h, s_{i}^{+}\right)$. The flow cost of research during the small interval of time is $a_{i} d t$.

The maximization problem on the right hand side of (1) is strictly concave in $a_{i}$. Let $a_{i}\left(s_{i}^{+}\right)$denote a solution to this problem. The first-order condition,

$$
\begin{equation*}
\alpha\left(a_{i}\left(s_{i}^{+}\right)\right)^{\alpha-1}\left(\left(1-s_{i}^{+}\right) V_{i}^{+}-V_{i}\left(h, s_{i}^{+}\right)\right)=1, \tag{2}
\end{equation*}
$$

is necessary and sufficient for the unique optimal choice of $a_{i}\left(s_{i}^{+}\right)$induced by the contract $\left\{F-W_{i}, s_{i}^{+}\right\}$. We can multiply both sides of condition (2) by $a_{i}\left(s_{i}^{+}\right)$and obtain the condition

$$
\begin{equation*}
\alpha\left(a_{i}\left(s_{i}^{+}\right)\right)^{\alpha}\left(\left(1-s_{i}^{+}\right) V_{i}^{+}-V_{i}\left(h, s_{i}^{+}\right)\right)=a_{i}\left(s_{i}^{+}\right) . \tag{3}
\end{equation*}
$$

If we substitute condition (3) into the asset equation (1) we can solve for the value of the entrepreneur's claim in firm $i$

$$
\begin{equation*}
V_{i}\left(h, s_{i}^{+}\right)=\left(1-s_{i}^{+}\right) \frac{(1-\alpha)\left(a_{i}\left(s_{i}^{+}\right)\right)^{\alpha} V_{i}^{+}}{(1-\alpha)\left(a_{i}\left(s_{i}^{+}\right)\right)^{\alpha}+h+r} . \tag{4}
\end{equation*}
$$

Let $B_{i}\left(h, s_{i}^{+}\right)$denote the value of the investor's claim in the firm. The investor receives the profits $\pi_{i}$ as long as no firm innovates and receives the value $V_{i}^{-} \equiv \frac{\pi_{i}}{r}$ from the time of innovation onwards if any firm $j \neq i$ innovates. Moreover, the investor receives a share $s_{i}^{+}$ of the profit $\bar{\pi}_{i}$ from the time of innovation onwards. $B_{i}\left(h, s_{i}^{+}\right)$satisfies

$$
r B_{i}\left(h, s_{i}^{+}\right) d t=\left\{a_{i}\left(s_{i}^{+}\right)^{\alpha}\left(s_{i}^{+} V_{i}^{+}-B_{i}\left(h, s_{i}^{+}\right)\right)+h\left(V_{i}^{-}-B_{i}\left(h, s_{i}^{+}\right)\right)+\pi_{i}\right\} d t .
$$

Dividing by $d t$ and rearranging, we can solve for $B_{i}\left(h, s_{i}^{+}\right)$and get

$$
B_{i}\left(h, s_{i}^{+}\right)=\frac{a_{i}\left(s_{i}^{+}\right)^{\alpha} s_{i}^{+} V_{i}^{+}+h V_{i}^{-}+\pi_{i}}{a_{i}\left(s_{i}^{+}\right)^{\alpha}+h+r} .
$$

Individual rationality of the investor requires that $B_{i}\left(h, s_{i}^{+}\right) \geq F-W_{i}$. Perfect competition in the market for funds drives the investor's profits to zero, so

$$
\begin{equation*}
\frac{a_{i}\left(s_{i}^{+}\right)^{\alpha} s_{i}^{+} V_{i}^{+}+h V_{i}^{-}+\pi_{i}}{a_{i}\left(s_{i}^{+}\right)^{\alpha}+h+r}=F-W_{i} . \tag{5}
\end{equation*}
$$

The investor's problem is to maximize $V_{i}\left(h, s_{i}^{+}\right)$with respect to $s_{i}^{+}$subject to (2) and (5). We can use (2) and (5) to eliminate $s_{i}^{+}$and characterize the solution in terms of the induced effort level. Let $\hat{a}_{i}$ denote a level of research effort by firm $i$ as induced by a contract that satisfies (2) and (5). Substituting (4) and (5) into (22) we conclude that $\hat{a}_{i}$ must satisfy the condition

$$
\begin{equation*}
\Omega \equiv \alpha\left(\hat{a}_{i}^{\alpha} V_{i}^{+}+h V_{i}^{-}+\pi_{i}-\left(\hat{a}_{i}^{\alpha}+h+r\right)\left(F-W_{i}\right)\right)(h+r)-\hat{a}_{i}\left((1-\alpha) \hat{a}_{i}^{\alpha}+h+r\right)=0 . \tag{6}
\end{equation*}
$$

$\Omega\left(\hat{a}_{i} ; \cdot\right)$ is strictly concave in $\hat{a}_{i}$. Hence (6) has at most two distinct solutions. Let $a_{i}^{*}$ denote an effort level induced by an optimal contract. It is now easy to see that $a_{i}^{*}$ is the largest solution of (6) . The reason is as follows. The investor just breaks even, so the firm receives all of the surplus. The firm's effort is distorted downwards (which can be seen from (2)). Hence, it is desirable to induce the highest possible effort level. Note also that this implies that the optimal contract is unique and moreover at $\hat{a}_{i}=a_{i}^{*}$ we have $\frac{\partial \Omega\left(a_{i}^{*} ; \cdot\right)}{\partial \hat{a}_{i}}<0$. Since we look at the case where the first-best level of effort is not implementable, we have $\Omega(0 ; \cdot)=$ $\alpha\left(h V_{i}^{-}+\pi_{i}-(h+r)\left(F-W_{i}\right)\right)(h+r)<0$ (see Lemma 1, for a proof that strict inequality holds). So, given that $\Omega\left(\hat{a}_{i} ; \cdot\right)$ is concave in $\hat{a}_{i}$, it must be downward-sloping at $a_{i}^{*}$ whenever (6) has a solution.

### 1.1.2 Existence and acceptance of contracts

The existence of an optimal contract, depends on the aggressiveness of the rival firms, as measured by $h$. One can show that for all $W_{i} \geq 0$ and $F$ there exists $\bar{h} \equiv \bar{h}\left(V_{i}^{+}, W_{i}, V_{i}^{-}, \pi_{i}, F\right)$ such that a unique optimal contract exists if and only if $h \leq \bar{h}$. The threshold $\bar{h}$ is nondecreasing in the first four arguments and non-increasing in the last one. The intuition for these results is straightforward. The higher the research effort chosen by the rival firms, the smaller the expected value of the prize for a given effort level by firm $i$. As a result, the value of the investor's claim is decreasing in $h$ for fixed $s_{i}^{+}$, and the investor requires a larger share of profits the higher is $h$. But an increase in $s_{i}^{+}$decreases firm $i$ 's incentive to provide effort.

For a large enough $h$, this discouragement effect is so strong that an optimal contract ceases to exist. On the other hand, an increase in $V_{i}^{+}, V_{i}^{-}, \pi_{i}, W_{i}$, or a reduction of $F$ balances these effects, so that the higher is the value of the race, the larger is the critical level of the rival firms' aggregate likelihood of winning, $\bar{h}$, that chokes off firm $i$ 's innovative efforts. Likewise, the higher is the firm's wealth, the smaller is the amount of money needed from the investor and the less discouraging is an increase in the other firms' aggregate research.

Consider now firm $i$ 's decision whether or not to accept the contract. Let the optimal sharing rule if firm $i$ wins be denoted by $s_{i}^{+*} \equiv s_{i}^{+*}\left(h, V_{i}^{+}, V_{i}^{-}, \pi_{i}, W_{i}, F\right)$. The firm accepts the optimal contract if and only if the net present value of its investment is nonnegative, that is if

$$
V_{i}\left(h, s_{i}^{+*}\right)-W_{i} \geq 0
$$

Suppose $V_{i}^{+}$is sufficiently large so that firm $i$ engages in research for $h=0$. Then, one can show that for all $W_{i} \geq 0$ and $F$, there exists $\overline{\bar{h}}>0$ such that firm $i$ accepts the optimal contract if and only if $h \leq \overline{\bar{h}}\left(V_{i}^{+}, V_{i}^{-}, \pi_{i}, W_{i}, F\right) . \overline{\bar{h}}$ has essentially the same comparative statics properties as $\bar{h}$ has, so we omit a further discussion.

### 1.1.3 Induced behavior in the race

Let the function $b_{i}\left(h ; W_{i}, \cdot\right)$ denote the effort level induced by the optimal contract as a function of $h$, the rival firms' aggregate likelihood of winning, and the firm's wealth (and further parameters of the contracting problem). We note that $b_{i}\left(h ; W_{i}, \cdot\right)$ is positive and increasing in $h$ for all $h \leq \min \{\bar{h}, \overline{\bar{h}}\}$ and is equal to zero otherwise. Applying the implicit function theorem to condition (6), we have that

$$
\frac{d a_{i}^{*}}{d W_{i}}=\frac{\frac{\partial \Omega\left(a_{i}^{*}, W_{i} ; \cdot\right)}{\partial W_{i}}}{-\frac{\partial \Omega\left(a_{i}^{*}, W_{i} ; \cdot\right)}{\partial \hat{a}_{i,}}},
$$

where $\frac{\partial \Omega\left(a_{i}^{*}, W_{i} ; \cdot\right)}{\partial W_{i}}=\alpha\left(a_{i}^{* \alpha}+h+r\right)(h+r)>0$ and the denominator is positive because $a_{i}^{*}$ is the larger one of the solutions to equation (6). Thus, whenever $b_{i}\left(h ; W_{i}, \cdot\right)>0$ and the effort level is second-best, $\frac{d b_{i}\left(h ; W_{i}, \cdot\right)}{d W_{i}}>0$.

If the first-best level of effort is implementable, then an increase in $W_{i}$ has no effect whatsoever on the firm's best response. The best-response function in this case coincides
with the one in Reinganum's model. However, in the second best, the larger is $F-W_{i}$, the larger is the repayment share to the investor and the smaller the firm's effort choice. Intuitively, an increase in $F-W_{i}$ increases the agency costs of finance and increases the firm's marginal costs of innovative activity.

### 1.2 Equilibrium comparative statics and testable implications

We now show that equilibria of our game display natural comparative statics. We present these results first for the special case where there are two firms, and then present a generalization to the case of an arbitrary number of firms.

### 1.2.1 The case of two firms

For two firms, our game admits two kinds of equilibria for different parameter constellations. First, there exist equilibria where both firms are active and the equilibrium research efforts, $a_{i}^{*}$ for $i=1,2$, are both positive. Second, there exist also equilibria where only one firm enters the research race and the other firm stays out. When the prizes the firms can win, $V_{i}^{+}$, are sufficiently large relative to the cost of entering the race, $F$, then both firms must be active in any equilibrium. Whenever such an equilibrium exists, it has the following properties:

Proposition 1 Consider a stable, interior equilibrium. Formally, suppose that for $i=1,2$ and $j \neq i,\left(a_{i}^{*}, a_{j}^{*}\right) \gg 0$ and $\left|\frac{d b_{i}\left(a_{j} ; W_{i}\right)}{d a_{j}}\right|<1$ around $\left(a_{i}^{*}, a_{j}^{*}\right)$. If in addition
i) $F>\max \left\{W_{i}+\frac{a_{j}^{* \alpha} V_{i}^{-}+\pi_{i}}{a_{j}^{* \alpha}+r}, W_{j}+\frac{a_{i}^{* \alpha} V_{j}^{-}+\pi_{j}}{a_{i}^{* \alpha}+r}\right\}$, then $\frac{d a_{i}^{*}}{d W_{i}}>0$; moreover, $\frac{d a_{i}^{*}}{d W_{i}}>\frac{d a_{j}^{*}}{d W_{i}}>0$.
ii) $F<W_{i}+\frac{a_{j}^{* \alpha} V_{i}^{-}+\pi_{i}}{a_{j}^{* \alpha}+r}$, then $a_{i}^{*}$ and $a_{j}^{*}$ are independent of $W_{i}$.

Proposition 2 In a stable, interior equilibrium, the probability that firm $i$ wins the race is non-decreasing in $W_{i}$ and strictly increasing in $W_{i}$ if $F>W_{i}+\frac{a_{j}^{* \alpha} V_{i}^{-}+\pi_{i}}{a_{j}^{* \alpha}+r}$.

The intuition for the results is quite simple. An increase in firm $i$ 's wealth improves the contracts that can be offered to this firm and hence increases this firm's research effort. In other words, the best reply of firm $i$ to any given research effort of firm $j$ is increased. Firm $j$ adjusts to this change by increasing its own research effort along its best reply function. While the first effect tends to increase the probability that firm $i$ wins the race, the second
effect tends to reduce it. However, in a stable equilibrium, the former effect always dominates the latter.

### 1.2.2 A case of $n>2$ firms

The general $n>2$ firms version of our race is difficult to treat analytically. While we conjecture that our main results hold in general, we confine ourselves here to develop a simplified $n$ firm version that remains analytically tractable. 8 Suppose firm $i$ 's level of wealth is low enough so that its level of research effort, for given effort levels of the other firms, is second-best optimal. Suppose further that all firms $j \neq i$ are wealthy enough so that their research efforts, for given efforts of the other firms, correspond to their first-best levels. Finally, let $V_{j}^{-}=\pi_{j}=0$ and $V_{j}^{+}=V^{+}$for all $j \neq i$. By construction, any firm $j \neq i$ faces exactly the same incentives at the margin where it chooses its research effort. For large enough values of $V^{+}$all such firms participate in the race and the overall game has an equilibrium where they all behave identically.

Let $a_{-i}^{*}$ denote the equilibrium effort level of any firm $j \neq i$. We have the following result: Proposition 3 Suppose that $W_{i}+\frac{(n-1) a_{-i}^{* \alpha} V_{i}^{-}+\pi_{i}}{(n-1) a_{-i}^{* \alpha+r}}<F<W_{j}$ for all $j \neq i$. Then, in a stable, interior equilibrium, the probability that firm $i$ wins the race is strictly increasing in $W_{i}$.

### 1.2.3 Testable implications

Propositions 1, 2, and 3 establish that improved financing conditions improve a firm's strategic position, and its chances of winning. While wealth is a one-dimensional measure in our theory, the empirical investigation will have to distinguish between inside and outside finance. The firm can either use its own generated cash to finance its R\&D expenditures internally or pledge its assets to reduce the cost of using external finance. The immediate testable implication is that, given a level of pledgeable assets, the firm's winning probability increases with the level of cash and that, given a level of cash holdings, the firm's winning probability increases with the level of pledgeable assets. Moreover, the winning probability of any other firm $j \neq i$ in the race decreases with the level of cash or assets of firm $i$.

The effects of the remaining parameters on the equilibrium research efforts are ambiguous. Anything that increases $\bar{\pi}_{i}$ (say, an increase in demand) will also increase $\bar{\pi}_{j}$. As a result
both reaction functions are shifted upwards by an increase in the value of the patent race as measured by $V_{i}^{+}$and $V_{j}^{+}$and the effect on the equilibrium efforts is unclear. Increases in $\underline{\pi}_{i}$ and $\pi_{i}$ have two effects. On the one hand it may become feasible to write first-best contracts so that the firm's best response function shifts up. On the other hand, an increase in operating profits makes the firm reluctant to destroy these profits, so that it reduces its research efforts and its best response function shifts downwards.

We now proceed to investigate whether the key predictions of our model as outlined in Propositions 1 through 3 are verified empirically. We start by describing how we construct our data set and how we define our observational unit, the race for a patent pool, from this data.

## 2 The data

We use two sources of data. The first is the NBER Patent Citations Data File developed by Hall, Jaffe and Trajtenberg (2002). This data set comprises all utility patents granted in the United States between 1963 and 1999 and records their technological category, the dates of award and their assignees. Each patent awarded after 1975 is linked to all the patents it cites and the assignee names in the patent records are matched to the name of the company as it appears in COMPUSTAT. From COMPUSTAT we get the financial information of the patent assignees whose stock is publicly traded in the U.S.

### 2.1 Patenting in the pharmaceutical industry

The NBER Patent Citations Data File is useful to identify racing behavior only in industries that rely heavily on patent protection to appropriate the returns of R\&D. It is well recognized that patenting is crucial to protect $R \& D$ in the pharmaceutical industry (see the survey conducted by Levin, Klevorick, Nelson and Winter (1987), and its follow-up by Cohen, Nelson, and Walsh (2000)). Moreover, the race for the patent is the best stage to test for strategic interactions during the drug discovery process. The exclusivity rights on a new drug are only contestable during the pre-clinical stage. After that, only the patent holder may conduct the clinical trials without the threat of imitation.

### 2.2 Measuring the value of a patent

It will prove useful to explore the predictions of our model conditional on the value of the patent by estimating our model across value quartiles. Since the value of a patent is not readily observable, we use the best available proxy; we follow Harhoff, Scherer, and Vopel (2003), who find a strong positive association between the number of citations received and the value of each patent reported by their owners in a survey of German firms. Because the raw count of citations is prone to biases due to time differences in the patent officers' propensity to add or drop citations, we adjust it using the coefficients provided by Hall et al. (2005) $!^{9}$

### 2.3 Patents versus patent pools as units of observation

Cohen et al. (2000) categorize industries into "discrete" and "complex" technologies. Discrete innovations comprise single patents that are used to prevent imitation. The pharmaceutical industry belongs to the discrete technology category. In contrast, firms that develop complex technologies (software, electrical equipment) accumulate bundles of patents to induce rivals to negotiate property rights over complementary technologies (Hall, 2004). To ensure that we meet our model's assumption of discrete type technologies, we restrict our sample to patents in the technological category 3, i.e., Drugs and Medical, and the subcategories 31, 33 and 39: Drugs, Biotechnology, and Miscellaneous Drugs, respectively (see Hall, et al. (2005) for a definition of these categories).

Even in this restricted sample, it is still debatable whether each patent can be treated as the outcome of a race. Although most authors argue in favour of one patent per race, to be sure, we explore the possibility that patents in our data may be pooled ${ }^{10}$ We group together all patents filed by the same firm on the same day, week or month that were subsequently also granted the same day, week or month, respectively. We find that there is significant clustering in the same week: $52 \%$ of the patents in subcategories 31,33 and 39 are filed together with at least one other patent in the same week, and then approved together in the same week (Figure 1). In fact, $50 \%$ of all patents are filed together by the same firm with another patent on the same day.

## <INSERT FIGURE 1 ABOUT HERE>

Table $\square$ shows the consequences of grouping individual patents into pools of all patents filed the same week. The 77,704 individual patents owned by corporations (Panel A) are grouped into 37,283 pools (Panel B). The average pool comprises two patents but an overwhelming majority comprises only one (median of 1 , max of 50 ). This grouping seems appropriate: of all patents grouped in the weekly pool, a single one receives most of the future citations. On average, the most cited patent in the pool gets $89 \%$ of the pool's total citations (median of $100 \%$ ). The citations received by the pool are strongly concentrated, with an average concentration index of 0.45 (Panel B).
<INSERT TABLE $\mathbb{1}$ ABOUT HERE>

The exercise above shows that the patents that are never cited are typically filed together with others that are. Austin (1993) uses the same weekly grouping for biotech, and obtains the same result. The weekly grouping seems to capture in each pool the essential patent that was being raced for and rules out many of the patents that are never cited as individual races. While the weekly grouping still yields many pools of single, non-cited patents, a broader definition of a pool, which includes all patents filed the same month, yields similar results. Indeed, the most cited patent in the pool still concentrates $72 \%$ of the total number of citations. Further, the monthly pooling reduces the number of pools to 28,430 and risks grouping different races into one. We choose the weekly grouping, which only risks having too many races of no value. By conducting our empirical tests across value quartiles - where value is measured by the pool's citations count - we ensure that the inference in the top quartiles is free of such a risk.

### 2.4 COMPUSTAT match

We cannot match all the patents to COMPUSTAT, primarily because not all winners are publicly traded firms. In fact, there is a large proportion of patents owned by universities, individuals and the public sector. Table $\rrbracket$ summarizes and compares the main characteristics of the matched patents to those of the patent universe $\sqrt{11}$

We find a COMPUSTAT match for the winners of $40 \%$ of the total number of patents awarded to corporations. Panel B shows that, on average, the COMPUSTAT-matched patent pool is cited twice as many times as the average patent pool awarded to a firm. In fact, as many as $24,302(84 \%)$ of the unmatched pools are never cited, whereas relatively few of the COMPUSTAT-matched pools (14\%) received no citations. Our match essentially drops a disproportionate amount of patents that seem to be of little or no value, for which it is extremely unlikely that an $\mathrm{R} \& \mathrm{D}$ race ever took place.

Our matching rate is higher for the pools of patents that receive more citations. Our overall matching rate of $22.5 \%$ is broken down into a rate of $13.35 \%$ in the first quartile of citations received, $15.84 \%$ in the second, $23.35 \%$ in the third, and a maximum rate of $38.15 \%$ in the fourth quartile. Again, because we estimate our model across all quartiles of citation counts, we can assess ex-post how the inference is affected by losing, on average, patents that are cited less after the COMPUSTAT match.

We acknowledge that there are still many potentially valuable patents that we haven't been able to match with COMPUSTAT. In fact, there are 7,622 unmatched pools that are cited as often as the upper half of the matched ones. Panel B also shows that the unmatched patents are won by firms with much less experience, clearly, firms that are public but small or private. Sections 5 and 7 interpret the inference we derive from the matched patents considering this omission.

We note finally that our matched sample of patents does not only include US firms. We have indeed matched most of the non US firms that are important players in the US races. These firms have securities traded in the US (e.g., through ADR programs) and are therefore covered in COMPUSTAT ${ }^{12}$ The following section derives an econometric model of a patent race from our theoretical model, and explains how we use it to test our theoretical predictions.

## 3 The econometric approach

### 3.1 Nash equilibrium winning probabilities

Let $\lambda_{i k} \equiv a_{i k}^{\alpha}$ denote the best response hazard rate of firm $i \in\left\{1,2, \ldots, n_{k}\right\} \equiv \mathcal{N}_{k}$ in race $k$. The Nash equilibrium is a vector of hazard rates $\boldsymbol{\lambda}_{k}^{*}$ that solves the system

$$
\begin{equation*}
\lambda_{i k}^{*}=\lambda\left(\mathbf{W}_{i k}, E_{i k}, \boldsymbol{\pi}_{i k}, \mathbf{C}_{i k} ; \boldsymbol{\lambda}_{-i k}^{*}\right) \forall i \in \mathcal{N}_{k}, \tag{7}
\end{equation*}
$$

where the vector $\mathbf{W}_{i k}$ includes our measures of financial wealth of firm $i$ before race $k, E_{i k}$ our measure of firm $i$ 's patenting experience before race $k, \boldsymbol{\pi}_{i k}$ the values of all the patent pools owned by firm $i$ that are being replaced by patent $k$ and $\mathbf{C}_{i k}$ the vector of other control variables. Conditional on $\mathbf{X}_{i k} \equiv\left(\mathbf{W}_{i k}, E_{i k}, \boldsymbol{\pi}_{i k}, \mathbf{C}_{i k}\right)$ and $\boldsymbol{\lambda}_{-i k}^{*}$, firm $i$ 's date of innovation, $T_{i k}$, follows a Poisson process. Therefore, the probability that $i$ wins race $k$ against all other racing firms $j \in \mathcal{N}_{k}$ is
$\operatorname{Pr}($ firm $i$ wins race $k)=\operatorname{Pr}\left(T_{i k} \leq T_{j k} \forall j \in \mathcal{N}_{k}\right)=\int_{0}^{\infty} e^{-\left(\lambda_{i k}^{*}+\sum_{j \neq i \in \mathcal{N}_{k}} \lambda_{j k}^{*}\right) t} \lambda_{i k}^{*} d t=\frac{\lambda_{i k}^{*}}{\sum_{j \in \mathcal{N}_{i}} \lambda_{j k}^{*}}$. Because the Nash Equilibrium of the race is the solution to the system (7), we can write each firm's hazard rate and winning probability as a function of its own and the other firms' characteristics as

$$
\begin{equation*}
\operatorname{Pr}(i \text { wins race } k)=\frac{\lambda_{i k}^{*}\left(\mathbf{W}_{k}, \mathbf{E}_{k}, \boldsymbol{\pi}_{k}, \mathbf{C}_{k}\right)}{\sum_{j \in \mathcal{N}_{k}} \lambda_{j k}^{*}\left(\mathbf{W}_{k}, \mathbf{E}_{k}, \boldsymbol{\pi}_{k}, \mathbf{C}_{k}\right)}, \tag{8}
\end{equation*}
$$

where $\mathbf{X}_{k} \equiv\left(\mathbf{W}_{k}, \mathbf{E}_{k}, \boldsymbol{\pi}_{k}, \mathbf{C}_{k}\right)$ is the full data vector for race $k$, collecting the characteristics of all firms in race $k$ before the race starts.

### 3.2 The empirical winning probabilities

If we approximate the equilibrium hazard rate function by a parametrized exponential function of the form $\lambda_{i k}^{*} \approx \exp \left(\boldsymbol{\beta}_{1}^{\prime} \mathbf{X}_{i k}+\boldsymbol{\beta}_{2}^{\prime} \mathbf{X}_{-i k}\right)$, then, for $\boldsymbol{\beta}_{1}-\boldsymbol{\beta}_{2} \equiv \boldsymbol{\beta}$, we have

$$
\begin{aligned}
\frac{\lambda_{i k}^{*}\left(\mathbf{X}_{k}\right)}{\sum_{j \in \mathcal{N}_{k}} \lambda_{j k}^{*}\left(\mathbf{X}_{k}\right)} & \approx \frac{\exp \left(\boldsymbol{\beta}_{1}^{\prime} \mathbf{X}_{i k}+\boldsymbol{\beta}_{2}^{\prime} \mathbf{X}_{-i k}\right)}{\sum_{j \in \mathcal{N}_{k}} \exp \left(\boldsymbol{\beta}_{1}^{\prime} \mathbf{X}_{i k}+\boldsymbol{\beta}_{2}^{\prime} \mathbf{X}_{-i k}\right)} \\
& =\frac{\exp \left(\boldsymbol{\beta}^{\prime} \mathbf{X}_{i k}\right)}{\sum_{j \in \mathcal{N}_{k}} \exp \left(\boldsymbol{\beta}^{\prime} \mathbf{X}_{j k}\right)}
\end{aligned}
$$

Expanding the product terms and adding noise terms $\eta_{i k}$ for all $i$, we obtain

$$
\begin{equation*}
\operatorname{Pr}(\text { firm } i \text { wins race } k)=\frac{\exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{i k}+\beta_{E} E_{i k}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{i k}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{i k}+\eta_{i k}\right)}{\sum_{j \in \mathcal{N}_{k}} \exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{j k}+\beta_{E} E_{j k}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{j k}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{j k}+\eta_{j k}\right)}, \tag{9}
\end{equation*}
$$

the multinomial logit function (MNL). $\boldsymbol{\beta}_{W}, \beta_{E}, \boldsymbol{\beta}_{\pi}$ and $\boldsymbol{\beta}_{C}$ are the parameters to estimate and $\eta_{i k}$ represents the characteristics of $i$ that are unobserved by the econometrician but known by all the firms.

The MNL is ideal to test the comparative statics of the equilibrium of the race because it maps the given characteristics of the game directly into the winning probabilities. As in equation (8), the MNL allows us to eliminate the equilibrium hazard rates and focus on the observable outcome, that is, who is the winner. Moreover, the MNL respects the fact that the winning probabilities are derived from the comparison of every competitor's vector of characteristics.

### 3.3 Specification

The main variables of interest in our model are the measures of financial wealth, W. The firm can use its own generated cash to finance its $R \& D$ expenditures internally or pledge its less liquid resources to reduce the cost of using external finance. It is therefore crucial to distinguish between the ability to use its own resources from the ability to borrow at a lower cost.

The vector, $\mathbf{W}$, includes the logarithm of the firm's cash holdings (COMPUSTAT item 36). The more cash available the more resources the firm can devote to R\&D and the more likely the firm is to win the race. W also includes the logarithm of the total value of the firm's assets as a measure of the firm's ability to finance its R\&D gap at lower borrowing costs: the larger the firm, the more it can pledge as collateral for a given amount to finance, and the more $\mathrm{R} \& \mathrm{D}$ it can undertake in equilibrium.

We include the total number of patents accumulated by the firm in the same class up to one year before the date of the award of the patent to control for the effectiveness of the firm in obtaining patents. We expect that players who have accumulated more patents in the past in the same class will be more experienced in the patenting process and thus be more likely to obtain a new patent, ceteris paribus.

To test whether the profits from the firm's pre-existing patents, which were denoted by $\pi_{i}$ in the model, increase or decrease the incentives to innovate we include proxies for $\pi_{i}$
into our empirical model. We term the effects of $\pi_{i}$ "incumbency" effects and measure them by an "incumbency index", which is a citation count of the cited patents constructed as follows. For each patent pool $k$ that we consider as a race we find all the patent pools that are cited by a patent in $k$ and consider the owners of these patents as incumbents to race $k$. We proxy the value of these cited patents by the number of citations they receive, that is by the citation count of the patents cited by a patent in pool $k$. To enrich our understanding of the incumbency effect, we distinguish the citations by vintages and include all vintages that are at most 20 years old into our specification. In addition, we also aggregate the citation counts of all vintages into an index for each firm in each race. Letting $\pi 0_{i k}, \pi 1_{i k}, \ldots, \pi 19_{i k}$ denote the number of citations received by all pools cited by pool $k$ that belong to firm $i$ that are $0,1, \ldots$, up to 20 years old, we define the incumbency index of firm $i$ in race $k$ as

$$
\begin{equation*}
I_{i k}=\sum_{a g e=0}^{19} \pi a g e_{i k} \times(20-a g e) \tag{10}
\end{equation*}
$$

Finally, we include in all specifications yearly dummies as controls. Yearly dummies capture exogenous aggregate changes in financing conditions or additional changes in procedures in the US Patent Office.

### 3.4 Estimation and instruments

In order to estimate the parameters of the model in (8) by maximum likelihood, we need to ensure that $\eta_{i k}$ is uncorrelated with the observable characteristics. While experience and the incumbency index are obviously given at the time the race starts, cash holdings are the result of cash management and are therefore endogenous. To estimate $\boldsymbol{\beta}_{W}$ consistently, we use a set of instruments for cash that are predetermined to the race, in order to rule out any residual correlation between $\eta_{i k}$ and the projection of cash on said instruments. We use i) the logarithms of cash, total debt, total assets and sales two and three years before the patent application; ii) the averages of each of the previous variables for all the other rival firms, $j \neq i$, in the same race; iii) the average patenting experience for all other rival firms, $j \neq i$, in the same race; and iv) the average incumbency index per firm per vintage for all other rival firms in the same race.

Following the literature on the demand for cash holdings (Opler et al., 1999; Almeida,
et al. 2004), we use the lags of cash and total assets to capture cross-sectional differences in the levels of cash and the lags of sales and debt to capture cross-sectional differences in the changes in cash holdings. Following the new empirical industrial organization tradition, we use the rivals' experience and incumbency indices as measures of their expected activity level in the race. Indeed, if cash is chosen to minimize the need for external finance and its costs, then this choice will ultimately depend on the rivals' average characteristics.

There is one major advantage from using as instruments measures of the competitiveness of rivals in the race. While the firm's total cash holdings will be the sum of cash pledged to each race the firm is simultaneouly in, the projection of the race-specific characteristics of rivals in race $k$ on the total cash holdings will capture the component of total cash that the firm pledges to race $k$ only. Therefore, we can interpret our estimates of $\beta_{W}$ as the sensitivity of innovation to the cash pledged to the given race.

We estimate our model using a control function approach proposed by Petrin and Train (2003, 2009). We cannot use standard instrumental variables techniques because the estimation is non-linear. The estimation proceeds in two steps. In the first step we estimate $\eta_{i k}$ with a first stage regression of the endogenous variables on their instruments. In the second stage we compute the maximum likelihood estimates of (9) after including the first stage residuals, $\hat{\eta}_{i k}$, in the specification. Identification of $\beta_{W}$ is achieved if the instruments provide enough exogenous variation in cash holdings: thus, the estimate $\hat{\eta}_{i k}$ will not be linearly dependent on cash because, by construction, it is the residual of the regression of cash on its instruments. Following also Petrin and Train (2003, 2009), we use a bootstrap estimator for the parameter estimates' standard errors.

The main comparative statics result of our theoretical model is that the winning probability of any firm in a given race should be positively associated with its own wealth and negatively associated with any other firm's wealth. A rejection of the null hypothesis that $\boldsymbol{\beta}=0$ implies that winning the race is determined jointly by all the competitors' wealth levels.

## 4 Selecting the participants of a race

The remaining challenge is that, except for the winner, we do not observe which firms participate in a race. We now address this problem. One possibility would be to include all firms that are potentially in the race, e.g., all COMPUSTAT firms that typically file patents of the same classification. This set is clearly too large to include and estimate equation (9) by maximum likelihood. Therefore, we choose a subset of firms in addition to the winner and estimate (9) for all firms in it.

There are various ways to choose the subset. McFadden (1978) has demonstrated that the maximum likelihood estimator of the multinomial logit based only on a random selection of fixed size from the (large) universe of alternatives produces consistent estimates ${ }^{[13}$ While easy to implement, this strategy carries the risk of including firms that may have decided not to participate in the race after evaluating their chances given the competition. Indeed, it does not exploit our model's prediction of which firms are most likely to race. According to our model firms decide to enter the race only if their chances of winning the race conditional on entry are sufficiently high. Below we propose a selection of firms for the estimation based precisely on that prediction. In unreported results, but available upon request, we show that our method produces results in line with the random selection method, if not more precisely estimated, and that our estimator satisfies McFadden's (1978) sufficient conditions for consistency.

### 4.1 A useful distinction: incumbents versus newcomers

We find it useful to partition, for any given patent, the set of all potential racing firms into two sets depending on whether a firm owns prior technology that is cited by the patent or not. We term the cited firms incumbents and the non-cited firms newcomers or entrants to this particular race ${ }^{14}$ Formally, an incumbent is a firm with a strictly positive incumbency index, an entrant has an incumbency index of zero.

The set of all cited firms is observable. Panel A of Table II shows that $95 \%$ of pools of patents cite fewer than 10 firms (left column). Some of these citations are insignificant because they are too old or receive no citations themselves. The right column shows the
cumulative relative contribution of each firm's incumbency index to the total incumbency index of patent $k$. From (10), the total incumbency index is simply the sum of all firms' incumbency values, i.e., $I_{k}=\sum_{i \in \mathcal{N}_{k}^{C}} I_{i k}$. The cumulative incumbency index of the first four incumbents already concentrates an average $94 \%$ (median 100\%) of the patent's total incumbency value. Therefore, the set of firms that have a significant incumbency stake in race $k$ is likely to be captured by the few firms that own the most often cited patents that are cited by a patent in pool $k$. Hence, every selection of firms in the race will include the four firms with the four highest citation counts of cited patents.

## $<$ INSERT TABLE [II ABOUT HERE>

### 4.2 Selecting the newcomers

We treat any firm that has won at least one Drug or Medical patent in the same five-year period the patent was awarded as a potential newcomer to each race. From this set we select the firms that have - according to our model - the highest chances to win a race. To estimate which firms have the highest chances of winning, we follow Berry's (1994) approach and transform the non-linear MNL probabilities in (9) into a linear model.

Another way to understand equation (9) , is to interpret the left-hand side as the aggregate share of patents won by a given firm over a period of time $t$. Let $\mathcal{N}^{C}$ and $\mathcal{N}^{N C}$ be the sets of firms cited and not cited by any patent at time $t$, respectively, where $\mathcal{N} \equiv \mathcal{N}^{C} \cup \mathcal{N}^{N C}$. Note that $\mathcal{N}^{C}$ is observable, while $\mathcal{N}^{N C}$ is not. Let $s_{i t}$ be the share of patent pools that firm $i \in \mathcal{N}^{N C}$ wins in period $t$. Let $s_{0 t}$ be the share of patents won by any of the firms in $\mathcal{N}^{C}$ in period $t$. We demonstrate in the appendix that the logarithm of the relative share, $\frac{s_{i t}}{s_{0 t}}$, can be written as

$$
\begin{equation*}
\ln s_{i t}-\ln s_{0 t}=\beta_{0}+\boldsymbol{\beta}_{Y}^{\prime} \mathbf{d}+\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{i t}+\beta_{E} E_{i t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{i t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{i t}+\eta_{i t}, \tag{11}
\end{equation*}
$$

where $\mathbf{d}$ is a vector of the four yearly dummy variables in each five-year estimation sample. This transformation is very intuitive. It says that the difference between a non-cited firm's share of patents won in a year relative to the share of patents won by the cited firms is explained by the former firm's characteristics in the same period. This is simply because the
set of cited firms, and hence their characteristics, are held constant by construction. Hence, if we treat the unobservable $\eta_{i t}$ as the structural error of unobservable firm characteristics, we can estimate the parameters, $\beta_{0}, \boldsymbol{\beta}_{Y}, \boldsymbol{\beta}_{W}, \beta_{E}$ and $\boldsymbol{\beta}_{C}$ from a regression of $\ln s_{i t}-\ln s_{0 t}$ on $\mathbf{W}_{i t}, E_{i t}$ and $\mathbf{C}_{i t}$ for all potential racing firms in $t$. Because cash holdings are likely to be correlated with $\eta_{i t}$, we use an instrumental variables estimator and the set of instruments defined above. Each estimation sample contains a panel of five yearly patent shares crosssections of all non-cited firms, for each patent subclass and each quartile of the number of citations received $\sqrt{15}$

This procedure assumes that any non-cited firm evaluates its chances for every race based on its characteristics and all the others, using our model. Firms with a low rank drop out of the race early enough, so that eventually the predicted equilibrium racing behavior is driven by the characteristics of the subset of firms who have a "fair" chance, that is, whose predicted probability of winning is positive. The main limitation of this approach is that firms with little or no past success will be included in races they won, but not in races where they lost despite having a good (unobservable) chance of winning. It is difficult to assess how this possible omission affects our results. On the one hand, we could be underestimating the effect of financing constraints if these firms were also young and with limited access to external finance. On the other hand, because it is likely that firms with good chances eventually become winners, the risk of omission will be smaller for the late sample periods, when these firms are more mature and their success is explained more by observable factors.

The approach has several virtues: (i) the dimensionality of the selection problem is transformed into the number of cross-sectional units in the panel, so that we can use a very large number of potential entrants every period; (ii) we can use a straightforward instrumental variables estimator to address the endogeneity of $\mathbf{W}$ because the model is estimable by linear methods; (iii) because the dependent variable, $\ln s_{i t}-\ln s_{0 t}$, depends only on firm $i$ 's characteristics, the instruments based on the characteristics of firm $j \neq i$ automatically satisfy the exclusion restriction; and (iv) the dependent variable is by itself the score we use to rank firms in terms of the likelihood of participating in each race. Indeed, the predicted difference $\ln s_{i t}-\ln s_{0 t}$ ranks all firms active in $t$ according to the probability that they might win against a given set of cited firms. As we have shown above, the best response effort level of
a firm facing very aggressive rivals is zero, and it opts out of the race.

### 4.3 Selection stage results

We compute the score $\hat{\beta}_{0}+\hat{\boldsymbol{\beta}}_{1}^{\prime} \mathbf{d}+\hat{\boldsymbol{\beta}}_{W}^{\prime} \mathbf{W}_{i t-1}+\hat{\beta}_{E} E_{i t-1}+\hat{\boldsymbol{\beta}}_{C}^{\prime} \mathbf{C}_{i t}$ for all firms in $t$. This score is the predicted probability that a firm wins a representative period $t$ patent from the set of all non cited firms. We rank firms according to their score within the year and within the citation count quartile. We generate 285 rankings: one for each year ( 25 years), subclass (between 2 and 3), and total citation count quartile. Panel B of Table $I$ reports the average cumulative scores for the top ranked firms. The predicted probability that the winner is within the top ten firms, given that the winner is a non-cited firm, is on average 0.88 (median of 1 ). The winner is almost surely within the top fifteen. Because there is little gain, and large computational costs, to include more firms, we select the top ten firms to be the set of non cited firms, $\mathcal{N}^{N C}$, that race for each patent pool in the same year, of the same subclass, and in the same total citation count quartile. As a robustness check, we have estimated the models that follow with fifteen non-cited firms in the last five year period and have observed very similar results. They are available to the reader upon request. We note too that our selection always includes the actual winner.

Based on our results above, we let the set $\mathcal{N}_{k}$ contain the four cited firms with the highest incumbency index and the ten entrants with the highest estimated winning scores in the same year, subclass, and total citation count quartile. Table III summarizes the main characteristics of this selection. It shows that firms hold between $10 \%$ and $12 \%$ of their assets in cash. While the proportion of cash to assets has not changed much over time, the skewness of the distribution of cash across players has increased over time.
<INSERT TABLE III ABOUT HERE>

## 5 Estimates of the racing model

This section describes our results from estimating the parameters in (9), using the set of 14 pre-selected firms (four from the citations list, ten from the non-cited set). The estimates
are obtained by maximum likelihood, and Petrin and Train's (2003, 2009) control function method to instrument for endogenous cash holdings.

### 5.1 Internal finance

Our model predicts that the probability that a firm wins an average patent in each period-category-citation count cluster depends positively on the firm's own cash holdings and negatively on the competitors', i.e., that $\beta_{W}>0$. Table IV confirms that prediction for all pools of patents in the three upper quartiles of citation counts as from 1985, and before that, for the pools in the fourth quartile of citation counts. The lack of significance in most estimation clusters before 1985 must be interpreted with caution: those years concentrate many more patents that receive relatively few citations, where it is less likely that the pools constructed effectively represent a technology race. As the patenting activity increases, and the patents' adjusted number of citations becomes larger this source of noise should become less important. Indeed, after 1985, we find a significant effect of cash holdings on the winning probability in all except the lowest citation count quartile.

## <INSERT TABLE IV] ABOUT HERE>

Patenting experience has a positive and significant effect in all cases, in line with our expectations. There is no clear pattern regarding the effect of the cited patents' citation counts. Whenever the effect is significant, the more valuable the firm's one year old or younger patents are, the less likely it is the firm wins the next race. We find an opposing effect for patents between 2 to 5 years in some cases. This is in line with an ambiguous effect of the same variable in our theoretical model. On the one hand, the firm is less financially constrained the more valuable the patents the firm currently owns. On the other hand, the more valuable the previous patents the smaller the incentive of a firm to make these patents obsolete by inventing new products. In addition to these effects that are present in our theory, there are also experience effects: previous innovations may create better technological opportunities to the previous winners (incumbents) than to the previous losers (entrants). We believe that our estimates are more likely to capture the first two effects. Indeed, the citation count coefficient will capture technological opportunity only to the
extent that it favours one type of firm more than the other because the left hand side of (8) is the probability of winning conditional on the fact that there is a winner. Hence, the component of technological opportunity common to all players cancels out. Further, some of the advantageous effects of technological opportunity through incumbency may disappear through the public disclosure of the new technology after the patent is announced.

Note that the first stage error component is significant almost everywhere. This implies that our first stage control function approach has effectively captured some of the important correlated unobservable components.

### 5.2 Internal vs. External Finance

Our model implies also that, given a level of cash, the firm's borrowing capacity should increase its probability of winning a patent pool and decrease that of its rivals. Table V shows the results of adding the logarithm of the total value of assets to our previous specification. The predicted effect is present in all top three citations quartiles since 1985, and in the fourth quartile since 1975. Moreover, the effect of cash has strengthened with respect to the previous specification.

## <INSERT TABLE V ABOUT HERE>

To interpret the economic significance of these coefficients, we have computed the predicted change in the probability of winning a patent pool with respect to an increase in one standard deviation about the mean of cash, total assets or patenting experience. Both cash and total assets have an economically significant effect on the winning probability. For example, between 1995 and 1999, a firm won a race for a patent pool in the highest citation count quartile with an average probability of 0.08 ; an increase of a one standard deviation amount of cash would have increased this probability by 0.047 , that is by almost $60 \%$. A similar increase in the amount of total assets would have doubled its chances. The winning probability is in general more sensitive to assets than to cash. This confirms our earlier point that COMPUSTAT firms have already been successful in obtaining external finance. Notably, the sensitivity of innovation to experience looks steady over time but in the case of cash and total assets, this sensitivity has increased.

## <INSERT TABLE VI ABOUT HERE>

Table A.II in the appendix, serves as a useful benchmark to compare the effects of our instruments on the estimates of the sensitivity of innovation to cash. Panel B shows the standard maximum likelihood estimates of (9), that is without instruments, for the same specification reported in Table V. The estimates without instruments slightly overestimate the sensitivity of cash. It seems therefore that, within the set of selected firms, the unobservable firm characteristics that make a racing firm more competitive may be positively, but weakly, correlated with total cash. Panel A shows that the OLS estimates of the selection stage underestimate the patenting sensitivity to cash with respect to the IV estimates (Table A.II).

## 6 Evidence from R\&D data

### 6.1 Method

Our model also has implications about the $R \& D$ intensity chosen by all firms in a race. Indeed, firms choose the hazard rate indirectly through their R\&D expenditures. Provided that this mapping is one to one, the comparative statics of the firm's winning probability with respect to changes in its characteristics are identical to the comparative statics of R\&D with respect to changes in the firm's characteristics. Under the null hypothesis that the firms engage in a race, $R \& D$ is determined in a system of equations like (7) where $R \& D$ is the dependent variable. As a result, the correlation of R\&D levels across players within the same race should be different from zero. We test these comparative statics by treating each race as a panel unit, $k$, where the observations in each unit are the firms in the race, i.e., all $i \in \mathcal{N}_{k}$. The regression model we use is

$$
\ln R \& D_{i k}=\gamma_{W}^{\prime} \mathbf{W}_{i k}+\gamma_{E} E_{i k}+\gamma_{\pi}^{\prime} \boldsymbol{\pi}_{i k}+\gamma_{C}^{\prime} \mathbf{C}_{i k}+\eta_{i k}+v_{k}+u_{i k},
$$

where the $v_{k}$ is the component in $\mathrm{R} \& \mathrm{D}$ that is common to all firms racing for the same pool of patents. We estimate $v_{k}$ as a random or a fixed effect, and compute the proportion of the variation in individual $\mathrm{R} \& \mathrm{D}$ that it is attributed to this effect. We also use an instrumental variables panel estimator, to account for the endogeneity of cash holdings, which are specified in $\mathbf{W}_{i k}$.

Cockburn and Henderson (1994) estimate the same model with project-specific data from a survey of ten large firms in the pharmaceutical industry and could not reject that $v_{k}=0$. While their level of disaggregation is ideal, the limited coverage of firms may have missed potentially important correlations between the R\&D expenditures of smaller entrants and the large firms. The difference here is that, as we have shown above, we devise a procedure that selects the firms most likely to be in $\mathcal{N}_{k}$ from the universe of publicly traded firms who have filed at least one pharmaceutical patent. While here we measures $v_{k}$ as the correlation in aggregate $\mathrm{R} \& \mathrm{D}$, we note that this correlation is (i) over and above the common time effects and (ii) between firms that we identify as being in that particular race only. For any two firms $i, j$ that have at least one race in common, a necessary condition for the residual correlation between their aggregate $\mathrm{R} \& \mathrm{D}$ levels to be zero is that the correlation between $R \& D$ at every race the two overlap is zero. Therefore, rejection of this hypothesis implies that there is at least one race where they race against each other and where their R\&D is correlated.

Table VII displays our results for the periods of 1990 to 1994 and 1995 to 1999. We report the efficient, random effect estimates whenever we cannot reject that the estimator is consistent. Otherwise, we report the fixed effects estimator. COMPUSTAT coverage for R\&D intensity in the early sample is limited, resulting in a significant loss of observations. We omit these results here. They are available to the reader upon request.

### 6.2 Results

Our estimates imply that an increase in the logarithm of the firm's cash holdings or an increase in the logarithm of total assets are associated with a significant increase in the logarithm of R\&D (Table VII). These estimates can be directly interpreted as elasticities. Because the instruments for cash holdings are based on the measures of competitiveness of the firms rivals in that given race, the coefficient of cash measures the conditional covariance between firm-level R\&D and cash holdings at the race level. The most striking result is the sharp increase in the sensitivity of $\mathrm{R} \& \mathrm{D}$ with respect to own cash holdings: a doubling of cash holdings increases total firm R\&D by at most $43 \%$ between 1990 and 1994. Between 1995 and 1999 a $100 \%$ increase in cash holdings doubles the total level of R\&D.

## <INSERT TABLE VII ABOUT HERE>

While the dependent variable is firm-level $R \& D$, our panel unit is race-specific. Therefore, once the set of firms in a race is defined, we are able to measure the race-specific R\&D component, $v_{k}$. Our results show that this component is very important: for patent pools in the upper half of the distribution of total citations received, the variation in the estimated common race component explains between $7.4 \%$ and up to $47 \%$ of the total variation in total firm R\&D explained by the model. This novel result must be interpreted with caution. Our estimate of $v_{k}$ is only accurate to the extent that our selection of firms considered as rivals in the same race is precise. Because our method tends to select either (i) firms that have been most successful in the given patent subclass or (ii) firms whose patents have been heavily cited, a more accurate interpretation of our evidence is that the R\&D intensity of firms that have been successfully patenting in the same line of technology is highly correlated.

## 7 Discussion

The empirical analysis above has shown that the cross-sectional variation in the ratio of cash holdings to total assets of publicly traded firms is a powerful determinant of the crosssectional variation in the probability of winning drugs and medical patents. We have identified this effect through the comparison of success rates across races and across incumbents and entrants to these races. Therefore, innovative success depends on how much more cash the firm has relative to its rivals.

The theoretical relationship tested by this data is itself very robust. Indeed, the empirical specification is derived directly from a Nash equilibrium where firms are optimally financed at any point on their best-response function. This feature distinguishes our approach from others in the literature that analyze best-response behavior keeping the financing contract fixed as the financing needs of the firm change (e.g., Chevalier, 1995; Jensen and Showalter 2004).

Our model distinguishes firms in an industry in terms of their technological standing. The empirical analysis isolates the effects of patenting experience from those of incumbency
by counting separately the cited and non-cited patents the firm has accumulated. We have shown that incumbents keep on innovating more often the more valuable their cited patents of age below two years are and the less valuable their older cited patents are (as measured by the number of citations these patents receive).

We end with an account of what we feel are limitations of our work. Our theory is arguably simple compared to the complexity of the firms in our sample. We are confident that a more complex theory would share the same comparative statics features, but we leave a detailed analysis of this case to future work. Our empirical analysis is based on our predictions of which firms will be in the race rather than actual data on whether they are in it or not. Future research could focus on collecting a comprehensive data set on project specific data. Another important step in this line of research is to repeat our exercise for the case of private firms. This paper identifies powerful effects of cash differences across COMPUSTAT firms only. While it is difficult to generalize our empirical results to private firms and startups, we would conjecture that financing constraints have an even more pronounced effect on the behavior of these firms.

Finally, we study sequences of races but not the evolution of particular firms within the industry. A further interesting question for future research is how the financing constraints of firms evolve over time as they accumulate patents and how this affects the dynamics of industry structure. We pursue these questions in ongoing research.

## Appendix 1: Proofs

Lemma 1 i) The first-best level of effort is implementable if and only if $\frac{h V_{i}^{-}+\pi_{i}}{h+r} \geq F-W_{i}$. ii) A second best contract takes the form $\mathbf{s}_{i}^{*} \equiv\left(1,1, s_{i}^{+}\right)$for some $s_{i}^{+} \in[0,1)$.

Proof of Lemma 1. i) Let $V_{i}(h)$ be the first-best value of firm $i . V_{i}(h)$ is defined by the asset equation

$$
r V_{i}(h) d t=\max _{a_{i}}\left\{a_{i}^{\alpha}\left(V_{i}^{+}-V_{i}(h)\right)+h\left(V_{i}^{-}-V_{I}(h)\right)+\pi_{i}-a_{i}\right\} d t .
$$

The problem on the right hand side of this asset equation is a strictly concave in $a_{I}$. The first-order condition is

$$
\begin{equation*}
\alpha a_{i}^{* \alpha-1}\left(V_{i}^{+}-V_{i}(h)\right)=1, \tag{12}
\end{equation*}
$$

If we multiply both sides of (12) by $a_{i}^{*}$, and substitute the resulting equality into the asset equation, we can solve for the value of the firm:

$$
\begin{equation*}
V_{i}(h)=\frac{(1-\alpha) a_{i}^{* \alpha} V_{i}^{+}+h V_{i}^{-}+\pi_{i}}{(1-\alpha) a_{i}^{* \alpha}+h+r} \tag{13}
\end{equation*}
$$

Substituting back into equation (12), we observe that $a_{i}^{*}$ is the unique solution to the equation

$$
\begin{equation*}
\alpha\left((h+r) V_{i}^{+}-\left(h V_{i}^{-}+\pi_{i}\right)\right)=a_{i}^{* 1-\alpha}\left((1-\alpha) a_{i}^{* \alpha}+h+r\right) \tag{14}
\end{equation*}
$$

With financing, the asset equation takes the form

$$
\begin{equation*}
r V_{i}(\cdot) d t=\max _{a_{i}}\left\{a_{i}^{\alpha}\left(\left(1-s_{i}^{+}\right) V_{i}^{+}-V_{i}(\cdot)\right)+h\left(\left(1-s_{i}^{-}\right) V_{i}^{-}-V_{i}(\cdot)\right)+\left(1-s_{i}\right) \pi_{i}-a_{i}\right\} d t . \tag{15}
\end{equation*}
$$

Let $\mathbf{s}_{i} \equiv\left(s_{i}, s_{i}^{-}, s_{i}^{+}\right)$. Since the right-hand-side of the asset equation is strictly concave in $a_{i}$, a solution to (15) must satisfy the first-order condition

$$
\begin{equation*}
\alpha a_{i}\left(\mathbf{s}_{i}\right)^{\alpha-1}\left(\left(1-s_{i}^{+}\right) V_{i}^{+}-V_{i}(\cdot)\right)=1 \tag{16}
\end{equation*}
$$

Multiplying condition (16) on both sides by $a_{i}\left(\mathbf{s}_{i}\right)$ and substituting the resulting expression into (15) we solve for the value of the firm's claim

$$
\begin{equation*}
V_{i}\left(h, \mathbf{s}_{i}\right)=\frac{(1-\alpha) a_{i}\left(\mathbf{s}_{i}\right)^{\alpha}\left(1-s_{i}^{+}\right) V_{i}^{+}+h\left(1-s_{i}^{-}\right) V_{i}^{-}+\left(1-s_{i}\right) \pi_{i}}{(1-\alpha) a_{i}\left(\mathbf{s}_{i}\right)^{\alpha}+h+r} \tag{17}
\end{equation*}
$$

In addition, investors must break even. Formally, it must be true that

$$
\begin{equation*}
\frac{a_{i}\left(\mathbf{s}_{i}\right)^{\alpha} s_{i}^{+} V_{i}^{+}+h s_{i}^{-} V_{i}^{-}+s_{i} \pi_{i}}{a_{i}\left(\mathbf{s}_{i}\right)^{\alpha}+h+r}=F-W_{i} . \tag{18}
\end{equation*}
$$

An optimal contract maximizes (17) subject to (18) and (16) .
We now show that a contract implementing the first-best level of effort provision is feasible if and only if

$$
\frac{h V_{i}^{-}+\pi_{i}}{h+r} \geq F-W_{i} .
$$

The first-best is feasible if and only if there exists a contract that allows investors to break even, and, at the same time, does not distort the marginal incentive to provide effort in research. That is, the differences in values on the left hand side of conditions (12) and (16) must be identical:

$$
\left(1-s_{i}^{+}\right) V_{i}^{+}-V_{i}\left(h, \mathbf{s}_{i}\right)=V_{i}^{+}-V_{i}(h) .
$$

Substituting from equations (17) and (13) we obtain

$$
\begin{aligned}
& \left(1-s_{i}^{+}\right) V_{i}^{+}-\frac{(1-\alpha) a_{i}\left(\mathbf{s}_{i}\right)^{\alpha}\left(1-s_{i}^{+}\right) V_{i}^{+}+h\left(1-s_{i}^{-}\right) V_{i}^{-}+\left(1-s_{i}\right) \pi_{i}}{(1-\alpha) a_{i}\left(\mathbf{s}_{i}\right)^{\alpha}+h+r} \\
= & V_{i}^{+}-\frac{(1-\alpha) a_{i}^{* \alpha} V_{i}^{+}+h V_{i}^{-}+\pi_{i}}{(1-\alpha) a_{i}^{* \alpha}+h+r} .
\end{aligned}
$$

Clearly, by the definition of first-best, $a_{i}^{*}=a_{i}\left(\mathbf{s}_{i}\right)$. Exploiting this fact we can simplify the condition on the equality of margins to the following simple condition

$$
\begin{equation*}
h s_{i}^{-} V_{i}^{-}+s_{i} \pi_{i}=s_{i}^{+} V_{i}^{+}(h+r) . \tag{19}
\end{equation*}
$$

In addition, investors must break even, i.e., condition 18 must be respected. Substituting condition (19) into condition (18) we obtain the relation

$$
\begin{equation*}
s_{i}^{+} V_{i}^{+}=F-W_{i} . \tag{20}
\end{equation*}
$$

Substituting condition (20) back into condition (19) we obtain

$$
\begin{equation*}
\frac{h s_{i}^{-} V_{i}^{-}+s_{i} \pi_{i}}{h+r}=F-W_{i} \tag{21}
\end{equation*}
$$

The first-best is thus feasible if and only if we are able to find nonnegative numbers $\mathbf{s}_{i}=$ $\left(s_{i}, s_{i}^{-}, s_{i}^{+}\right)$smaller or equal to one that satisfy conditions 20 and 21). If $W_{i} \geq 0$ and $V_{i}^{+}>F$ then it is always possible to find a $s_{i}^{+}<1$ such that $s_{i}^{+} V_{i}^{+}=F-W_{i}$. Hence condition (21) is the crucial one. We can find numbers $s_{i}^{-}$and $s_{i}$ both smaller or equal to one that satisfy the implementability condition if and only if

$$
\begin{equation*}
\frac{h V_{i}^{-}+\pi}{h+r} \geq F-W_{i} \tag{22}
\end{equation*}
$$

The derivative of the left-hand side of inequality (22) with respect to $h$ is equal to $\frac{V_{i}^{-} r-\pi_{i}}{(h+r)^{2}}$, which is negative. Since the left-hand side tends to zero as $h$ tends to infinity, there exists a strictly positive value of $\bar{h}^{F B}$ such that (22) holds with equality if and only if $\frac{\pi_{i}}{r}>F-W_{i}$. In that case $\bar{h}^{F B}$ is defined by the condition

$$
\left.\frac{h V_{i}^{-}+\pi_{i}}{h+r}\right|_{h=\bar{h}^{F B}}=F-W_{i} .
$$

ii) follows directly from (21) and (22).

Proof of Proposition 1. ii) is a direct consequence of the Lemma above; hence it suffices to prove i). An equilibrium satisfies the condition

$$
a_{i}=b_{i}\left(b_{j}\left(a_{i} ; W_{j}, \cdot\right) ; W_{i}, \cdot\right)
$$

Differentiating totally with respect to $a_{j}^{*}, W_{i}$, and $W_{j}$, we get

$$
\left(1-\frac{\partial b_{i}}{\partial a_{j}} \frac{\partial b_{j}}{\partial a_{i}}\right) d a_{i}^{*}=\frac{\partial b_{i}}{\partial a_{j}} \frac{\partial b_{j}}{\partial W_{j}} d W_{j}+\frac{\partial b_{i}}{\partial W_{i}} d W_{i}
$$

Setting $d W_{i}$ and $d W_{j}$, respectively, equal to zero we find

$$
\begin{equation*}
\frac{d a_{i}^{*}}{d W_{i}}=\frac{\frac{\partial b_{i}}{\partial W_{i}}}{\left(1-\frac{\partial b_{i}}{\partial a_{j}} \frac{\partial b_{j}}{\partial a_{i}}\right)} \tag{23}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d a_{i}^{*}}{d W_{j}}=\frac{\frac{\partial b_{i}}{\partial a_{j}} \frac{\partial b_{j}}{\partial W_{j}}}{\left(1-\frac{\partial b_{i}}{\partial a_{j}} \frac{\partial b_{j}}{\partial a_{i}}\right)} \tag{24}
\end{equation*}
$$

By the fact that $\left|\frac{d b_{i}\left(a_{j} ; W_{i}, \cdot\right)}{d a_{j}}\right|<1$ for $i=1,2$ and $j \neq i$, the denominators in these expressions are positive, and since $\frac{\partial b_{i}}{\partial W_{i}}>0$ for $i=1,2$ it follows that $\frac{d a_{i}^{*}}{d W_{i}}>0$. Switching indices, (24) gives an expression for $\frac{d a_{j}^{*}}{d W_{i}}$. In particular, we have $\frac{d a_{j}^{*}}{d W_{i}}=\frac{\frac{\partial b_{j}}{\partial a_{i}} \frac{\partial b_{i}}{\partial W_{i}}}{\left(1-\frac{\partial b_{j}}{\partial a_{i}} \frac{\partial b_{i}}{\partial a_{j}}\right)}$. Since $\left|\frac{d b_{j}\left(a_{i} ; W_{j}, \cdot\right)}{d a_{i}}\right|<1$, we have $\frac{d a_{j}^{*}}{d W_{i}}<\frac{d a_{i}^{*}}{d W_{i}}$.

Proof of Proposition 2. The probability that firm $i$ wins the race is equal to the probability that firm $i$ 's "first" innovation arrives before firm $j$ 's "first" innovation. The arrival times follow independent Poisson distributions with hazard rates $a_{i}^{* \alpha}$ and $a_{j}^{* \alpha}$, respectively. So the arrival time of the first innovation has probability distribution function $1-\exp \left(-a_{i}^{* \alpha} t\right)$ for $i=1,2$. Hence, the probability that firm $i$ innovates first is

$$
\int_{0}^{\infty} a_{i}^{* \alpha} \exp \left(-a_{i}^{* \alpha} t\right)\left(1-\left(1-\exp \left(-a_{j}^{* \alpha} t\right)\right)\right) d t=\frac{a_{i}^{* \alpha}}{a_{i}^{* \alpha}+a_{j}^{* \alpha}}
$$

Differentiating $\frac{a_{i}^{* \alpha}}{a_{i}^{* \alpha}+a_{j}^{* \alpha}}$ with respect to $W_{i}$ we obtain

$$
\begin{aligned}
\frac{\partial}{\partial W_{i}} \frac{a_{i}^{* \alpha}}{a_{i}^{* \alpha}+a_{j}^{* \alpha}} & =\frac{\alpha a_{i}^{* \alpha-1}\left(a_{i}^{* \alpha}+a_{j}^{* \alpha}\right) \frac{d a_{i}^{*}}{d W_{i}}-\left(\alpha a_{i}^{* \alpha-1} \frac{d a_{i}^{*}}{d W_{i}}+\alpha a_{j}^{* \alpha-1} \frac{d a_{j}^{*}}{d W_{i}}\right) a_{i}^{* \alpha}}{\left(a_{i}^{* \alpha}+a_{j}^{* \alpha}\right)^{2}} \\
& =\frac{\alpha a_{i}^{* \alpha} a_{j}^{* \alpha}}{\left(a_{i}^{* \alpha}+a_{j}^{* \alpha}\right)^{2}}\left(\frac{\frac{d a_{i}^{*}}{d W_{i}}}{a_{i}^{*}}-\frac{\frac{d a_{j}^{*}}{d W_{i}}}{a_{j}^{*}}\right)
\end{aligned}
$$

So, we have $\frac{\partial}{\partial W_{i}} \frac{a_{i}^{* \alpha}}{a_{i}^{* \alpha}+a_{j}^{* \alpha}}>0$ iff $\frac{d a_{i}^{*}}{d W_{i}}>\frac{a_{i}^{*}}{a_{j}^{*}} \frac{d a_{j}^{*}}{d W_{i}}$. Cancelling terms on both sides this is equivalent to $\frac{a_{j}^{*}}{a_{i}^{*}}>\frac{\partial b_{j}}{\partial a_{i}}\left(a_{i}^{*} ; W_{j}, \cdot\right)$. We now show that this condition is indeed verified: applying the implicit function theorem to condition (6), we have
$\frac{d a_{j}^{*}}{d a_{i}}=\frac{\left(\alpha\left(V_{j}^{-}-\left(F-W_{j}\right)\right)\left(a_{i}^{\alpha}+r\right)+\alpha\left(a_{j}^{* \alpha} V_{j}^{+}+a_{i}^{\alpha} V_{j}^{-}+\pi_{j}-\left(a_{j}^{* \alpha}+a_{i}^{\alpha}+r\right)\left(F-W_{j}\right)\right)-a_{j}^{*}\right) \alpha a_{i}^{\alpha-1}}{-\left(\alpha^{2} a_{j}^{* \alpha-1}\left(V_{j}^{+}-\left(F-W_{j}\right)\right)\left(a_{i}^{\alpha}+r\right)-\left(\left(1-\alpha^{2}\right) a_{j}^{* \alpha}+a_{i}^{\alpha}+r\right)\right)}$

Using condition (6) (and some straightforward manipulations) to simplify expression (25) we obtain

$$
\frac{d a_{j}^{*}}{d a_{i}}=\frac{a_{i}^{\alpha}}{a_{i}^{\alpha}+r} \frac{a_{j}^{*}}{a_{i}} \Gamma .
$$

where

$$
\Gamma \equiv \frac{\left(\alpha\left(V_{j}^{-}-\left(F-W_{j}\right)\right)\left(a_{i}^{\alpha}+r\right)+\alpha\left(a_{j}^{* \alpha} V_{j}^{+}+a_{i}^{\alpha} V_{j}^{-}+\pi_{j}-\left(a_{j}^{* \alpha}+a_{i}^{\alpha}+r\right)\left(F-W_{j}\right)\right)-a_{j}^{*}\right)}{-\left(\alpha a_{j}^{* \alpha}\left(V_{j}^{+}-\left(F-W_{j}\right)\right)-\frac{a_{j}^{*}\left(\left(1-\alpha^{2}\right) a_{j}^{* \alpha}+a_{i}^{\alpha}+r\right)}{\alpha\left(a_{i}^{\alpha}+r\right)}\right)}
$$

Since $\frac{a_{i}^{\alpha}}{a_{i}^{\alpha+r}}<1$, we have $\frac{a_{j}^{*}}{a_{i}^{*}}>\frac{\partial b_{j}}{\partial a_{i}}\left(a_{i}^{*} ; W_{j}, \cdot\right)$ if $\Gamma<1$. Using (6) again, and simplifying terms, we find $\Gamma<1$ if and only if

$$
\left(\alpha\left(V_{j}^{-}-\left(F-W_{j}\right)\right)\left(a_{i}^{\alpha}+r\right)\right)<\left(\hat{a}_{j}^{\alpha}(1-\alpha) V_{j}^{+}+a_{i}^{\alpha} V_{j}^{-}+\pi_{j}-\left(\hat{a}_{j}^{\alpha}(1-\alpha)+a_{i}^{\alpha}+r\right)\left(F-W_{j}\right)\right) .
$$

From (6) one can verify that the right-hand side of this expression is positive. The left-hand side must be negative. If it were positive, then first-best financing would be possible, because the value of a losing firm would be sufficient to cover the cost of the investment. Hence, we have shown that $\frac{a_{j}^{*}}{a_{i}^{*}}>\frac{\partial b_{j}}{\partial a_{i}}\left(a_{i}^{*} ; W_{j}, \cdot\right)$.

Likewise, $\frac{\partial}{\partial W_{j}} \frac{a_{i}^{* \alpha}}{a_{i}^{*}+a_{j}^{* \alpha}}<0$ iff $a_{j}^{*} \frac{d a_{i}^{*}}{d W_{j}}<a_{i}^{*} \frac{d a_{j}^{*}}{d W_{j}}$, which is after cancelling terms, equivalent to $\frac{\partial b_{i}}{\partial a_{j}}<\frac{a_{i}^{*}}{a_{j}^{*}}$. Up to an interchange of indices, exactly the same argument as given above can be used to show that indeed $\frac{\partial b_{i}}{\partial a_{j}}<\frac{a_{i}^{*}}{a_{j}^{*}}$; this is omitted.

Proof of Proposition 3. Denote the set of firms as $\mathcal{N}=\{1,2, \ldots, n\}$ and its partition $\{i, \mathcal{N} \backslash i\}$. Consider first any firm $j \in \mathcal{N} \backslash i$. Let $\tilde{h}=\sum_{k \neq j} a_{k}^{\alpha}$. From (14), we can write firm $j$ 's best reply as the solution to the equation

$$
\alpha a_{j}^{* \alpha}(\tilde{h}+r) V^{+}=a_{j}^{*}\left((1-\alpha) a_{j}^{* \alpha}+\tilde{h}+r\right),
$$

where we have used $V_{j}^{-}=\pi_{j}=0$. Imposing symmetry among firms $j \in \mathcal{N} \backslash i$, we can write

$$
\tilde{h}=(n-2) a_{j}^{\alpha}+a_{i}^{\alpha} .
$$

Substituting back, we obtain

$$
\alpha a_{j}^{* \alpha}\left((n-2) a_{j}^{* \alpha}+a_{i}^{\alpha}+r\right) V^{+}=a_{j}^{*}\left((n-1-\alpha) a_{j}^{* \alpha}+a_{i}^{\alpha}+r\right) .
$$

Changing variables to $h \equiv(n-1) a_{j}^{\alpha}$ and rearranging, we can write

$$
\begin{equation*}
\alpha \frac{h^{*}}{n-1}\left(\frac{n-2}{n-1} h^{*}+a_{i}^{\alpha}+r\right) V^{+}-\left(\frac{h^{*}}{n-1}\right)^{\frac{1}{\alpha}}\left(\frac{n-1-\alpha}{n-1} h^{*}+a_{i}^{\alpha}+r\right)=0, \tag{26}
\end{equation*}
$$

which corresponds to the best response function of the set of firms $j \in \mathcal{N} \backslash i$. Denote the solution of this function for given $a_{i}$ as $\tilde{b}\left(a_{i}\right)$.

Firm $i$ 's best reply is still given by (6)

$$
\begin{equation*}
\alpha\left(a_{i}^{* \alpha} V_{i}^{+}+h V_{i}^{-}+\pi_{i}-\left(a_{i}^{* \alpha}+h+r\right)\left(F-W_{i}\right)\right)(h+r)-a_{i}^{*}\left((1-\alpha) a_{i}^{* \alpha}+h+r\right)=0 . \tag{27}
\end{equation*}
$$

The solution to this equation is denoted $b_{i}\left(h ; W_{i}\right)$.
To prove our result, we need to show that

$$
\frac{\partial}{\partial W_{i}} \frac{a_{i}^{* \alpha}}{a_{i}^{* \alpha}+h^{*}}=\frac{\alpha a_{i}^{* \alpha} h^{*}}{\left(a_{i}^{* \alpha}+a_{j}^{* \alpha}\right)^{2}}\left(\frac{\frac{d a_{i}^{*}}{d W_{i}}}{a_{i}^{*}}-\frac{\frac{d h^{*}}{d W_{i}}}{\alpha h^{*}}\right)>0
$$

From the equilibrium condition, $a_{i}^{*}=b_{i}\left(\tilde{b}\left(a_{i}^{*}\right) ; W_{i}\right)$ we get $\frac{d a_{i}^{*}}{d W_{i}}=\frac{\frac{\partial b_{i}}{\partial W_{i}}}{\left(1-\frac{\partial b_{i}}{\partial h} \frac{\partial \bar{b}}{\partial a_{i}}\right)}$ and from $h^{*}=$ $\tilde{b}\left(b_{i}\left(h^{*} ; W_{i}\right)\right)$ we get $\frac{d h^{*}}{d W_{i}}=\frac{\frac{\partial \tilde{b}}{\partial a_{i}} \frac{\partial b_{i}}{\partial W_{i}}}{\left(1-\frac{\partial b_{i}}{\partial h} \frac{\partial \bar{b}}{\partial a_{i}}\right)}$. Stability implies that $\frac{\partial b_{i}}{\partial h} \frac{\partial \tilde{b}}{\partial a_{i}}<1$. So, $\frac{\frac{d a_{i}^{*}}{d W_{i}}}{a_{i}^{*}}-\frac{\frac{d h^{*}}{d W_{i}}}{\alpha h^{*}}>0$ if and only if $\frac{\partial \tilde{b}}{\partial a_{i}}<\frac{\alpha h^{*}}{a_{i}^{*}}$. By straightforward calculus, we have

$$
\frac{d h}{d a_{i}}=\frac{\left[\alpha^{2} \frac{h^{*}}{n-1} a_{i}^{\alpha-1} V^{+}-\alpha a_{i}^{\alpha-1}\left(\frac{h^{*}}{n-1}\right)^{\frac{1}{\alpha}}\right]}{-\left[\frac{\alpha}{n-1}\left(2 \frac{n-2}{n-1} h^{*}+a_{i}^{\alpha}+r\right) V^{+}-\left(\frac{1}{n-1}\right)^{\frac{1}{\alpha}} \frac{1}{\alpha} h^{* \frac{1-\alpha}{\alpha}}\left(\frac{n-1-\alpha}{n-1} h^{*}+a_{i}^{\alpha}+r\right)-\left(\frac{h^{*}}{n-1}\right)^{\frac{1}{\alpha}} \frac{n-1-\alpha}{n-1}\right]}
$$

By a similar reasoning as for the case of two firms, the denominator is positive. Using this insight, and condition (26) one can show that $\frac{\partial \tilde{b}}{\partial a_{i}}<\frac{\alpha h^{*}}{a_{i}^{*}}$ if and only if

$$
\begin{equation*}
-\alpha \frac{h^{*}}{n-1} r V^{+}+\alpha\left(\frac{h^{*}}{n-1}\right)^{\frac{1}{\alpha}} r<(1-\alpha)\left(\frac{h^{*}}{n-1}\right)^{\frac{1}{\alpha}}\left(\frac{n-1-\alpha}{n-1} h^{*}+a_{i}^{\alpha}+r\right) \tag{28}
\end{equation*}
$$

The right-hand side of 28 ) is positive; so we need to show that the left-hand side is negative. This is the case if and only if

$$
\left(\frac{h^{*}}{n-1}\right)^{\frac{\alpha-1}{\alpha}} V^{+}>1
$$

Substituting for $\frac{h^{*}}{n-1}=a_{j}^{* \alpha}$, this is equivalent to

$$
a_{j}^{* \alpha-1} V^{+}>1
$$

Let $V(\tilde{h})$ denote the value of firm $j$ before the innovation is found. From the first-order condition of firm $j, 12$, we know that

$$
a_{j}^{* \alpha-1} V^{+}=\frac{1}{\alpha}+a_{j}^{* \alpha-1} V(\tilde{h})>1,
$$

which proves the proposition.

## Appendix 2: Selection of Entrants

Derivation of Equation (11). From (9) define

$$
s_{i t} \equiv \frac{\exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{i t}+\beta_{E} E_{i t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{i t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{i t}+\eta_{i t}\right)}{\sum_{j \in \mathcal{N}} \exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{j t}+\beta_{E} E_{j t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{j t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{j t}+\eta_{j t}\right)}
$$

and

$$
s_{0 t} \equiv \frac{\sum_{h \in \mathcal{N}^{C}} \exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{h t}+\beta_{E} E_{h t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{h t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{h t}+\eta_{h t}\right)}{\sum_{j \in \mathcal{N}} \exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{j t}+\beta_{E} E_{j t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{j t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{j t}+\eta_{j t}\right)}
$$

Taking logarithms and subtracting we obtain

$$
\begin{aligned}
\ln s_{i t}-\ln s_{0 t}= & \boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{i t}+\beta_{E} E_{i t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{i t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{i t}+\eta_{i t} \\
& -\ln \sum_{j \in \mathcal{N}} \exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{j t}+\beta_{E} E_{j t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{j t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{j t}+\eta_{j t}\right) \\
& -\ln \sum_{h \in \mathcal{N}^{C}} \exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{h t}+\beta_{E} E_{h t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{h t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{h t}+\eta_{h t}\right) \\
& +\ln \sum_{j \in \mathcal{N}} \exp \left(\boldsymbol{\beta}_{W}^{\prime} \mathbf{W}_{j t}+\beta_{E} E_{j t}+\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{j t}+\boldsymbol{\beta}_{C}^{\prime} \mathbf{C}_{j t}+\eta_{j t}\right)
\end{aligned}
$$

Note that $\boldsymbol{\beta}_{\pi}^{\prime} \boldsymbol{\pi}_{i t}=0$ for all $i \in \mathcal{N}^{N C}$. Note too that the second and fourth term cancel out, and that the third term, $\ln \sum_{j \in \mathcal{N}^{C}} \exp ($.$) , is constant across i$ and varies only across time. Hence, this term can be written as a constant plus yearly dummies, simplifying the model to expression (11).

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## Footnotes

1. Gilbert and Newbery (1982) show that incumbents can preempt entrants from racing for incremental innovations if the incumbent benefits more from persisting as a monopolist than the entrant from coexisting as a duopolist. Reinganum (1983) shows how this result is reversed if innovation is stochastic: incumbent firms will have less incentives to innovate than entrants because additional investments in R\&D will only speed up the erosion of their own current monopoly profits.
2. It is widely acknowledged that firms in most other industries use other mechanisms to protect the competitive advantages of R\&D (e.g., superior marketing, customer service, client switching costs) and in these industries patent records are not a good indicator for innovations and the races for them. Despite our focus on pharmaceutical patents, our method can be directly applied to any race in any industry provided that a satisfactory measure of success is available.
3. The authors state that the firms they sample account for approximately 25 to $30 \%$ of the worldwide sales and R\&D of the Ethical Drugs Industry and claim that these firms are not markedly unrepresentative of the industry in terms of size, or of technical and commercial performance.
4. Note that this result is diametrically opposed to the results of Blundell, et al. (1999): technology laggards have more incentives to innovate because, unlike leaders, their innovative efforts do not erode the profits of "shelving" current innovations.
5. Another advantage of our approach is that we do not have to control for technological opportunity. Since we focus on races that have actually occurred and been won by someone, our observations are conditional on there being a technological opportunity to explore.
6. We could allow for a technology where the hazard rate is $f\left(a_{i}, k_{i}\right)$, where $k_{i}$ is a variable investment complementary to effort. However, this introduces further technical complications without adding insights.
7. This formulation gives all the bargaining power to the firm. This is not crucial; all our results go through if the investor has all the bargaining power, or for any linear surplus sharing rule between investor and firm.
8. The extension to the case of an arbitrary number of firms could be done along the lines suggested by Dixit (1986).
9. Additional evidence suggesting the use of patent citations as a measure of private value is provided by Hall, Jaffe, and Trajtenberg (2005), who show that an extra citation per patent is on average associated with a $3 \%$ increase in the firm's market value. The citation count has been traditionally used as a measure of the social value of a patent (e.g., Trajtenberg 1990).
10. Hall (2003) and Hall and Ziedonis (2001) argue that a pharmaceutical patent is clearly linked to a unique, new, chemical composition. Therefore, it clearly defines a potential new product market. As a result, Kremer (1998) singles out pharmaceutical patents as the ideal candidate for social welfare maximizing patent buy-outs. Bessen and Hunt (2003) show that the pharmaceutical industry is the only industry where the propensity to patent is insensitive to time variation in the US Patent Office's patenting standards. Their interpretation is that an easier approval of patents creates incentives to file patents that increase the firm's litigation bargaining power and not to file patents that block imitation. Because pharmaceutical firms typically don't accumulate patents for reasons other than to block imitation, their patenting intensity does not react to changes in the patenting standards.
11. Bronwyn Hall's webpage http://elsa.berkeley.edu/ bhhall/pat/namematch.html provides the code that corrects any misspelling by the USPTO of the assignee's names. This code enhances the matching of the NBER to COMPUSTAT by company name and CUSIP in the NBER database significantly.
12. E.g., our sample includes all patents awarded to firms such as Hoechst, Hoffman-LaRoche, Pfizer, Schering, Ciba, among others.
13. We thank an anonymous referee for pointing out this result to us.
14. Note that an entrant may be an existing firm in the industry that has so far not obtained any patents in this particular category, but potentially many patents in other categories.
15. A summary of the results of this step is included Table A All estimations also include dummy variables for each year, and $\mathbf{C}_{i k}$ includes 2-digit SIC code fixed effects. We show there the elasticities implied by the estimates. The full details of the results are available upon request.
Table I: Summary of the Patents in the NBER Database Before and After the Match to COMPUSTAT
This table summarizes the main characteristics of all US patents in the NBER Database between 1975 and 1999, in the technological category 3 (Drugs and Medical), subcategories 31, 33 and 39. It shows the results of matching the awarded patents, or pools of such patents, to their citations and to their assignees' financial data in COMPUSTAT. A patent pool groups all patents awarded to the same firm that were filed in the same week. All citation counts are corrected for yearly differences in the propensity to cite using the adjustment factors provided by Hall et al. (2002). The concentration index is the sum of the squares of the relative contribution of each patent's number of citations to the total number of citations received by all patents in the same pool.

| Panel A: Citations received by patents matched and not matched to COMPUSTAT |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Number of Observations | Mean | Standard deviation | Minimum | First quartile | Median | Third quartile | Maximum |
| All patents in NBER | 91,656 | 0.753 | 2.534 | 0 | 0 | 0.216 | 0.810 | 150 |
| All patents owned by corporations (A) | 77, 704 | 0.754 | 2.495 | 0 | 0 | 0.217 | 0.793 | 150 |
| Patents where CUSIP is available (B) | 31,039 | $0.825^{* * *}$ | * 2.390 | 0 | 0 | 0.296 | 0.917 | 150 |
| Panel B: Summary of patent pools matched and not matched to COMPUSTAT |  |  |  |  |  |  |  |  |
|  | Number of Observations | Mean | Standard deviation | Minimum | First quartile | Median | Third quartile | Maximum |
| Number of patents in the pool |  |  |  |  |  |  |  |  |
| All pools owned owned by corporations (A) | 37, 283 | 2.101 | 3.356 | 1 | 1 | 1 | 2 | 50 |
| Pools where CUSIP is available (B) | 8,399 | $3.696^{* * *}$ | * 5.480 | 1 | 1 | 2 | 4 | 50 |
| Total citations received by the pool |  |  |  |  |  |  |  |  |
| All pools owned by corporations (A) | 37, 283 | 1.583 | 4.414 | 0 | 0 | 0.432 | 1.479 | 162 |
| Pools where CUSIP is available (B) | 8,399 | $3.049^{* * *}$ | * 6.396 | 0 | 0.298 | 1.064 | 3.030 | 162 |
| Within-pool citations concentration |  |  |  |  |  |  |  |  |
| All pools owned by corporations (A) | 37, 283 | 0.586 | 0.451 |  |  | 0.901 |  | 1 |
| Pools where CUSIP is available (B) | 8,399 | $0.634^{* * *}$ | * 0.389 | 0 | 0.284 | 0.722 | 1 | 1 |
| All pools with at least one citation (A) | 11,769 | 0.740 | 0.310 | 0.057 | 0.479 | 1. | 1 | 1 |
| All pools with at least one citation, with available CUSIP (B) | 7,187 | 0.741 | 0.313 | 0.057 | 0.484 | 1 | 1 | 1 |
| Patent pools accumulated by the winner |  |  |  |  |  |  |  |  |
| All pools owned by corporations (A) | 37, 283 | 43.667 | 124.751 | 1 | 2 | 7 | 28 | 1936 |
| Pools where CUSIP is available (B) | 8,399 | $100.971^{* * *}$ | * 202.935 | 1 | 7 | 27 | 89 | 1936 |

[^1]
## Table II: Selection of Firms Competing in a Patent Race

This table describes the statistic of the selection of cited and non-cited firms for every patent race. All COMPUSTAT firms that have won a patent in each five year period are ranked each year by their predicted probability of winning a patent pool of a given patent subclass in a given quartile of the number of citations received. The probability is predicted using the model and the estimates in Table A If a patent in the pool, $k$, cites a patent in pool, $l$, which by firm $i$, then the citations count of all patents cited by $k$ is given by the weighted average, $I_{k}$,

$$
I_{k}=\sum_{\forall i} \sum_{\substack{\forall l \text { cited by } k \\ \text { owned by } i}} \#(\text { citationsof }) \times\left(20-\text { age }_{l}\right),
$$

where $l$ is at most 20 years old and has been itself cited \#(citations ${ }_{l}$ ) times. Each cited firm's relative contribution to $I_{k}$ is given by

$$
\frac{\sum_{\forall l \text { cited by } k} \begin{array}{c}
\#\left(\text { citations }_{l}\right) \times\left(20-\text { age }_{l}\right) \\
\text { owned by } i
\end{array}}{I_{k}} .
$$

All citation counts are corrected for yearly differences in the propensity to cite using the adjustment factors provided by Hall et al. (2002).

## Panel A: Universe of cited firms

Number of patent pools $=37,283$

| Number of firms cited by patent pool |  | Relative contribution of the $n$-th or better ranked firm to the index, $I_{k}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Number | Cumulative frequency | Top $n$ firms, by citations index | Mean | Median |
| 1 | 23.21 | 1 | 0.659 | 0.659 |
| 2 | 42.54 | 2 | 0.837 | 0.939 |
| 3 | 57.04 | 3 | 0.906 | 1.000 |
| 4 | 74.15 | 4 | 0.939 | 1.000 |
| 5 | 81.21 | 5 | 0.958 | 1.000 |
| 10 | 95.60 | 10 | 0.983 | 1.000 |

Panel B: Selection of non-cited firms

$$
\text { Number of selections }=285
$$

Predicted probability that the winner is the $n$-th or higher ranked non-cited firm, given that a non-cited firm wins

| Top $n$ firms, by <br> winning probability |  | Mean <br> probability |
| :---: | :---: | :---: |
|  |  | Median <br> probability |
|  |  |  |
| 5 | 0.399 | 0.293 |
| 5 | 0.755 | 0.999 |
| 10 | 0.884 | 1.000 |
| 15 | 0.909 | 1.000 |
| 20 | 0.916 | 1.000 |

Table III: Summary Statistics of the Selection
This table summarizes the main characteristics of the firms selected as the most likely participants in every given race. The selection includes
the firms that own the four most valuable cited patent pools portfolios, and the ten firms most likely to win any given pool each year, from among the set of all non-cited firms. The probabilities of winning a pool each year are predicted using the model and the estimates reported in Table A.I.

|  | Number of Observations | Mean | Standard deviation | Minimum | First quartile | Median | Third quartile | Maximum |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sample Period: 1975 to 1979 |  |  |  |  |  |  |  |  |
| Cash holdings, 1 year before the filing date (\$ Millions) | 6, 840 | 306.12 | 1,449.31 | 1.27 | 37.33 | 97.00 | 212.87 | 15,328.79 |
| Total assets, 2 years before the filing date (\$ Millions) | 6, 840 | 2,523.33 | 6, 053.10 | 6.04 | 398.17 | 1,041.26 | 1,993.39 | 52,557.91 |
| Patent pools accumulated, up to 1 year before the filing date | 6, 840 | 0.09 | 0.45 | 0.00 | 0.00 | 0.02 | 0.07 | 14.15 |
| Sample Period: 1980 to 1984 |  |  |  |  |  |  |  |  |
| Cash holdings, 1 year before the filing date (\$ Millions) | 5, 832 | 375.30 | 1,529.46 | 0.01 | 38.28 | 119.05 | 317.37 | 15,328.79 |
| Total assets, 2 years before the filing date ( $\$$ Millions) | 5, 832 | 3,810.99 | 7, 034.68 | 4.64 | 369.63 | 1,511.20 | 3,609.60 | 52,557.91 |
| Patent pools accumulated, up to 1 year before the filing date | 5,832 | 0.27 | 1.14 | 0.00 | 0.00 | 0.05 | 0.21 | 18.64 |
| Sample Period: 1985 to 1989 |  |  |  |  |  |  |  |  |
| Cash holdings, 1 year before the filing date (\$ Millions) | 7,354 | 544.70 | 1,859.46 | 0.11 | 29.77 | 110.44 | 472.00 | 15,328.79 |
| Total assets, 2 years before the filing date ( $\$$ Millions) | 7,354 | 4,661.38 | 9, 021.03 | 3.39 | 202.50 | 1,208.51 | 5, 095.10 | 66, 710.02 |
| Patent pools accumulated, up to 1 year before the filing date | 7,354 | 0.36 | 1.90 | 0.00 | 0.00 | 0.04 | 0.18 | 36.61 |
| Sample Period: 1990 to 1994 |  |  |  |  |  |  |  |  |
| Cash holdings, 1 year before the filing date (\$ Millions) | 9, 801 | 616.65 | 1,542.21 | 0.17 | 51.81 | 153.70 | 671.71 | 20, 760.20 |
| Total assets, 2 years before the filing date ( $\$$ Millions) | 9,801 | 7,360.02 | 10, 981.54 | 6.59 | 573.84 | 3,599.61 | 9,215.00 | 98,627.88 |
| Patent pools accumulated, up to 1 year before the filing date | 9,801 | 0.54 | 2.68 | 0.00 | 0.01 | 0.07 | 0.25 | 44.79 |
| Sample Period: 1995 to 1999 |  |  |  |  |  |  |  |  |
| Cash holdings, 1 year before the filing date (\$ Millions) | 15, 231 | 1,161.05 | 2, 084.02 | 0.01 | 92.60 | 410.85 | 1,794.00 | 24, 760.89 |
| Total assets, 2 years before the filing date ( $\$$ Millions) | 15,231 | 10,787.12 | 12,540.87 | 8.02 | 1,688.67 | 6,340.30 | 16, 012.07 | 102, 714.00 |
| Patent pools accumulated, up to 1 year before the filing date | 15,231 | 1.81 | 9.39 | 0.00 | 0.04 | 0.25 | 0.49 | 171.20 |

Table IV: Estimates of the Model of a Patent Race Winner
This table shows the parameter estimates of the model that selects the winner of each patent pool from the set of pre-selected competitors. The estimates estimable model is

$$
\operatorname{Pr}(\text { firm } i \text { wins race } k)=\frac{\exp \left(\boldsymbol{\beta}^{\prime} \mathbf{x}_{i k}+\eta_{i k}\right)}{\sum_{j} \exp \left(\boldsymbol{\beta}^{\prime} \mathbf{x}_{j k}+\eta_{j k}\right)}
$$

where the regressors are listed below, and $\eta_{i k}$ represents the unobserved firm characteristics that are correlated with cash. The instruments for cash holdings are the two and three year lags of the logarithm of cash, sales, total assets and outstanding debt, and the averages of cash, sales, debt and accumulated patent pools of all other rival firms in the same race. The standard errors of the parameter estimates are computed using a bootstrap estimator. They are
shown in brackets underneath the parameter estimate. ${ }^{a}$ The estimation uses all US patent pools won by COMPUSTAT firms from 1975 to 1999 , in the techonological category 3 (Drugs and Medical), subcategories 31, 33 and 39. Patent pools are classified into quartiles according to the number of citations received. The number of citations is adjusted for time differences in the propensity to cite, using the factors provided by Hall et al. (2002).

|  | Estima | Estimation Period: 1975 to 1979 |  | 979 | Estimati | Estimation Period: 1980 to 1984 |  |  | Estimat | tion Period ent Citation | d: 1985 to ns Quartile | 1989 es |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| Logarithm of cash holdings, 1 year before the filing date | $\begin{gathered} -0.383^{* * *} \\ (0.105) \end{gathered}$ | $\begin{gathered} 0.063 \\ (0.090) \end{gathered}$ | $\begin{gathered} 0.148 \\ (0.146) \end{gathered}$ | $\begin{aligned} & 0.261^{* * *} \\ & (0.091) \end{aligned}$ | $\begin{gathered} 0.029 \\ (0.080) \end{gathered}$ | $\begin{gathered} 0.220^{*} \\ (0.119) \end{gathered}$ | $\begin{gathered} 0.047 \\ (0.133) \end{gathered}$ | $\begin{aligned} & 0.270^{* *} \\ & (0.108) \end{aligned}$ | $\begin{gathered} 0.078 \\ (0.092) \end{gathered}$ | $\begin{aligned} & 0.324^{* * *} \\ & (0.082) \end{aligned}$ | $\begin{aligned} & 0.188^{* *} \\ & (0.091) \end{aligned}$ | $\begin{aligned} & 0.236^{* * *} \\ & (0.068) \end{aligned}$ |
| Total patent pools accumulated, up to 1 year before the filing date | $\begin{gathered} 0.567^{*} \\ (0.322) \end{gathered}$ | $\begin{aligned} & 0.548^{* * *} \\ & (0.117) \end{aligned}$ | $\begin{aligned} & 0.364^{* *} \\ & (0.162) \end{aligned}$ | $\begin{aligned} & 0.498^{* * *} \\ & (0.082) \end{aligned}$ | $\begin{aligned} & 0.188^{* * *} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & \quad 0.166^{* * *} \\ & (0.039) \end{aligned}$ | $\begin{aligned} & 0.226^{* * *} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.252^{* * *} \\ & (0.043) \end{aligned}$ | $\begin{aligned} & 0.091^{* * *} \\ & (0.027) \end{aligned}$ | $\begin{aligned} & 0.084^{* * *} \\ & (0.021) \end{aligned}$ | $\begin{aligned} & 0.108^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.134^{* * *} \\ & (0.021) \end{aligned}$ |
| Citations count of citations, by vintages: |  |  |  |  |  |  |  |  |  |  |  |  |
| $\text { Age }<1$ | $\begin{array}{r} -0.794 \\ (0.515) \end{array}$ | $\begin{array}{r} -1.149 \\ (0.796) \end{array}$ | $\begin{gathered} -0.393^{*} \\ (0.214) \end{gathered}$ | $\begin{gathered} -0.063^{* *} \\ (0.032) \end{gathered}$ | $\begin{gathered} -26.130^{* * *} \\ (2.566) \end{gathered}$ | $\begin{array}{r} *-0.935 \\ (0.590) \end{array}$ | $\begin{gathered} -0.193^{*} \\ (0.104) \end{gathered}$ | $\begin{gathered} -0.523^{* * *} \\ (0.166) \end{gathered}$ | $\begin{gathered} -0.888^{* *} \\ (0.412) \end{gathered}$ | $\begin{gathered} -0.066 \\ (0.078) \end{gathered}$ | $\begin{array}{r} -0.153 \\ (0.093) \end{array}$ | $\begin{gathered} -0.108^{* *} \\ (0.052) \end{gathered}$ |
| $1 \leq$ Age $<2$ | $\begin{gathered} 0.053 \\ (0.037) \end{gathered}$ | $\begin{gathered} 0.027 \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.008 \\ (0.031) \end{gathered}$ | $\begin{aligned} & 0.074^{* * *} \\ & (0.022) \end{aligned}$ | $\begin{aligned} & 1.154^{* * *} \\ & (0.194) \end{aligned}$ | $\begin{gathered} 0.037 \\ (0.171) \end{gathered}$ | $\begin{gathered} 0.37 \\ (0.290) \end{gathered}$ | $\begin{aligned} & 0.159^{* *} \\ & (0.070) \end{aligned}$ | $\begin{gathered} 0.317 \\ (0.281) \end{gathered}$ | $\begin{gathered} 0.056 \\ (0.093) \end{gathered}$ | $\begin{gathered} 0.246^{* *} \\ (0.099) \end{gathered}$ | $\begin{gathered} 0.019 \\ (0.015) \end{gathered}$ |
| $2 \leq$ Age $<3$ | $\begin{gathered} -8.053^{* * *} \\ (0.309) \end{gathered}$ | $\begin{gathered} -0.147 \\ (0.123) \end{gathered}$ | $\begin{gathered} -0.112^{* *} \\ (0.044) \end{gathered}$ | $\begin{gathered} 0.008 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.027 \\ (0.182) \end{gathered}$ | $\begin{aligned} & 0.648^{* *} \\ & (0.290) \end{aligned}$ | $\begin{gathered} 0.14 \\ (0.190) \end{gathered}$ | $\begin{aligned} & 0.433^{* * *} \\ & (0.116) \end{aligned}$ | $\begin{gathered} 0.575 \\ (0.922) \end{gathered}$ | $\begin{aligned} & 0.344^{* * *} \\ & (0.121) \end{aligned}$ | $\begin{array}{r} -0.038 \\ (0.070) \end{array}$ | $\begin{aligned} & 0.114^{* * *} \\ & (0.037) \end{aligned}$ |
| $3 \leq$ Age $<4$ | $\begin{gathered} 0.021 \\ (0.051) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.035) \end{gathered}$ | $\begin{array}{r} -0.314^{*} \\ (0.182) \end{array}$ | $\begin{gathered} 0.028 \\ (0.017) \end{gathered}$ | $\begin{gathered} -1.725^{* * *} \\ (0.598) \end{gathered}$ | $\begin{gathered} 0.032 \\ (0.065) \end{gathered}$ | $\begin{gathered} 0.030 \\ (0.053) \end{gathered}$ | $\begin{aligned} & 0.086^{* *} \\ & (0.039) \end{aligned}$ | $\begin{gathered} -0.147 \\ (0.220) \end{gathered}$ | $\begin{gathered} -0.09 \\ (0.168) \end{gathered}$ | $\begin{gathered} 0.06 \\ (0.110) \end{gathered}$ | $\begin{gathered} 0.062 \\ (0.042) \end{gathered}$ |
| $4 \leq$ Age $<5$ | $\begin{gathered} 0.022 \\ (0.030) \end{gathered}$ | $\begin{gathered} 0.006 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.015) \end{gathered}$ | $\begin{aligned} & 0.030^{* * *} \\ & (0.011) \end{aligned}$ | $\begin{gathered} 0.060 \\ (0.092) \end{gathered}$ | $\begin{array}{r} -0.352 \\ (0.417) \end{array}$ | $\begin{array}{r} -0.096 \\ (0.125) \end{array}$ | $\begin{gathered} 0.093 \\ (0.097) \end{gathered}$ | $\begin{array}{r} -0.005 \\ (0.202) \end{array}$ | $\begin{gathered} 0.044 \\ (0.073) \end{gathered}$ | $\begin{gathered} 0.046 \\ (0.069) \end{gathered}$ | $\begin{gathered} 0.058^{*} \\ (0.035) \end{gathered}$ |
| $5 \leq$ Age $<10$ | $\begin{gathered} 0.022 \\ (0.031) \end{gathered}$ | $\begin{array}{r} -0.032 \\ (0.022) \end{array}$ | $\begin{gathered} -0.084^{* *} \\ (0.040) \end{gathered}$ | $\begin{array}{r} -0.013 \\ (0.009) \end{array}$ | $\begin{gathered} 0 \\ (0.018) \end{gathered}$ | $\begin{array}{r} -0.024 \\ (0.025) \end{array}$ | $\begin{gathered} 0.004 \\ (0.007) \end{gathered}$ | $\begin{array}{r} -0.007 \\ (0.007) \end{array}$ | $\begin{gathered} -0.205^{*} \\ (0.105) \end{gathered}$ | $\begin{gathered} 0.018 \\ (0.062) \end{gathered}$ | $\begin{gathered} 0.012 \\ (0.039) \end{gathered}$ | $\begin{array}{r} -0.005 \\ (0.027) \end{array}$ |
| $10 \leq$ Age $<20$ | $\begin{gathered} -10.734^{* * *} \\ (1.370) \end{gathered}$ | $\begin{gathered} -11.203^{* * *} \\ (0.717) \end{gathered}$ | $\begin{gathered} -11.879^{* * *} \\ (0.598) \end{gathered}$ | $\begin{gathered} -11.803^{* * *} \\ (0.632) \end{gathered}$ | $\begin{gathered} -0.438^{* * *} \\ (0.107) \end{gathered}$ | $\begin{aligned} & 0.006^{* *} \\ & (0.003) \end{aligned}$ | $\begin{array}{r} -0.004 \\ (0.010) \end{array}$ | $\begin{array}{r} -0.012 \\ (0.012) \end{array}$ | $\begin{aligned} & 0.014^{* * *} \\ & (0.004) \end{aligned}$ | $\begin{gathered} -0.091^{* *} \\ (0.043) \end{gathered}$ | $\begin{gathered} 0.001 \\ (0.007) \end{gathered}$ | $\begin{array}{r} -0.011 \\ (0.016) \end{array}$ |
| First stage error ( $\hat{\eta}_{i k}$ ) | $\begin{aligned} & 0.657^{* * *} \\ & (0.187) \end{aligned}$ | $\begin{gathered} -0.138 \\ (0.142) \end{gathered}$ | $\begin{gathered} -0.353^{*} \\ (0.195) \end{gathered}$ | $\begin{gathered} -0.655^{* * *} \\ (0.101) \end{gathered}$ | $\begin{aligned} & 0.401^{* *} \\ & (0.167) \end{aligned}$ | $\begin{gathered} 0.150 \\ (0.175) \end{gathered}$ | $\begin{aligned} & 0.512^{* * *} \\ & (0.166) \end{aligned}$ | $\begin{gathered} -0.632^{* * *} \\ (0.134) \end{gathered}$ | $\begin{gathered} 0.046 \\ (0.117) \end{gathered}$ | $\begin{gathered} -0.203^{*} \\ (0.104) \end{gathered}$ | $\begin{aligned} & 0.285^{* * *} \\ & (0.106) \end{aligned}$ | $\begin{gathered} -0.422^{* * *} \\ (0.075) \end{gathered}$ |
| Number of observations | 1,249 | 1,700 | 2,243 | 3,224 | 1,244 | 1,151 | 1,829 | 3,037 | 1,519 | 1,619 | 2,194 | 4,186 |
| $\chi^{2, b}$ statistic | 951.089 | 287.234 | 517.843 | 500.638 | 179.068 | 40.112 | 76.715 | 93.444 | 35.423 | 54.023 | 106.264 | 105.776 |
| $p$-value of $\chi^{2}$ statistic | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Pseudo $\mathrm{R}^{2}$ | 0.077 | 0.062 | 0.068 | 0.126 | 0.103 | 0.086 | 0.120 | 0.118 | 0.052 | 0.079 | 0.132 | 0.071 |

[^2]Table IV : continued.

|  | Estimation Period: 1990 to 1994 |  |  |  | Estimation Period: 1995 to 1999 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| Logarithm of cash holdings, 1 year before the filing date | $\begin{gathered} 0.146^{*} \\ (0.081) \end{gathered}$ | $\begin{aligned} & 0.531^{* * *} \\ & (0.071) \end{aligned}$ | $\begin{gathered} \quad 0.333^{* * *} \\ (0.084) \end{gathered}$ | $\begin{aligned} & 0.352^{* * *} \\ & (0.047) \end{aligned}$ | $\begin{gathered} 0.011 \\ (0.046) \end{gathered}$ | $\begin{aligned} & 0.326^{* * *} \\ & (0.081) \end{aligned}$ | $\begin{gathered} 0.390^{* * *} \\ (0.067) \end{gathered}$ | $\begin{aligned} & 0.406 * * * \\ & (0.075) \end{aligned}$ |
| Total patent pools accumulated, up to 1 year before the filing date | $\begin{aligned} & 0.063^{* *}, \\ & (0.017) \end{aligned}$ | $\begin{gathered} \quad 0.095^{* * *} \\ (0.026) \end{gathered}$ | $\begin{aligned} & 0.059^{* * *} \\ & (0.013) \end{aligned}$ | $\begin{aligned} & 0.105^{* * *} \\ & (0.016) \end{aligned}$ | $\begin{aligned} & 0.016^{* * *} \\ & (0.003) \end{aligned}$ | $\begin{gathered} \left.\quad \begin{array}{c} 0.026 * * * \\ (0.005) \end{array}\right) . \end{gathered}$ | $\begin{aligned} & 0.027^{* * *} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.031^{* *} \\ & (0.002) \end{aligned}$ |
| Citations count of citations, by vintages: |  |  |  |  |  |  |  |  |
| Age $<1$ | $\begin{gathered} -0.828^{* * *} \\ (0.294) \end{gathered}$ | $\begin{gathered} -0.222^{* *} \\ (0.087) \end{gathered}$ | $\begin{gathered} -0.311^{*} \\ (0.183) \end{gathered}$ | $\begin{gathered} -0.057^{* *} \\ (0.023) \end{gathered}$ | $\begin{gathered} -0.002 \\ (0.012) \end{gathered}$ | $\begin{gathered} -0.072 \\ (0.053) \end{gathered}$ | $\begin{gathered} -0.082^{* *} \\ (0.039) \end{gathered}$ | $\begin{gathered} -0.018 \\ (0.014) \end{gathered}$ |
| $1 \leq$ Age $<2$ | $\begin{gathered} 0.424^{*} \\ (0.257) \end{gathered}$ | $\begin{aligned} & 0.366^{* * *} \\ & (0.141) \end{aligned}$ | $\begin{gathered} 0.289^{*} \\ (0.152) \end{gathered}$ | $\begin{aligned} & 0.247^{* * *} \\ & (0.076) \end{aligned}$ | $\begin{gathered} 0.012 \\ (0.023) \end{gathered}$ | $\begin{gathered} 0.033 \\ (0.052) \end{gathered}$ | $\begin{aligned} & 0.279^{* *} \\ & (0.116) \end{aligned}$ | $\begin{aligned} & 0.131^{* *} \\ & (0.051) \end{aligned}$ |
| $2 \leq$ Age $<3$ | 0.225 | 0.329*** | ${ }_{\text {0 }} 0.437^{* * *}$ | 0.010 | 0.021 | 0.040 | 0.134** | 0.110*** |
|  | (0.280) | (0.091) | (0.113) | ${ }^{(0.080)}{ }^{0.09 * * *}$ | (0.021) | (0.058) | (0.062) | (0.041) |
| $3 \leq$ Age $<4$ | $\begin{gathered} 0.170 \\ (0.109) \end{gathered}$ | $\begin{gathered} -0.02 \\ (0.062) \end{gathered}$ | $\begin{array}{r} 0.006 \\ (0.015) \end{array}$ | $\begin{aligned} & 0.098^{* * *} \\ & (0.037) \end{aligned}$ | $\begin{array}{r} -0.084^{*} \\ (0.049) \end{array}$ | $\begin{gathered} 0.058^{*} \\ (0.034) \end{gathered}$ | $\begin{gathered} 0.001 \\ (0.050) \end{gathered}$ | $\begin{gathered} -0.008 \\ (0.026) \end{gathered}$ |
| $4 \leq$ Age $<5$ | $-0.076$ | 0.126 | $0.147^{* *}$ | -0.018 | 0.030 | 0.031 | 0.059 | 0.039 |
| $5 \leq$ Age $<10$ | $(0.202)$ 0.061 | ${ }^{(0.077)} 0$ | ${ }_{-0.001}^{(0.061)}$ | ${ }^{(0.050)} 0$ | ${ }^{(0.063)} 0$ | ${ }_{*}^{(0.042)} 0$ | ${ }^{(0.052)} 0$ | ${ }^{(0.029)} 0$ |
| $5 \leq$ Age $<10$ | (0.072) | (0.027) | (0.026) | (0.013) | (0.015) | (0.015) | (0.028) | $(0.007)$ |
| $10 \leq$ Age $<20$ | $\begin{array}{r} -0.015 \\ (0.034) \end{array}$ | $\begin{gathered} 0.012 \\ (0.022) \end{gathered}$ | $\begin{gathered} -0.010^{* *} \\ (0.004) \end{gathered}$ | $\begin{array}{r} -0.006 \\ (0.010) \end{array}$ | $\begin{gathered} -0.065^{*} \\ (0.037) \end{gathered}$ | $\begin{array}{r} -0.018 \\ (0.015) \end{array}$ | $\begin{gathered} -0.040 \\ (0.026) \end{gathered}$ | $\begin{gathered} 0.004 \\ (0.006) \end{gathered}$ |
| First stage error ( $\hat{\eta}_{i k}$ ) | $\begin{gathered} 0.053 \\ (0.096) \end{gathered}$ | $\begin{gathered} -0.481^{* * *} \\ (0.092) \end{gathered}$ | $\begin{gathered} *-0.089 \\ (0.082) \end{gathered}$ | $\begin{aligned} & 0.468^{* * *} \\ & (0.070) \end{aligned}$ | $\begin{gathered} 0.001 \\ (0.072) \end{gathered}$ | $\begin{gathered} -0.219^{* *} \\ (0.093) \end{gathered}$ | $\begin{aligned} & -0.256^{* * *} \\ & (0.082) \end{aligned}$ | $\begin{gathered} -0.316^{* * *} \\ (0.086) \end{gathered}$ |
| Number of observations | 1,839 | 2,899 | 3,278 | 6,115 | 2,963 | 3,735 | 4,574 | 11,935 |
| $\chi^{2, b}$ statistic | 49.198 | 99.399 | 70.190 | 141.632 | 65.763 | 58.792 | 102.104 | 282.525 |
| $p$-value of $\chi^{2}$ statistic | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Pseudo R ${ }^{2}$ | 0.060 | 0.115 | 0.078 | 0.086 | 0.027 | 0.046 | 0.088 | 0.110 |

[^3]Table V: Estimates of the Model of a Patent Race Winner
This table shows the parameter estimates of the model that selects the winner of each patent pool from the set of pre-selected competitors. The estimates estimable model is
$$
\operatorname{Pr}(\text { firm } i \text { wins race } k)=\frac{\exp \left(\boldsymbol{\beta}^{\prime} \mathbf{x}_{i k}+\eta_{i k}\right)}{\sum_{j} \exp \left(\boldsymbol{\beta}^{\prime} \mathbf{x}_{j k}+\eta_{j k}\right)} .
$$
where the regressors are listed below, and $\eta_{i k}$ represents the unobserved firm characteristics that are correlated with cash. The instruments for cash holdings patent pools of all other rival firms in the same race. The standard errors of the parameter estimates are computed using a bootstrap estimator. They are shown in brackets underneath the parameter estimate. ${ }^{a}$ The estimation uses all US patent pools won by COMPUSTAT firms from 1975 to 1999 , in the techonological category 3 (Drugs and Medical), subcategories 31, 33 and 39. Patent pools are classified into quartiles according to the number of citations received. The number of citations is adjusted for time differences in the propensity to cite, using the factors provided by Hall et al. (2002).

|  | Estimation Period: 1975 to 1979 <br> Patent Citations Quartiles |  |  |  | Estimation Period: 1980 to 1984 <br> Patent Citations Quartiles |  |  |  | Estimation Period: 1985 to 1989 <br> Patent Citations Quartiles |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| Logarithm of cash holdings, 1 year before the filing date Logarithm of total assets, 2 years before the filing date | $\begin{gathered} -0.210^{*} \\ (0.116) \\ -1.311^{* * *} \\ (0.383) \end{gathered}$ | $\begin{gathered} 0.229^{* *} \\ (0.097) \\ -0.797^{* * *} \\ (0.291) \end{gathered}$ | $\begin{array}{r} 0.326^{*} \\ (0.183) \\ -\quad-0.585^{*} \\ (0.307) \end{array}$ | $\begin{gathered} 0.200^{* *} \\ (0.095) \\ 0.228^{* *} \\ (0.107) \end{gathered}$ | $\begin{aligned} & 0.004 \\ & (0.077) \\ & 1.163^{* * *} \\ & (0.369) \end{aligned}$ | $\begin{gathered} 0.264^{* *} \\ (0.109) \\ -0.265 \\ (0.537) \end{gathered}$ | $\begin{aligned} & 0.255^{*} \\ & (0.150) \\ & 1.322^{* * *} \\ & (0.254) \end{aligned}$ | $\begin{aligned} & 0.395^{* * *} \\ & (0.114) \\ & 1.475^{* * *} \\ & (0.407) \end{aligned}$ | $\begin{gathered} 0.163^{*} \\ (0.091) \\ -1.182^{* *} \\ (0.530) \end{gathered}$ | $\begin{gathered} 0.444^{* * *} \\ (0.085) \\ 0.292 \\ (0.254) \end{gathered}$ | $\begin{aligned} & 0.372^{* * *} \\ & (0.097) \\ & 1.451^{* * *} \\ & (0.279) \end{aligned}$ | $\begin{gathered} 0.326^{* * *} \\ (0.069) \\ 0.300 \\ (0.211) \end{gathered}$ |
| Total patent pools accumulated, up to 1 year before the filing date | $\begin{aligned} & 0.598^{* *} \\ & (0.256) \end{aligned}$ | $\begin{aligned} & 0.556^{* * *} \\ & (0.113) \end{aligned}$ | $\begin{aligned} & 0.357^{* *} \\ & (0.163) \end{aligned}$ | $\begin{aligned} & 0.478^{* * *} \\ & (0.081) \end{aligned}$ | $\begin{aligned} & 0.220^{* * *} \\ & (0.044) \end{aligned}$ | $\begin{aligned} & \quad 0.229^{* * *} \\ & (0.038) \end{aligned}$ | $\begin{aligned} & 0.281^{* * *} \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.285^{* * *} \\ & (0.048) \end{aligned}$ | $\begin{aligned} & 0.114^{* * *} \\ & (0.029) \end{aligned}$ | $\begin{aligned} & 0.086^{* * *} \\ & (0.020) \end{aligned}$ | $\begin{aligned} & 0.132^{* * *} \\ & (0.019) \end{aligned}$ | $\begin{aligned} & 0.145^{* * *} \\ & (0.019) \end{aligned}$ |
| Citations count of citations, by vintages: |  |  |  |  |  |  |  |  |  |  |  |  |
| $\text { Age }<1$ | $\begin{array}{r} -0.500 \\ (0.339) \end{array}$ | $\begin{gathered} -0.826 \\ (0.644) \end{gathered}$ | $\begin{array}{r} -0.336^{*} \\ (0.194) \end{array}$ | $\begin{gathered} -0.084^{* *} \\ (0.034) \end{gathered}$ | $\begin{gathered} -24.428^{* * *} \\ (2.760) \end{gathered}$ | $\begin{array}{r} *-0.569 \\ (0.418) \end{array}$ | $\begin{gathered} -0.091 \\ (0.069) \end{gathered}$ | $\begin{gathered} -0.369^{* *} \\ (0.157) \end{gathered}$ | $\begin{gathered} -0.608^{*} \\ (0.342) \end{gathered}$ | $\begin{gathered} -0.039 \\ (0.063) \end{gathered}$ | $\begin{gathered} -0.102 \\ (0.077) \end{gathered}$ | $\begin{gathered} -0.078^{* *} \\ (0.036) \end{gathered}$ |
| $1 \leq$ Age $<2$ | $\begin{gathered} 0.067 \\ (0.043) \end{gathered}$ | $\begin{gathered} 0.038^{*} \\ (0.020) \end{gathered}$ | $\begin{gathered} 0.018 \\ (0.030) \end{gathered}$ | $\begin{aligned} & 0.069^{* * *} \\ & (0.021) \end{aligned}$ | $\begin{aligned} & 1.183^{* * *} \\ & (0.215) \end{aligned}$ | $\begin{gathered} 0.191 \\ (0.173) \end{gathered}$ | $\begin{aligned} & 0.631^{* *} \\ & (0.255) \end{aligned}$ | $\begin{aligned} & 0.210^{* * *} \\ & (0.071) \end{aligned}$ | $\begin{gathered} 0.549^{*} \\ (0.320) \end{gathered}$ | $\begin{gathered} 0.101 \\ (0.092) \end{gathered}$ | $\begin{aligned} & 0.319^{* * *} \\ & (0.108) \end{aligned}$ | $\begin{gathered} 0.022 \\ (0.016) \end{gathered}$ |
| $2 \leq$ Age $<3$ | $\begin{gathered} -7.207^{* * *} \\ (0.324) \end{gathered}$ | $\begin{gathered} -0.081 \\ (0.079) \end{gathered}$ | $\begin{gathered} -0.084^{* *} \\ (0.043) \end{gathered}$ | $\begin{gathered} 0.007 \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.111 \\ (0.186) \end{gathered}$ | $\begin{aligned} & 0.956^{* * *} \\ & (0.292) \end{aligned}$ | $\begin{gathered} 0.288 \\ (0.195) \end{gathered}$ | $\begin{aligned} & 0.515^{* * *} \\ & (0.113) \end{aligned}$ | $\begin{gathered} 0.743 \\ (1.210) \end{gathered}$ | $\begin{aligned} & 0.408^{* * *} \\ & (0.119) \end{aligned}$ | $\begin{gathered} -0.017 \\ (0.063) \end{gathered}$ | $\begin{aligned} & 0.124^{* * *} \\ & (0.034) \end{aligned}$ |
| $3 \leq$ Age $<4$ | $\begin{gathered} 0.065 \\ (0.055) \end{gathered}$ | $\begin{gathered} 0.018 \\ (0.036) \end{gathered}$ | $\begin{gathered} -0.272 \\ (0.167) \end{gathered}$ | $\begin{gathered} 0.026 \\ (0.014) \end{gathered}$ | $\begin{gathered} -1.814^{* * *} \\ (0.607) \end{gathered}$ | $\begin{gathered} 0.009 \\ +(0.056) \end{gathered}$ | $\begin{gathered} 0.067 \\ (0.060) \end{gathered}$ | $\begin{aligned} & 0.165^{* * *} \\ & (0.041) \end{aligned}$ | $\begin{gathered} -0.092 \\ (0.281) \end{gathered}$ | $\begin{gathered} -0.059 \\ (0.180) \end{gathered}$ | $\begin{gathered} 0.116 \\ (0.120) \end{gathered}$ | $\begin{gathered} 0.083 \\ (0.055) \end{gathered}$ |
| $4 \leq$ Age $<5$ | $\begin{gathered} 0.035 \\ (0.037) \end{gathered}$ | $\begin{gathered} 0.013 \\ (0.011) \end{gathered}$ | $\begin{gathered} 0.009 \\ (0.015) \end{gathered}$ | $\begin{aligned} & 0.028^{* * *} \\ & (0.010) \end{aligned}$ | $\begin{gathered} 0.06 \\ (0.090) \end{gathered}$ | $\begin{gathered} -0.281 \\ (0.345) \end{gathered}$ | $\begin{array}{r} -0.076 \\ (0.192) \end{array}$ | $\begin{gathered} 0.144 \\ (0.087) \end{gathered}$ | $\begin{gathered} 0.115 \\ (0.219) \end{gathered}$ | $\begin{gathered} 0.052 \\ (0.078) \end{gathered}$ | $\begin{gathered} 0.089 \\ (0.066) \end{gathered}$ | $\begin{aligned} & 0.081^{* *} \\ & (0.037) \end{aligned}$ |
| $5 \leq$ Age < 10 | $\begin{gathered} 0.037^{*} \\ (0.022) \end{gathered}$ | $\begin{array}{r} -0.016 \\ (0.018) \end{array}$ | $\begin{gathered} -0.069^{* *} \\ (0.034) \end{gathered}$ | $\begin{gathered} -0.016^{*} \\ (0.009) \end{gathered}$ | $\begin{gathered} 0.008 \\ (0.019) \end{gathered}$ | $\begin{array}{r} -0.008 \\ (0.023) \end{array}$ | $\begin{gathered} 0.004 \\ (0.012) \end{gathered}$ | $\begin{array}{r} -0.001 \\ (0.006) \end{array}$ | $\begin{array}{r} -0.141 \\ (0.119) \end{array}$ | $\begin{gathered} 0.055 \\ (0.056) \end{gathered}$ | $\begin{gathered} 0.049 \\ (0.033) \end{gathered}$ | $\begin{gathered} 0.018 \\ (0.026) \end{gathered}$ |
| $10 \leq$ Age $<20$ | $\begin{gathered} -9.426^{* * *} \\ (1.216) \end{gathered}$ | $\begin{gathered} -10.015^{* * *} \\ (0.736) \end{gathered}$ | $\begin{gathered} -10.838^{* * *} \\ (0.595) \end{gathered}$ | $\begin{gathered} -12.363^{* * *} \\ (0.646) \end{gathered}$ | $\begin{gathered} -0.416^{* * *} \\ (0.121) \end{gathered}$ | $\begin{aligned} & 0.007^{* *} \\ & (0.003) \end{aligned}$ | $\begin{gathered} 0.005 \\ (0.005) \end{gathered}$ | $\begin{array}{r} -0.007 \\ (0.008) \end{array}$ | $\begin{aligned} & 0.014^{* * *} \\ & (0.004) \end{aligned}$ | $\begin{gathered} -0.078^{* *} \\ (0.039) \end{gathered}$ | $\begin{gathered} 0.006 \\ (0.005) \end{gathered}$ | $\begin{gathered} -0.002 \\ (0.011) \end{gathered}$ |
| First stage error $\left(\hat{\eta}_{i k}\right)$ | $\begin{aligned} & 4.497^{* * *} \\ & (1.661) \end{aligned}$ | $\begin{gathered} 0.214 \\ (0.247) \end{gathered}$ | $\begin{gathered} -0.848 \\ (0.540) \end{gathered}$ | $\begin{gathered} -0.174 \\ (0.293) \end{gathered}$ | $\begin{gathered} 0.022 \\ (0.330) \end{gathered}$ | $\begin{gathered} 1.117 \\ (0.907) \end{gathered}$ | $\begin{gathered} 0.468 \\ (0.479) \end{gathered}$ | $\begin{gathered} -1.334^{* * *} \\ (0.456) \end{gathered}$ | $\begin{aligned} & 2.104^{* * *} \\ & (0.547) \end{aligned}$ | $\begin{gathered} -0.013 \\ (0.168) \end{gathered}$ | $\begin{gathered} -0.572^{*} \\ (0.347) \end{gathered}$ | $\begin{gathered} -0.354 \\ (0.237) \end{gathered}$ |
| Number of observations | 1,249 | 1,700 | 2,243 | 3,224 | 1,244 | 1,151 | 1,829 | 3,037 | 1,519 | 1,619 | 2,194 | 4,186 |
| $\chi^{2, b}$ statistic | 856.888 | 290.842 | 512.524 | 538.203 | 192.604 | 79.160 | 115.357 | 121.214 | 58.936 | 70.883 | 147.339 | 142.008 |
| $p$-value of $\chi^{2}$ statistic | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Pseudo $\mathrm{R}^{2}$ | 0.100 | 0.093 | 0.077 | 0.132 | 0.129 | 0.158 | 0.175 | 0.176 | 0.092 | 0.098 | 0.182 | 0.090 |

[^4]Table V : continued.

|  | Estimation Period: 1990 to 1994 |  |  |  | Estimation Period: 1995 to 1999 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| Logarithm of cash holdings, | $0.271{ }^{* * *}$ | * 0.637 ${ }^{* * *}$ | 0.369*** | $0.172^{* * *}$ | 0.110*** | 0.364*** | $0.636{ }^{* * *}$ | * 0.634*** |
| 1 year before the filing date | (0.083) | (0.077) | (0.076) | (0.048) | (0.041) | (0.079) | (0.076) | (0.078) |
| Logarithm of total assets, | 0.361 | $1.426^{* * *}$ | 0.478*** | 0.651** | $1.587^{* * *}$ | 0.947*** | $1.147^{* * *}$ | * $1.500^{* * *}$ |
| 2 years before the filing date | (0.271) | (0.268) | (0.135) | (0.305) | (0.262) | (0.301) | (0.226) | (0.379) |
| Total patent pools accumulated, up to 1 year before the award | $\begin{aligned} & 0.075^{* * *} \\ & (0.017) \end{aligned}$ | $\begin{aligned} & 0.134^{* * *} \\ & (0.026) \end{aligned}$ | $\begin{aligned} & 0.088^{* * *} \\ & (0.013) \end{aligned}$ | $\begin{aligned} & 0.124^{* * *} \\ & (0.020) \end{aligned}$ | $\begin{aligned} & 0.028^{* * *} \\ & (0.004) \end{aligned}$ | $\begin{aligned} & 0.036^{* * *} \\ & (0.006) \end{aligned}$ | $\begin{aligned} & 0.037^{* * *} \\ & (0.006) \end{aligned}$ | $\begin{aligned} & 0.035^{* * *} \\ & (0.003) \end{aligned}$ |
| Citations count of citations, by vintages: |  |  |  |  |  |  |  |  |
| Age $<1$ | $-0.629^{* *}$ | $-0.187^{* *}$ | $-0.248$ |  |  |  |  |  |
| $1 \leq$ Age $<2$ | $(0.247)$ | $(0.084)$ | $(0.193)$ | $(0.016)$ | $(0.010)$ | $(0.039)$ | $(0.021)$ | $(0.026)$ |
|  | $0.585^{* *}$ | $0.546^{* * *}$ | $0.359^{* *}$ | $0.294^{* * *}$ | $0.035$ | $0.053$ | $0.254^{*}$ | 0.136* |
|  | $(0.287)$ | $(0.143)$ | $(0.169)$ | $(0.062)$ | $(0.029)$ | $(0.064)$ | $(0.133)$ | (0.072) |
| $2 \leq$ Age $<3$ | $0.374$ | $0.463^{* * *}$ | $0.51^{* * *}$ | $0.017$ | $0.004$ | $0.078$ | $0.291^{* * *}$ | $0.148^{* *}$ |
|  | $(0.398)$ | $(0.109)$ | $(0.124)$ | (0.161) | $(0.027)$ | (0.062) | $(0.076)$ | $(0.061)$ |
| $3 \leq$ Age $<4$ | $0.238^{*}$ | $0.024$ | $0.009$ | $0.150^{* * *}$ | $-0.020$ | $0.092^{* *}$ | $-0.023$ | $0.033$ |
|  | (0.130) | (0.077) | $(0.015)$ | $(0.035)$ | $(0.049)$ | $(0.038)$ | (0.077) | $(0.027)$ |
| $4 \leq$ Age $<5$ | $-0.132$ | $0.167$ | $0.183^{* * *}$ | $0.003$ | $0.060$ | $0.044$ | $0.112^{*}$ | $0.062$ |
|  | $(0.246)$ | $(0.106)$ | (0.069) | $(0.055)$ | $(0.106)$ | $(0.060)$ | $(0.058)$ | $(0.041)$ |
| $5 \leq$ Age $<10$ | $0.085$ | $0.145^{* * *}$ | $0.025$ |  |  |  |  |  |
|  | $(0.068)$ | $(0.042)$ | $(0.025)$ | $(0.012)$ | $(0.021)$ | $(0.019)$ | $(0.032)$ | $(0.008)$ |
| $10 \leq$ Age $<20$ | $0.016$ | $0.026^{* *}$ | $-0.012^{* * *}$ | $0.000$ | $-0.034^{*}$ | $-0.017$ | $-0.009$ | $0.014^{* *}$ |
|  | $(0.025)$ | $(0.011)$ | $(0.004)$ | (0.005) | $(0.020)$ | $(0.015)$ | $(0.024)$ | $(0.006)$ |
| First stage error ( $\hat{\eta}_{i k}$ ) | 0.351** | $-0.992^{* * *}$ | 0.151 | 0.741* | $\begin{array}{rr} -0.668^{* * *} & -0.120 \\ (0.253) & (0.150) \end{array}$ |  | $\begin{array}{lr} 0.583^{* * *} & -0.461 \\ (0.154) & (0.402) \end{array}$ |  |
|  | (0.179) | (0.274) | (0.153) | (0.405) |  |  |  |  |
| Number of observations $\chi^{2, b}$ statistic | 1,839 | 2,899 | 3,278 | 6,115 | 2,963 | 3,735 | 4,574 | 11,935 |
|  | 85.498 | 150.342 | 174.844 | 219.968 | 75.006 | 91.115 | 286.927 | 454.064 |
| $\chi^{2, b}$ statistic $p$-value of $\chi^{2}$ statistic | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Pseudo $\mathrm{R}^{2}$ | 0.109 | 0.199 | 0.146 | 0.152 | 0.114 | 0.099 | 0.273 | 0.200 |

[^5]Table VI: Economic Significance of the Estimates of the Model of a Patent Race Winner
This table shows the predicted change in the probability of winning a patent pool in a given year with respect to an increase of one standard deviation of a given regressor, evaluated at the sample mean of all the data. These changes are computed using the parameter estimates of the model that selects the winner of each patent pool from the set of pre-selected competitors, which are reported in lable


[^6]Table VII：Determinants of R\＆D in the Patent Race
This table shows the parameter estimates of the determinants of $R \& D$ expenditures by firm．The model is： $\ln R \& D_{i k}=\gamma^{\prime} \mathbf{x}_{i k}+\eta_{i k}+v_{k}+u_{i k}$,
where the regressors are listed below．The panel unit，$k$ ，is a patent pool and the term $v_{k}$ is the R\＆D component that is common to every firm，$i$ ，
in the set of racing firms．All specifications include year－specific effects．The parameters are estimated by random or fixed effects and the standard
errors are shown in brackets underneath their estimate．${ }^{a}$ The instruments for cash holdings are the two and three year lags of the logarithm of
cash，sales，total assets and outstanding debt，and the averages of cash，sales，debt，and accumulated patent pools of all other rival firms in the
same race．The estimation uses all US patent pools won by COMPUSTAT firms from 1975 to 1999 ，in the techonological category 3 （Drugs and
Medical），subcategories 31,33 and 39．Patent pools are classified into quartiles according to the number of citations received．The number of
citations is adjusted for time differences in the propensity to cite，using the factors provided by Hall et al．（2002）．

> Estimation Period from 1995 to 1999 Quartiles for Numbers of Citations by Patent (1)
$0.494^{* * *} \quad 0.345^{* * *} \quad 0.590^{* * *} \quad 1.333^{* * *}$
$\begin{array}{ll}0.590^{* * *} & 1.333^{* * *} \\ 0.076) & (0.173)\end{array}$
 $\begin{array}{cccc} & & & (0.147) \\ 0.004^{* *} & 0.014^{* *} & 0.009^{* * *} & 0.003^{*} \\ (0.002) & (0.006) & (0.003) & (0.002)\end{array}$ $6000^{\circ} \quad 6000$
 0.002
$(0.007)$
$0.127^{* *}$ 0.009 $(0.041)$
-0.001 $(0.035)$
0.008 （0．038） － 0.006
$(0.009)$
-0.202
$(0.200)$ O $*$
$\stackrel{*}{*}$
$\stackrel{0}{2}$
0
0
0
0
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0

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国易

-0.174
$(0.113)$
2,963
$631.84^{* * *}$
0.550

ヘ | 10 |  |
| :--- | :--- |
| 0 |  |
| 0 | 0 |
| 0 |  | Estimation Period from 1990 to 1994

Quartiles for Numbers of Citations by Patent （ஏ）（६）（ $\quad$（ ）
$0.136^{* * *} \quad 0.429^{* *}$
$\stackrel{*}{*} \stackrel{*}{*}$
9）
$5^{* * *}$
$0.035^{* * *}$
$(0.013)$ $800^{\circ} 0$

 $\begin{array}{cccc}1,399.94^{* * *} & 407.73^{* * *} & 4,325.81^{* * *} & 19,977.74^{* * *} \\ 0.054 & 0.166 & 0.138 & 0.305\end{array}$红 Nic

[^7]Table A.I: Estimates of the Model of a Patent Race Winner
This table shows the estimates of the parameters of the model that selects the winner of a race for a patent pool from the set of all non-cited firms that won at least one patent in the same five-year period. The estimates were computed using an instrumental variables estimator, following
$$
\ln s_{i t}-\ln s_{0 t}=\boldsymbol{\beta}^{\prime} \mathbf{x}_{i t}+\eta_{i t}
$$
where $s_{i t}$ is the share of pools won by firm $i$ in year $t$, and $s_{0 t}$ is the share of patents with self-cited winners. The regressors are listed below. All patent pools by all other firms in the same period; as well as the logarithms of sales, cash, assets and outstanding debt, all in years $t-2$ and $t-3$. patent pools by all other firms in the same period; as well as the logarithms of sales, cash, assets and outstanding debt, all in years $t-2$ and $t-3$.
The estimates' standard errors are computed using a covariance matrix estimator robust to correlation within the same 2 -digit SIC code. They are shown in brackets under the parameter estimate. ${ }^{a}$ The estimation uses all US patent pools won by COMPUSTAT firms from 1975 to 1999 , in
 (2002).

|  | Estim $\mathrm{Pa}$ | Estimation Period: 1975 to 1979 |  | 979 | Estimation Period: 1980 to 1984 |  |  |  | Estimation Period: 1985 to 1989 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| Logarithm of cash holdings, 1 year before the filing date | $\begin{gathered} -0.144 \\ (0.310) \end{gathered}$ | $\begin{gathered} 4.100 \\ (4.075) \end{gathered}$ | $\begin{gathered} 1.436 \\ (1.460) \end{gathered}$ | $\begin{gathered} 0.517 \\ (0.525) \end{gathered}$ | $\begin{gathered} -0.008 \\ (0.469) \end{gathered}$ | $\begin{gathered} 0.963 \\ (0.614) \end{gathered}$ | $\begin{gathered} 0.316 \\ (0.117) \end{gathered}$ | $\begin{aligned} & 0.187^{* *} \\ & (0.034) \end{aligned}$ | $\begin{gathered} 0.227 \\ (0.898) \end{gathered}$ | $\begin{gathered} 0.587 \\ (0.733) \end{gathered}$ | $\begin{gathered} 0.319 \\ (0.484) \end{gathered}$ | $\begin{gathered} 1.112^{*} \\ (0.625) \end{gathered}$ |
| Logarithm of total assets, 2 years before the filing date | $\begin{gathered} -0.107 \\ (0.161) \end{gathered}$ | $\begin{gathered} 0.264 \\ (0.234) \end{gathered}$ | $\begin{gathered} -0.071 \\ (0.202) \end{gathered}$ | $\begin{gathered} 0.069 \\ (0.106) \end{gathered}$ | $\begin{gathered} -0.021 \\ (0.224) \end{gathered}$ | $\begin{gathered} 0.047 \\ (0.140) \end{gathered}$ | $\begin{gathered} 0.166 \\ (0.202) \end{gathered}$ | $\begin{gathered} 0.250^{*} \\ (0.126) \end{gathered}$ | $\begin{gathered} -0.007 \\ (0.166) \end{gathered}$ | $\begin{gathered} 0.288 \\ (0.270) \end{gathered}$ | $\begin{gathered} 0.231 \\ (0.191) \end{gathered}$ | $\begin{gathered} 0.206^{*} \\ (0.121) \end{gathered}$ |
| Total pools of patents accumulated, up to 1 year before the filing date | $\begin{gathered} 0.182^{*} \\ (0.107) \end{gathered}$ | $\begin{gathered} 0.205^{*} \\ (0.117) \end{gathered}$ | $\begin{aligned} & 0.274^{* *} \\ & (0.118) \end{aligned}$ | $\begin{aligned} & 0.195^{* * *} \\ & (0.071) \end{aligned}$ | $\begin{gathered} 0.171 \\ (0.146) \end{gathered}$ | $\begin{aligned} & 0.103^{* *} \\ & (0.022) \end{aligned}$ | $\begin{gathered} 0.090 \\ (0.088) \end{gathered}$ | $\begin{aligned} & 0.180^{* *} \\ & (0.088) \end{aligned}$ | $\begin{gathered} 0.434 \\ (0.636) \end{gathered}$ | $\begin{array}{r} -0.051 \\ (0.425) \end{array}$ | $\begin{gathered} 0.203 \\ (0.355) \end{gathered}$ | $\begin{gathered} 0.896^{*} \\ (0.520) \end{gathered}$ |
| Average pools per firm | 1.156 | 1.139 | 1.261 | 1.302 | 1.142 | 1.128 | 1.128 | 1.306 | 1.128 | 1.139 | 1.105 | 1.225 |
| Number of observations | 51 | 62 | 77 | 92 | 41 | 33 | 55 | 55 | 52 | 58 | 72 | 110 |
| $\mathrm{R}^{2}$ | $0.570$ | $0.300$ | $0.180$ | $0.360$ | $0.330$ | $0.370$ | $0.380$ | $0.320$ | $0.280$ | $0.230$ | $0.190$ | $0.040$ |
| $F^{b}$ statistic | $638.58^{* * *}$ | $1,699.21^{* * *}$ | $13.33^{* * *}$ | $58.28^{* * *}$ | $13.93^{* * *}$ | $460.18^{* * *}$ | $25.28^{* * *}$ | $809.23^{* * *}$ | $666.93^{* * *}$ | $37.11^{* * *}$ | $1,119.82^{* * *}$ | $150.72^{* * *}$ |
| P value OIR | 0.998 | 0.841 | 0.309 | 0.514 | 0.859 | 0.788 | 0.532 | 0.953 | 0.501 | 0.174 | 0.472 | 0.861 |

[^8]${ }^{a}$ Estimates followed by ${ }^{* * *},{ }^{* *}$ and * are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respec-
tively. ${ }^{b}$ The $F$ statistic is computed under the null hypothesis that all the model's parameters are zero.
Table A.I : continued.

|  | Estimation Period: 1990 to 1994 <br> Patent Citations Quartiles |  |  |  | Estimation Period: 1995 to 1999 <br> Patent Citations Quartiles |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| Logarithm of cash holdings, 1 year before the filing date | $\begin{gathered} 0.109 \\ (0.238) \end{gathered}$ | $\begin{gathered} -0.014 \\ (0.102) \end{gathered}$ | $\begin{gathered} 0.437 \\ (0.276) \end{gathered}$ | $\begin{gathered} 0.326^{*} \\ (0.192) \end{gathered}$ | $\begin{gathered} 1.454 \\ (0.917) \end{gathered}$ | $\begin{gathered} 0.678^{*} \\ (0.391) \end{gathered}$ | $\begin{aligned} & 0.389^{* *} \\ & (0.188) \end{aligned}$ | $\begin{aligned} & 0.440^{* *} \\ & (0.188) \end{aligned}$ |
| Logarithm of total assets, 2 years before the filing date | $\begin{gathered} 0.018 \\ (0.180) \end{gathered}$ | $\begin{gathered} 0.222 \\ (0.162) \end{gathered}$ | $\begin{gathered} 0.198 \\ (0.197) \end{gathered}$ | $\begin{gathered} 0.089^{*} \\ (0.051) \end{gathered}$ | $\begin{gathered} 0.924^{*} \\ (0.541) \end{gathered}$ | $\begin{aligned} & 0.628^{* *} \\ & (0.303) \end{aligned}$ | $\begin{gathered} 0.257^{*} \\ (0.147) \end{gathered}$ | $\begin{aligned} & 0.315^{* *} \\ & (0.136) \end{aligned}$ |
| Total pools of patents accumulated by the firm up to 1 year before the filing date | $\begin{gathered} 0.172 \\ (0.266) \end{gathered}$ | $\begin{gathered} 0.329^{*} \\ (0.188) \end{gathered}$ | $\begin{gathered} 0.321^{*} \\ (0.166) \end{gathered}$ | $\begin{aligned} & 0.243^{* *} \\ & (0.110) \end{aligned}$ | $\begin{aligned} & 2.164^{* * *} \\ & (0.494) \end{aligned}$ | $\begin{aligned} & 1.674^{* * *} \\ & (0.166) \end{aligned}$ | $\begin{aligned} & 1.383^{* * *} \\ & (0.095) \end{aligned}$ | $\begin{aligned} & 1.874^{* * *} \\ & (0.120) \end{aligned}$ |
| Average pools per firm | 1.173 | 1.190 | 1.154 | 1.231 | 1.519 | 1.283 | 1.353 | 1.778 |
| Number of observations | 66 | 92 | 106 | 153 | 39 | 64 | 91 | 164 |
| $\mathrm{R}^{2}$ | 0.190 | 0.150 | 0.170 | 0.110 | 0.330 | 0.280 | 0.190 | 0.120 |
| $F^{\text {b }}$ statistic | 7.91 | 2.16 | 1.09 | 3.90* | 3.02* | 2.19 | 5.74*** | 3.50 ** |
| P value OIR | 0.263 | 0.108 | 0.075 | 0.573 | 0.312 | 0.527 | 0.101 | 0.910 |

Table A.II: Benchmark Estimates of the Model: OLS and Standard ML
This table reports the benchmark estimates of the model that selects the firms in the race (Panel A) and the model that selects the winner of the race (Panel B). The selection model in Panel A is the same as the model in Table A.I, but the estimates are computed by Ordinary Least All specifications in Panel A include year-specific dummies, and all specifications in Panel B include the incumbency indeces by vintages. The estimates' standard errors are computed using a covariance matrix estimator robust to correlation within the same 2 -digit SIC code. They are shown in brackets under the parameter estimate. ${ }^{a}$ The estimation uses all US patent pools won by COMPUSTAT firms from 1975 to 1999, in the
techonological category 3 (Drugs and Medical), subcategories 31,33 and 39 . Patent pools are classified into quartiles according to the number of citations received. The number of citations is adjusted for time differences in the propensity to cite, using the factors provided by Hall, et al., (2002).


[^9]Table A.II : continued.

|  | Estimation Period: 1990 to 1994 |  |  |  | Estimation Period: 1995 to 1999 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (1) | (2) | (3) | (4) | (1) | (2) | (3) | (4) |
| Panel A: Entrant Selection Stage |  |  |  |  |  |  |  |  |
| Logarithm of cash holdings, 1 year before the filing date | $\begin{gathered} 0.039 \\ (0.105) \end{gathered}$ | $\begin{gathered} 0.028 \\ (0.069) \end{gathered}$ | $\begin{aligned} & 0.155^{* * *} \\ & (0.057) \end{aligned}$ | $\begin{gathered} 0.221^{*} \\ (0.092) \end{gathered}$ | $\begin{gathered} 0.104 \\ (0.116) \end{gathered}$ | $\begin{gathered} 0.012 \\ (0.105) \end{gathered}$ | $\begin{gathered} 0.047^{* *} \\ (0.021) \end{gathered}$ | $\begin{aligned} & 0.145^{* *} \\ & (0.068) \end{aligned}$ |
| Logarithm of total assets, | $-0.023$ | 0.054 | $-0.060$ | 0.055 | 0.046 | 0.106 | 0.171* | $0.064^{* * *}$ |
| 2 years before the filing date | (0.103) | (0.062) | (0.055) | (0.061) | (0.112) | (0.083) | (0.056) | (0.029) |
| Total patents pools accumulated, up to 1 year before the filing date | $\begin{gathered} 0.293 \\ (0.472) \end{gathered}$ | $\begin{gathered} 0.178 \\ (0.246) \end{gathered}$ | $\begin{gathered} 0.041 \\ (0.281) \end{gathered}$ | $\begin{gathered} 0.320 \\ (0.213) \end{gathered}$ | $\begin{gathered} 0.208 \\ (0.231) \end{gathered}$ | $\begin{gathered} -0.335 \\ (0.274) \end{gathered}$ | $\begin{gathered} 0.103 \\ (0.173) \end{gathered}$ | $\begin{gathered} 0.302^{*} \\ (0.135) \end{gathered}$ |
| Number of observations | 66 | 92 | 106 | 153 | 39 | 64 | 91 | 164 |
| $\mathrm{R}^{2}$ | 0.20 | 0.25 | 0.19 | 0.12 | 0.26 | 0.20 | 0.15 | 0.14 |
| $F$ statistic | $78.92^{* * *}$ | 13.80 *** | $6.34^{* * *}$ | $25.62^{* * *}$ | $16.20^{* * *}$ | 2.63 *** | 82.30*** | $29.16^{* * *}$ |
| Panel B: Patent Race Model |  |  |  |  |  |  |  |  |
| Logarithm of cash holdings, | 0.381 *** | 0.633*** | $0.518^{* * *}$ | 0.560*** | 0.129** | $0.426^{* * *}$ | 0.706*** | 0.834*** |
| 1 year before the filing date | (0.072) | (0.061) | $(0.065) ~\left({ }^{\text {( }}\right.$ | (0.065) | (0.054) | (0.061) | (0.053) | (0.061) |
| Logarithm of total assets, | $0.292{ }^{* * *}$ | $0.458^{* * *}$ | $0.300^{* * *}$ | 0.037 | $0.155^{* * *}$ | $0.322^{* * *}$ | $0.704^{* * *}$ | $0.727^{* * *}$ |
| 2 years before the filing date | (0.074) | (0.050) | (0.043) | (0.053) | (0.054) | (0.056) | (0.049) | (0.054) |
| Total patents pools accumulated, | $0.067^{* * *}$ | $0.115^{* * *}$ | $0.074^{* * *}$ | $0.106^{* * *}$ | $0.018^{* * *}$ | $0.03^{* * *}$ | $0.034^{* * *}$ | $0.034^{* * *}$ |
| up to 1 year before the filing date | (0.017) | (0.028) | (0.013) | $(0.016)$ | (0.003) | $(0.005)$ | (0.005) | (0.003) |
| Number of observations | 1,839 | 2,899 | 3,278 | 6,115 | 2,963 | 3,735 | 4,574 | 11,935 |
| $\chi^{2, b}$ statistic | 59.218 | 141.782 | 96.976 | 87.867 | 66.595 | 84.751 | 275.021 | 374.551 |
| P value of $\chi^{2}$ statistic | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Pseudo $\mathrm{R}^{2}$ | 0.080 | 0.158 | 0.116 | 0.060 | 0.035 | 0.068 | 0.193 | 0.167 |

[^10] ${ }_{b}^{\text {tively. }}$ The $F$ and $\chi^{2}$ statistics are computed under the null hypothesis that all the model's parameters are zero.


Figure 1: Distribution of the time, in weeks, between the filing dates of each patent and the next by the same firm


[^0]:    *We would like to thank Sudipto Bhattacharya, Bronwyn Hall, Martin Hellwig, Robert Hunt, Vojislav Maksimovic, Benny Moldovanu, Tiago Ribeiro, Kurt Schmidheiny, Armin Schmutzler, Margaret Slade, Elu Von Thadden, Mike Waterson, An Yan and seminar audiences at the Universities of Frankfurt, Zürich and Lausanne, the 2007 AEA Meetings (Chicago), the 2007 ESSFM (Gerzensee), the 2006 FIRS Meetings (Shanghai) and the 2005 GAEL IO and Innovaiton Conference (Grenoble).
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[^1]:    $a$ Estimates followed by ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ indicate that the p -value for the differences of means test between groups A and B are statistically different
    from zero with $0.01,0.05$ and 0.1 significance levels, respectively.

[^2]:    ${ }^{a}$ Estimates followed by ${ }^{* * *}$, ${ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respectively. ${ }^{b}$ The $\chi^{2}$ statistic is computed under the null hypothesis that all the model's parameters are zero.

[^3]:    ${ }^{a}$ Estimates followed by ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respec-
    ${ }^{\text {tively. }}$ The $\chi^{2}$ statistic is computed under the null hypothesis that all the model's parameters are zero.

[^4]:    ${ }^{a}$ Estimates followed by ${ }^{* * *}$, ** and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respectively. ${ }^{b}$ The $\chi^{2}$ statistic is computed under the null hypothesis that all the model's parameters are zero.

[^5]:    ${ }^{a}$ Estimates followed by ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respec${ }_{b}^{\text {tively. }}$ The $\chi^{2}$ statistic is computed under the null hypothesis that all the model's parameters are zero.

[^6]:    ${ }^{a}$ Estimates followed by ${ }^{* * *}$, ${ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respectively. ${ }^{b}$ The $\chi^{2}$ statistic is computed under the null hypothesis that all the model's parameters are zero.

[^7]:    ${ }^{b}$ The $\chi^{2}$ statistic is computed under the null hypothesis that all the model＇s parameters are zero．
    ${ }^{c}$ The $\chi^{2}$ statistic is computed under the null hypothesis that the random effects is both efficient and consistent．
    ${ }^{a}$ Estimates followed by ${ }^{* * *}$ ，${ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels，respectively．

[^8]:    ${ }^{a}$ Estimates followed by ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respectively.
    ${ }^{b}$ The $F$ statistic is computed under the null hypothesis that all the model's parameters are zero.

[^9]:    ${ }^{a}$ Estimates followed by ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respectively. ${ }^{b}$ The $F$ and $\chi^{2}$ statistics are computed under the null hypothesis that all the model's parameters are zero.

[^10]:    ${ }^{a}$ Estimates followed by ${ }^{* * *},{ }^{* *}$ and ${ }^{*}$ are statistically different from zero with $0.01,0.05$ and 0.1 significance levels, respec-

